Ron W. Nielsen

Evidence-based

UNIFIED GROWTH THEORY

Vol.1

Mechanism of the growth of population and of economic growth in the past 2,000,000 years explained

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Preface

The true laboratory is the mind, where behind illusions we uncover the laws of truth.
— Sir Jagadish Chandra Bose

Duration is not a test of true or false.
— Anne Morrow Lindbergh

If they don't depend on true evidence, scientists are no better than gossips.
— Penelope Fitzgerald

In science it often happens that scientists say, 'You know that's a really good argument; my position is mistaken,' and then they would actually change their minds and you never hear that old view from them again. They really do it. It doesn't happen as often as it should, because scientists are human and change is sometimes painful. But it happens every day.
— Carl Sagan

Unified Growth Theory[1] published by Oded Galor is called unified because it puts together earlier attempts to explain the historical economic growth and the historical growth of population. These attempts were made over many years and by now they form the established knowledge in economics and in demography.

Unfortunately, the past research was difficult because (1) access to data was strongly limited and (2) growth turns out to be represented by strongly deceptive distributions. They create an illusion of stagnation followed by a sudden explosion, while in fact they increase monotonically all the time and there is no sudden transition from a slow to fast growth. Data represented by these distributions have to be carefully and methodically analysed; otherwise conclusions are based on illusions.

Galor was in a far better position than many of the past researchers because he had access to superb and extensive sets of data made available by the world renown economist, Angus Maddison. These data describe economic growth and the growth of population, global, regional and even in individual countries. They

[1] Galor, O. (2011). *Unified Growth Theory*. Princeton, New Jersey: Princeton University Press.

are a rich source of information, which Galor failed to use. He made no attempt to analyse them.

There is no explanation for his neglect to analyse data mathematically because (1) he uses mathematics in his theory and thus he is familiar with mathematical procedures and (2) because trajectories describing growth of population and economic growth, while being deceptive, are trivially easy to analyse. No great skill is needed to analyse these distributions. Indeed, there is even no need to analyse them mathematically. Reliable conclusions can be reached just by using different plots of data. However, mathematical analysis, which is simple and easy, helps in a better understanding of the mechanism of growth.

Galor ignored also the earlier evidence published in 1960 that the growth of population during the AD section of time was hyperbolic. Using this information, the obvious next step would be to check whether the same type of growth is applicable to the economic growth.

Rather than using the previously published evidence, he systematically presented data in a suitably distorted way to support preconceived ideas. He could have made an important discovery but he did not. His theory presents nothing new. It is just a repetition of old interpretations of the growth of population and of economic growth, incorrect interpretations because they are contradicted by data. Unified Growth Theory is repeatedly contradicted even by the same data, which were used during its formulation.

The presented here *Evidence-based Unified Growth Theory* is firmly supported by a rigorous, mathematical analysis of data describing economic growth and the growth of population. It is also called *unified* because it presents a unified explanation of the growth of population and of economic growth in the past 2,000,000 years.

The terms *Malthusian stagnation, Malthusian regime* and *Malthusian trap* will be used in the presented here discussion but it should be remembered that they are incorrect, because Malthus never claimed that his positive checks were causing stagnation or creating a certain regime of growth or a trap. On the contrary, he observed that they stimulated growth and he even suggested that this curious phenomenon should be further investigated. Unfortunately, his observation was ignored, dubious concepts were later introduced and the name of Malthus was questionably attached to them, which Malthus would probably not approve. These phrases are used only because in this form, they are repeatedly used in the published literature.

This book is a compilation of my articles describing the investigation of the growth of population and of economic growth. I start by showing why the established knowledge is scientifically unacceptable. I follow this chapter by the introduction of a simple method of reciprocal values, which makes the analysis of hyperbolic distributions trivially simple. These two introductory chapters are followed by the explanation how the Unified Growth Theory is contradicted by data. These chapters are in turn followed by a detailed study of the growth of human population and of economic growth in the past 2,000,000 years; by the discussion of earlier attempts to explain the mechanism of hyperbolic growth; by the examination of the impacts of Malthusian positive checks; by the examination of impacts of demographic catastrophes; by the examination of the relation between the growth rate and growth trajectories, the essential step leading to the explanation of the mechanism of growth; by the formulation of the general law of growth; and by the explanation of the mechanism of the hyperbolic growth of human population and of the economic growth.

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Gold Coast, Australia
July, 2018

Contents

Introduction

This introduction is designed as a guide to the topics discussed in this book.

The spontaneous (unconstrained and undisturbed) growth of human population is not exponential, as was expected by Malthus, but hyperbolic. The same applies to the economic growth. This conclusion is in harmony with the earlier investigation carried out by von Foerster, Mora and Amiot[2] who studied the growth of the world population during the AD section of time. However, the study presented here extends the analysis to the BC time and to the economic growth. It also includes the analysis of regional growth of population and regional economic growth.

Results presented here are also in harmony with the earlier study of Deevey[3], who observed that growth of human population in the past 1,000,000 years was in three stages. However, he postulated that each stage was reaching an equilibrium. Results presented here confirmed the three stages of growth but demonstrated that each stage was hyperbolic. Rather than reaching an equilibrium, each stage had a potential to increase to infinity and was at a certain time terminated.

Two well-known theories, the Unified Growth Theory and the Demographic Transition Theory[4], are contradicted by the same data, which were used in their support.

[2] von Foerster, H., Mora, P., & Amiot, L. (1960). Doomsday: Friday, 13 November, A.D. 2026. *Science, 132*, 1291-1295.
[3] Deevey, E. S. Jr (1960). The human population. *Scientific American, 203*(9), 195-204.
[4] For references see Vol. 2, Chapter 5.

In the case of the Demographic Transition Theory, data, which appeared to be in support of this theory, were never analysed. Conclusions are based on impressions. However, in addition, contradicting data are systematically ignored.

In the case of the Unified Growth Theory, data were also never analysed but they were suitably distorted to support preconceived ideas. This deliberately distorted and misleading presentation of data is used in many other related publications.

There is no convincing explanation why the Author of the Unified Growth theory failed to analyse data mathematically and why he was systematically presenting them in a distorted way, because (1) he used mathematics in his theory and thus he is familiar with mathematical procedures, (2) hyperbolic growth was demonstrated as early as in 1960, (3) it is hard to imagine that he is not familiar with the fundamental properties of hyperbolic distributions, that they increase slowly over a long time and fast over a short time but that they increase monotonically, and (4) mathematical analysis of hyperbolic distributions is trivially simple.

Precisely the same data, which in their deliberately distorted way were used to support the Unified Growth Theory, are in fact in its direct contradiction. It is hard to understand why so much work was devoted to support the earlier erroneous interpretations of the mechanism of growth and why data were not properly analysed to check whether these interpretations, which were earlier based on limited data and on illusions, could be still supported.

Income per capita distributions show puzzling characteristics. They show that over a long time, income per capita was approximately constant but then, most recently, it was increasing extremely rapidly. The analysis of data presented here explains these puzzling characteristic features. They reflect nothing more than mathematical properties of dividing two hyperbolic distributions. They do not represent some peculiar mechanism applicable only to the economic growth but the feature, which applies to any two hyperbolic distributions, with only one condition that the singularity of the numerator is earlier than the singularity of the denominator.

Galor describes certain mysteries of growth in his Unified Growth Theory and indicates that they should be studied and explained. They have now been explained. They have nothing to do with the growth of population or with the economic growth. They were created by his distorted representations of data.

Galor describes a puzzling phenomenon of great divergence. This claimed phenomenon is also nothing more than a feature created by his distorted representations of data. There was no great divergence and there is nothing to explain except to explain how the great divergence was created by Galor.

Industrial Revolution had no impact on changing growth trajectories describing growth of population and economic growth, even in Western Europe and even in the United Kingdom. Forces associated with the Industrial Revolution are reflected in changing socio-economic conditions but they did not shape growth trajectories of the growth of population and of economic growth.

With the exception of just one event, demographic catastrophes had no impact on shaping the growth of population. The one and only exceptional event in the past 2,000,000 years, as presented by data, was an unusual convergence of five strong demographic catastrophes between AD 1195 and 1470. However, even this unusual event caused only a minor disturbance in the growth trajectory. When this exceptionally strong crisis was over, growth of population was even faster than before.

Survey of demographic catastrophes indicated that they were, in general, too weak to cause a major disruption in the growth of the world population even if they had strong local impacts. Analysis of Malthusian positive checks also added to the explanation why demographic catastrophes did not shape the growth of the world population.

It is interesting that Malthus noticed the dichotomous property of his positive checks, i.e. their destructive and regenerating effects. He even suggested that the regenerating effects should be further investigated. Unfortunately, the original observation of Malthus was ignored and the destructive aspect of his positive checks was blown out of proportion and used to explain the claimed prolonged stagnation, that never happened, while no effort was made to understand their regenerating property, which is in fact common in nature.

Mathematical analysis of the effects of Malthusian positive checks has now been carried out and it demonstrated that Malthus was right. His positive checks increase mortality rates but they also increase fertility rates, with the combined effect of increasing the growth rate. The regeneration process, or the growth stimulating property, is so efficient that the growth is even faster. This is a well-known phenomenon but it is an inconvenient property for those who created the concept of the prolonged epoch of stagnation used in the Demographic Growth Theory and in the Unified Growth Theory.

As a part of the presented here investigations, general law of growth was formulated and used to explain the mechanism of hyperbolic growth of population and of economic growth. It turns out that the mechanism is exceptionally simple, which is hardly surprising because hyperbolic growth is described by an exceptionally simple mathematical formula.

With the exception of two major transitions (46,000 - 27,000 BC and 425 BC – AD 510) and one minor disturbance (AD 1195 – 1470), growth of the world population in the past 2,000,000 years was consistently hyperbolic. It was steadily increasing without any signs of a random behaviour or of a sudden rapid increase towards the end of this long time. There was no stagnation and no sudden explosion. The same applies to the economic growth, which for the most part of the past 2,000,000 years was directly proportional to the size of human population. Explanation of the dynamics of growth is much simpler than presented in the Unified Growth Theory or in the Demographic Growth Theory or in many other published discussions, which ignore the earlier evidence of hyperbolic growth and which are not supported by a rigorous analysis of data but by impressions and conjectures.

1. Scientifically unacceptable established knowledge in demography and in economic research

Introduction

Two fields of research, economic growth and the growth of population, which might appear to be distinctly different, are in fact closely related for at least three reasons. *First*, there is obviously no economic growth without humans. *Second*, there is a close correlation between economic growth and the growth of human population (Nielsen, 2016a; 2016b). *Third*, in order to understand the growth of income per capita, measured by the Gross Domestic Product per capita (GDP/cap), it is obviously necessary to study not only the economic growth but also the growth of human population. It is *inter alia* for these reasons, that the best source of information about the historical economic growth, compiled by the world-renown economist, includes not only the data describing the growth of the GDP but also the growth of population (Maddison, 2001; 2010).

The established knowledge

The established knowledge in demography and in the economic research revolves around two fundamental concepts: the concept of Malthusian stagnation and the concept of a sudden explosion, which is supposed to have marked a dramatic escape from the assumed Malthusian trap. Gradually and by accretion, in the process extending over many years, these two fundamental concepts were fortified by various additional explanations, speculations and conjectures all adding to the now established knowledge based on the scientifically unacceptable doctrines and beliefs. These two fundamental regimes of growth, stagnation and explosion, are described as Stage 1 and Stage 2, respectively, in the Demographic Transition Theory (see Nielsen, 2016c and references therein). The epoch of stagnation was supposed to have lasted for many thousands of years and was allegedly strongly controlled by the Malthusian positive checks (Malthus, 1798) generating an unstable stage of growth characterised by irregular Malthusian oscillations. The mechanism of growth is claimed to have changed dramatically at the time of the supposed population explosion when the growth was supposed to have changed abruptly from slow to fast. The transition from stagnation to explosion is described as the great escape from the Malthusian trap.

Established knowledge is convincingly contradicted established knowledge is convincingly contradicted by the relevant data and by their analyses (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Lehmeyer, 2004; Livi-Bacci, 1997; Maddison, 2001, 2010; Mauritius, 2015; McEvedy & Jones, 1978; Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i; Podlazov, 2002; Shklovskii, 1962, 2002; Statistics Mauritius, 2014; Statistics Sweden, 1999; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973; 1999; 2013; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960; Wrigley & Schofield, 1981). The aim of this chapter is (1) to outline briefly the origin of the established knowledge, (2) to explain why the established knowledge is so strongly established, (3) to explain the deceptive evidence in data, which can be used in support of the established knowledge, (4) to give a few examples of how strongly the established knowledge is established and (5) to explain why the established knowledge as illustrated by these examples is scientifically unacceptable.

Evidence in data

Data describing historical growth of population and historical economic growth are hardly ever analysed. Recently, attempts were made to use some of these data (Maddison, 2001) but they were presented in grossly distorted and misleading diagrams, which appear to be supporting the established knowledge (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Data were not analysed to learn from them but manipulated to support preconceived ideas. Such approach to research is scientifically unacceptable. Data have to be carefully and methodically analysed to avoid drawing incorrect conclusions.

Their superficial examination creates strong impression of stagnation followed by explosion but when closely analysed they show that the apparent explosion was just the natural continuation of the past hyperbolic growth.

Global population in 10,000 BC is estimated at only between 1 and 10 million (McEvedy & Jones, 1978; Thomlinson, 1975). Now the population of this size can be located in just a single city. By AD 1, global population increased to only a few hundred million. The estimated values vary between 170 and 400 million (Biraben, 1980; Durand, 1974; Haub, 1995; McEvedy & Jones, 1978; Thomlinson, 1975; United Nations, 1973; 1999). Now, the population of this size or even larger can be found in just a single country.

The first billion of global population was reached around AD 1800 (Biraben, 1980; Durand, 1974; Haub, 1995; McEvedy & Jones, 1978; Thomlinson, 1975; United Nations, 1973; 1999) and from that time on growth was progressing exceedingly fast. The origin of *Homo Sapiens* is usually claimed at around 200,000 years ago but it might have been even earlier (Weaver, Roseman & Stringer, 2008). Thus, it took many thousands of years for the world population to increase to one billion but after reaching the first billion, the second billion was added in just only about 130 years (United Nations, 1999). The process of many hundreds of thousands of years was compressed to just over 100 years. The consumption of natural resources and the stress on the environment was increasing rapidly.

If adding one billion in just 130 years sounds too fast, the next billion was added in just 29 years, the next in 15 years, the next in 13 years, and the next in 12 years, increasing the size of global population to 6 billion (US Census Bureau, 2016). The last billion, which increased global population to 7 billion, was added in 13 years (US Census Bureau, 2016). We call it the slowing-down growth but obviously the slowing down process is still too slow.

Assuming a medium-intensity growth, the size of the world population is projected to increase to 8.39 billion in 2030 and 9.63 billion in 2050 reaching a maximum of 10.48 billion around 2080 (Nielsen, 2006). These projections are in good agreement with the US Census Bureau (2016) projections of 8.34 billion in 2030 and 9.41 billion in 2050. It is what we hope for, but the high intensity growth could lead to 12.26 billion by the end of the current century (Nielsen, 2006), assuming that such a growth can be supported by the availability of natural resources.

Similar surprising pattern of a slow growth in the past and a fast growth in recent years is for the growth of the Gross Domestic

Product (Maddison, 2001; 2010). The first trillion dollars ($\$10^{12}$) of the GDP (expressed in the 1990 international Geary-Khamis dollars) was reached in 1870. The next trillion was added in just 51 years, the next in 19 years and the next in only 10 years, increasing global GDP to $4 trillion in 1950. By 1998, global GDP increased to $34 trillion. The latest estimate for 2014 is $91 trillion (World Bank, 2016) and the projected value for 2050 is $118 trillion (Nielsen, 2015b).

Using such numbers, it would be easy to conclude that there was a long epoch of stagnation in the past economic growth and in the growth of human population and that this stagnation was followed by a sudden explosion. However, such a conclusion, which is the corner stone of the established knowledge in demography and in the economic research, would be unscientific because impressions can be misleading. Scientific research has to be conducted scientifically. If economic and demographic research is supposed to be recognised as science they have to adhere to the scientific rules of investigation.

In science, data have to be methodically analysed. This fundamental requirement in scientific research appears to have been ignored in the economic and demographic research. Hasty conclusion about stagnation followed by explosion is clearly incorrect and scientifically unacceptable because over 50 years ago, von Foerster, Mora & Amiot (1960) demonstrated that the growth of population during the AD era was monotonically hyperbolic. This crucial contribution to science should not have been ignored. It should have been further investigated because hyperbolic growth rules out the interpretations based on the assumption of stagnation followed by explosion.

Postulates of the established knowledge are also unacceptable because hyperbolic growth have been recognised and confirmed by other independent investigations (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975). Accepting the fundamental postulates of established knowledge is scientifically unjustified because for a long time now there was a large body of data describing the growth of population not only during the AD era but also during the BC era (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013). These data should have been analysed to check the earlier claims about the hyperbolic growth.

Fundamental postulates of the established knowledge are now contradicted by the excellent new data describing economic growth and the growth of population (Maddison, 2001; 2010). These postulates are scientifically unacceptable because they are consistently contradicted by the analysis of relevant data (Nielsen, 2013a; 2013b; 2013c; 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i).

Data describing birth *and* death rates and the associated growth of population are limited (Lehmeyer, 2004; Mauritius, 2015; Statistics Mauritius, 2014; Statistics Sweden, 1999; Wrigley & Schofield, 1981) but they also show consistently that the established knowledge, as expressed in the Demographic Transition Theory, is contradicted by their analysis (Nielsen, 2016c). We do not even have to analyse these data mathematically to see that they are in contradiction of the established knowledge because even though the birth and death rates and the associated growth rates were fluctuating, their time-dependence does not fit into the patterns claimed by the Demographic Transition Theory. Furthermore, the corresponding distributions describing the growth of population do not display any form of stagnation during the supposed Stage 1 or a transition to the supposed Stage 2, which is supposed to represent the explosion. Data show no such patterns.

Demographic Transition Theory is based on persistent disregard of relevant data. This theory is supported by largely meaningless presentations of data for birth *or* death rates. These rates have to be studied *together* and they should show the expected behaviour, as claimed by the Demographic Transition Theory, that the gap between them is approximately zero during the assumed Stage 1 and that it increases during the supposed Stage 2. These patterns are not confirmed by the best available data (Lehmeyer, 2004; Mauritius, 2015; Statistics Mauritius, 2014; Statistics Sweden, 1999; Wrigley & Schofield, 1981), which show that the Demographic Transition Theory is contradicted by the data describing birth and death rates and by the associated data describing the growth of population. Paradoxically, when closely analysed, data used in support of the Demographic Transition Theory are in fact in its clear contradiction.

A theory contradicted by just a single set of data is scientifically unacceptable and the Demographic Transition Theory was first contradicted by the results of von Foerster, Mora & Amiot (1960) who demonstrated that the growth of human population during the AD era was monotonically hyperbolic and thus that Stages 1 and 2 claimed by this theory did not exist. Demographic Transition Theory should have been rejected or at least fundamentally modified over 50 years ago. Its continuing use over such a long time has been scientifically unjustified.

Postulates of Malthusian stagnation followed by explosion, and all other associated postulates and explanations of the historical economic growth and of the historical growth of population followed by a mythical escape from the Malthusian trap have no scientific support. They may, however, have a place in the history of science.

Hyperbolic growth

Hyperbolic distributions are strongly deceptive and it is easy to make a mistake with their interpretation. Fortunately, however, analysis of hyperbolic distributions is also trivially simple (Nielsen, 2014) and it is easy to avoid making an easy mistake.

Examples of two hyperbolic distributions, a hyperbolic distribution describing the growth of the world population during the AD era and the distribution describing the world economic growth, are shown in Figures 1 and 2. Their analysis is based on using the method of reciprocal values (Nielsen, 2014). For a sufficiently wide range of data, hyperbolic distributions can be *uniquely* identified using this method because if the reciprocal values are convincingly decreasing linearly, then the growth is hyperbolic. There is no other option. It is something similar to the unique identification of the exponential growth. For a sufficiently large range of good quality data, exponential growth can be uniquely identified by the linear distribution of the logarithm of the size of a growing entity.

Figures 1 and 2 show that growth of human population and economic growth were indeed slow over a long time, but it was hyperbolic growth, which *is* slow over a long time and fast over a short time. It is still the same, *monotonically*-increasing, growth. It is *impossible* to divide such a growth into distinctly-different components and the best way to see it, is to examine the reciprocal values of the size of the growing entity, in our case the reciprocal values of the GDP or of the size of the population (Nielsen, 2014).

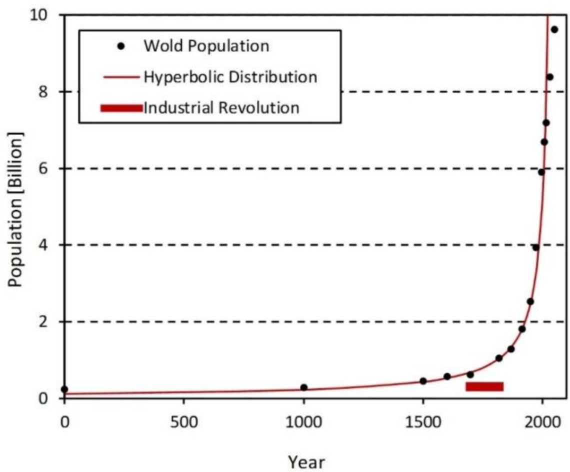

Figure 1. *Data describing the growth of the world population (Maddison, 2010) are compared with hyperbolic distribution.*

Hyperbolic distributions have to be analysed and interpreted as a whole. The same mechanism has to be applied to the slow and fast growth. If we apply the mechanism of Malthusian stagnation to the slow growth, we have to apply precisely the same mechanism to the fast growth. If we apply the mechanism of explosion to the fast growth, then precisely the same mechanism should be applied to the slow growth, which obviously is incorrect because explosion has to be triggered by something and there was clearly no explosion along the slow growth.

The usually assumed event that was supposed to have triggered population explosion or a sudden takeoff in economic growth or in the growth of population is the Industrial Revolution but as we can see in Figures 1 and 2, there was no sudden explosion during the Industrial Revolution or at any other time. The growth was increasing monotonically. Transition from slow to fast growth takes place all the time. We could demonstrate this monotonic growth even more clearly by using reciprocal values of data or by the semilogarithmic display (Nielsen, 2014; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i) but the primary aim of presenting these two diagrams is to illustrate the deceptive character of hyperbolic distributions. They can easily lead to incorrect interpretations particularly when they are not analysed but only used to quote certain, well-selected numbers or when they are deliberately manipulated and distorted (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a;

2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008) to support preconceived ideas. Hyperbolic distributions have to be analysed.

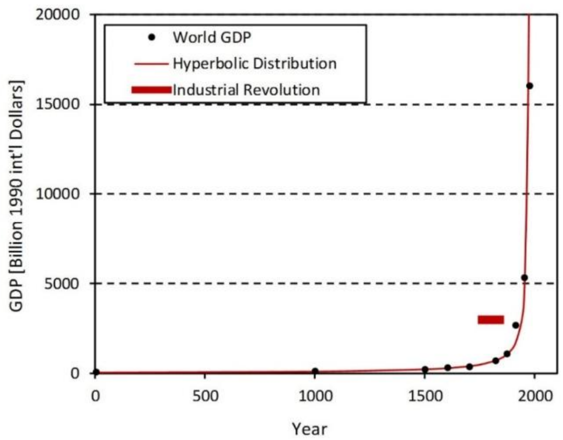

Figure 2. *Data describing the growth of the world Gross Domestic Product (Maddison, 2010) are compared with hyperbolic distribution.*

Figure 1 shows also that the growth of population is not yet levelling off. It is still following closely the fast-increasing historical hyperbolic distribution. Maddison's data end in 2008. The point representing the size of the population in 2014 is from the US Bureau of Census (2016) while the last two points are the predicted values (Nielsen, 2006). Not until 2030 or maybe even until 2050 could we expect a clear departure from the historical hyperbolic trend. The future of the population growth is uncertain, in much the same way as the future of the world economic growth (Nielsen, 2015b).

The origin of the concept of stagnation

Two features make the concept of the epoch of Malthusian stagnation deceptively attractive: (1) it is strongly believable and (2) it is supposed to have originated over 200 years ago. It is believable because the growth of human population and the economic growth over thousands of years were indeed slow, so slow that they appear to have been stagnant. It is also an old concept because its origin is traced, to Malthus (1798), perhaps inaccurately because Malthus never used the word *stagnation* in his book.

> The Malthusian theory, as was outlined initially by Malthus (1978), captures the main attributes of the epoch of Malthusian stagnation that had characterized most of human existence… (Galor, 2005, p. 221).
>
> The idea of multiple equilibria, or poverty traps, can be retraced back to Malthus (Wang, 2005, p. 36).

The work of Malthus was the first well-documented attempt to understand and explain the mechanism of growth of human population but it appears that this is also precisely where it ended. Considering the time when Malthus was writing his book, it was a remarkable achievement, but his work should have been not only checked but also extended using a large body of data, which were not available to Malthus but which are readily available to us.

> The history of population theory can be summarized in three words: pre-Malthusian, Malthusian, and post-Malthusian. Hardly ever in intellectual history does one man so dominate a field as does the Reverend Thomas Robert Malthus in demographic theory. To paraphrase a quotation attributed to Newton, Malthus' shoulders *must* be climbed (Thomlinson, 1965, p. 47. Italics in the original text.).
>
> …the demographic transition experiences three regimes: the 'Malthusian Regime,' the 'Post-Malthusian Regime,' and the 'Modern Growth Regime.' *Any theory* attempts (sic) to describe the process of demographic transition *must* include these three periods (Wang, 2005, p. 3. Italics added.).

Claiming, suggesting or assuming that something *must* be accepted just because it comes from a certain source is not acceptable in science. Any theory can be questioned and even should be questioned, and if necessarily corrected or rejected. The sooner it is done, the better it is for science. If Malthus's shoulders must be climbed it is only for the same reason as climbing the shoulders of any giant of human intellect: to see better and further ahead. It is not just to have a comfortable ride.

However, we are not even climbing Malthus's shoulders. Attaching his name to the concept of stagnation and calling it Malthusian stagnation sounds like defamation. It is questionable whether Malthus would be pleased with such a dubious distinction. We are putting our interpretation into his work and we are claiming that he did it.

If we read his publication carefully, we can find that he was writing not only about the devastating effects of positive checks but also about their regenerating effects (Nielsen, 2013b). Given enough time he would have probably studied this issue further. Descriptions of destructive effects of positive checks, which we label rather inaccurately as Malthusian stagnation should be balanced by descriptions of regeneration, which Malthus mentions in his book.

The name "Malthusian stagnation" is a misnomer because Malthus never claimed that positive checks would produce prolonged and wide-spread stagnations in the growth of population and because we know now that Malthusian positive checks, even if present, were not producing such effects (Kapitza, 2006; Kremer, 1993; Nielsen, 2013a; 2014; 2016b; 2016c; 2016d; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960). They appear to have been generally either too weak or their destructive impacts were effectively compensated by the well-known, natural process of regeneration (Nielsen, 2013a; 2013b; 2013c).

It would be interesting to search for impacts of Malthusian positive checks on the growth of population by investigating the growth of local populations. Generally, there appears to have been no impact. The only known example (Nielsen, 2016d) is a minor distortion in the growth of the world population between AD 1200 and 1400, which appears to be correlated with the convergence of *five* major demographic catastrophes: Mongolian Conquest (1260-1295) with the total estimated death toll of 40 million; Great European Famine (1315-1318), 7.5 million; the 15-year Famine in China (1333-1348), 9 million; Black Death (1343-1352), 25 million; and the Fall of Yuan Dynasty (1351-1369), 7.5 million. In general, demographic catastrophes were too weak to disturb the growth of global population (Nielsen, 2013c).

Looking for convincing evidence of impacts of Malthusian positive checks on the growth of population would not be easy because we would have to demonstrate not only clear discontinuities in the growth of population but also that these discontinuities are correlated with the records of demographic catastrophes. We would have to know the intensity of these demographic catastrophes not just in the number of deaths but in their relative impact. However, even then we would have to be aware of the possibility of spurious correlations.

Malthus never claimed that his concepts must be accepted. On the contrary, he was open to new ideas. Referring to himself in the third person he wrote:

> If he should succeed in drawing the attention of more able men to what he conceives to be the principal difficulty in the way to the improvement of society and should, in consequence, see this difficulty removed, even in theory, *he will gladly retract his present opinions and rejoice in a*

> *conviction of his error* (Malthus, 1798, p. viii. Italics added.)

It is interesting that Malthus used arithmetic and geometric progressions to support his arguments but it is not certain whether he was familiar with the hyperbolic growth, let alone that he appreciated the difference between hyperbolic and exponential (geometric) types of growth. Even now, hyperbolic distributions are repeatedly misinterpreted and exponential growth is used to explain the growth of just about anything.

Malthus claimed that "Population, when unchecked, increases in a geometrical ratio" (Malthus, 1978, p. 4). Now we know that this is not true. Population, when unchecked does not increase in a geometrical ratio (exponentially) but hyperbolically (Kapitza, 2006; Kremer, 1993; Nielsen, 2016b; 2016d; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960).

Malthus did not base his claims on a rigorous analysis of data. If he lived long enough to have better data, he would have probably discovered that the growth of population is not characterised by a constant doubling time and consequently that it could not have been increasing exponentially. If he were familiar with hyperbolic growth, he would have probably discovered that population increases hyperbolically. However, Malthus did not live long enough, he did not have access to good data and he was probably unfamiliar with hyperbolic growth. Those who lived after him and those who live now are more privileged.

Examples of questionable claims

The supposed Law of Population

During the assumed but non-existent epoch of Malthusian stagnation, birth rates are claimed to have been high because new generations were needed to support many tiresome and mundane activities such as hunting, gathering, cultivating crops, caring for children and generally for coping with harsh living conditions.

> According to Classical economists, and early Neo-Classical economists as well, population size was determined by the demand for labor. This was the Law of Population which constantly operated behind the seemingly random variations in fertility and mortality induced by epidemic, famine, and war (Lee, 1997, p. 1063).

Claims:

1. Population size was determined by the demand for labour.
2. This is the Law of Growth.

3. This law has been accepted by Classical and early Neo-Classical economists.
4. There were seemingly random variations in fertility and mortality.
5. Random variations were caused by epidemics, famine and war.
6. This law operated constantly behind these seemingly random variations.

It is interesting how much is claimed in this single paragraph and it does not matter whether Lee agrees with all these claims or just describes them. This quotation represents a typical set of questionable claims often encountered in publications related to the concept of the epoch of Malthusian stagnation. Can we prove them or do we have to accept them by faith?

To prove this "Law of Population" we would have to have data about the demand for labour and about the growth of population extending over thousands of years, and we would have to prove that there is a correlation between the demand for labour and the size of human population. We would have to prove that population size was determined by the demand for labour. We cannot prove it because we do not have such data, but we can show that the population data (Nielsen, 2016b; 2016d) do not display any features that could be linked with this supposed "Law of Population." This law has to be accepted by faith but this law is also in contradiction with data and with their analysis.

It is easy to imagine and claim, without a proof, that there were random variations in the fertility and mortality. It would be probably more difficult to expect that there were no variations but we have no information about these variations. We can only imagine them but we cannot analyse them.

We have reliable data about the *size* of human population (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013) over thousands of years but we have *no matching data for fertility and mortality (birth rates and death rates)*. We also have no matching data about epidemics, famines and wars to study how they were correlated with "random variations in fertility and mortality." We have absolutely no way of proving that "the Law of Population" "constantly operated behind the seemingly random variations in fertility and mortality induced by epidemic, famine, and war." This claim is *unscientific* because we can never expect to verify it by data but also it is scientifically unacceptable because data and their analysis give no support for such declarations.

It should be also noted that growth of population is not determined directly by birth and death rates but by the *difference* between these two quantities. This difference determines the *growth rate*. More precisely, it determines the rate of natural increase but generally migrations rates are relatively small and consequently the difference between birth and death rates can be taken as determining the growth rate.

A constant (non-zero) difference (constant growth rate) produces *exponential* growth. A zero difference produces *constant* population. However, variable difference between birth and death rates (i.e. the variable growth rate) does not necessarily produce a variable *size* of the population. In fact, even large fluctuations in the growth rate are not readily reflected in the growth of population. They might be reflected only as small and negligible variations (Nielsen, 2016c).

Fluctuations in birth and death rates have no impact on the mechanism of growth because they do not change population growth trajectories. We can see it even without analysing data. We can easily check that even for data characterised by large fluctuations in birth and death rates, and consequently by large fluctuations in the growth rate, the corresponding data, which describe the growth of population are not affected by such fluctuations. Fluctuations in birth and death rates do not change the general character of the distributions describing the growth of population (Lehmeyer, 2004; Mauritius, 2015; Statistics Mauritius, 2014; Statistics Sweden, 1999; Wrigley & Schofield, 1981). These data are well known. Some of them are even repeatedly used to defend the erroneous Demographic Transition Theory but no-one cared to check the population data published in the same sources, which list the fluctuating birth and death rates. While the fluctuating birth and death rates are taken as the confirmation of the established knowledge, the data describing the growth of population, data coming from precisely the same sources as the data for birth and death rates, are methodically ignored. Data describing the growth of population are in contradiction of the Demographic Transition Theory and in contradiction of the established knowledge.

The claimed losing battle

According to the concept of the epoch of Malthusian stagnation, as soon as the population started to increase, it was significantly reduced by numerous factors associated with severe living conditions.

> During the first [stage of the demographic transition], fertility is assumed to have been sufficiently high to allow a population to grow slowly even in the face of a rather high level of mortality. However, periodic epidemics of plague, cholera, typhoid and other infectious diseases would *in one or two years wipe out the gains made over decades. Over long periods of time there would, consequently, be almost no population growth at all* (van de Kaa, 2010, p. 87. Italics added.).

Claims:

1. During the first stage of the demographic transition, fertility and mortality are assumed to have been high.
2. Population was growing slowly.
3. Population growth was strongly controlled by periodic epidemics of plague, cholera, typhoid and other infectious diseases.
4. Periodic epidemics of plague, cholera, typhoid and other infectious diseases would in *one or two years* wipe out the gains made over *decades.*
5. Over long periods of time there was no population growth at all.

Van de Kaa describes the first of the four stages of growth claimed by the classical Demographic Transition Theory, the stage corresponding to the mythical but non-existent epoch of Malthusian stagnation (Nielsen 2016b; 2016c; 2016d).

Here we have a vivid description of what was happening so long ago and over a long time; not only a vivid description but also an explanation. In science, one would have to do a lot of solid work in order to be able to make such a sweeping declaration. We would have to prove that our conclusions are supported by data. We would have to give frequent examples that *the growth of population* was indeed *controlled* by "periodic epidemics of plague, cholera, typhoid and other infectious diseases." We would have to demonstrate convincingly that there were *frequent correlations* between "periodic epidemics of plague, cholera, typhoid and other infectious diseases" and the growth of population. Ideally, we would also have to prove that these frequent irregularities were *caused* by "periodic epidemics of plague, cholera, typhoid and other infectious diseases" because even observed correlations could be spurious.

Van de Kaa produces no such proof. He does not even give reference to such research. As far as we can tell, no-one has ever carried out such systematic and well-documented research.

His claims have to be accepted by faith and even more importantly, by faith contradicted by data (Biraben, 1980;

Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013). With only one exception in the past 12,000 years, between AD 1200 and 1400 (Nielsen, 2016d), there is no convincing evidence of generally occurring "long periods of time" when there was "almost no population growth at all" and that the growth was controlled by "periodic epidemics of plague, cholera, typhoid and other infectious diseases." The only way we could hope to give support to his claims would be to find exceptions to the generally observed regularities in the growth of population but even then his claims would not have a general application. The established knowledge may sound plausible and convincing but it has to be accepted by faith.

It is scientifically incorrect to take an easy way out by assuming that something happened, which we *think* could have happened and claim with such absolute certainty that it *did* happen. We might feel or think that our descriptions are true; we might wish for them to be true, but we should test them by following the generally accepted process of scientific investigation.

The supposed food-controlled homeostatic equilibrium

Harsh living conditions, and in particular the availability of food, are supposed to have a suppressive influence on the growth of human population but these intuitive expectations are again contradicted by data (UNDP, 2011) showing that growth rate is not directly proportional to the level of affluence but to the level of deprivation (Nielsen, 2013b). There is also convincing evidence that harsh living conditions in the distant past did not shape the growth of population (Nielsen, 2016b; 2016d; von Foerster, Mora & Amiot, 1960). Again, it is scientifically inexcusable to take an easy way out, to ignore data and try to mould science in the image of our wished-for interpretations.

> …the food-controlled homeostatic equilibrium had prevailed since time immemorial (Komlos, 2000, p. 320).
>
> …the population tends to oscillate in a homeostatic mechanism resulting from the conflict between the population's natural tendency to increase and the limitations imposed by the availability of food (Artzrouni & Komlos, 1985, p. 24).

Claims:

1. There was a food-controlled homeostatic equilibrium.
2. This equilibrium prevailed since time immemorial.
3. Population tends to oscillate in a homeostatic mechanism.

4. Oscillations are caused by the natural tendency of the population to increase and by the limitations imposed by the availability of food.

It is easy to *assume* that "the food-controlled homeostatic equilibrium had prevailed since time immemorial" but it is more difficult to *prove* it. It is easy to *claim* that "the population tends to oscillate in a homeostatic mechanism resulting from the conflict between the population's natural tendency to increase and the limitations imposed by the availability of food" but it is more difficult to *prove* it.

Authors of these confident declarations do not prove anything nor do they give reference to such a proof because such a proof does not exist. These declarations are in harmony with the established knowledge but the established knowledge is in conflict with science (Kapitza, 2006; Kremer, 1993; Nielsen, 2016b; 2016d; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960).

In order to have these declarations supported by science we would have to work a little harder. We would have to design a model with the homeostatic equilibrium. We would have to have data for the availability of food "since time immemorial." We would have to have corresponding data describing the growth of population. These data would have to be at small time intervals in order to detect the postulated oscillations. We would have to demonstrate convincingly that there were oscillations in the growth of population and that there was a correlation between the recorded oscillations in the growth of population and the oscillations in the availability of food. We would have to prove that the oscillations in the growth of population were *caused* by the oscillations in the availability of food. Acceptable evidence would have to be in demonstrating that our mathematical model reproduces all these oscillations. This would have been science but what we are offered is just a story, which has to be accepted by faith.

It is easy to claim many things but it is more difficult to prove them. Our postulates and explanations might sound plausible but they would have to be verified by the rigorous process of scientific investigation. Data (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013) give no support for the existence of the claimed fluctuations or oscillations.

There is no scientific basis for claiming that "food-controlled homeostatic equilibrium had prevailed since time immemorial."

This claim has to be accepted by faith. We have to accept by faith that "population tends to oscillate in a homeostatic mechanism resulting from the conflict between the population's natural tendency to increase and the limitations imposed by the availability of food." It all might sound plausible but we cannot prove it. However, even if it sounds plausible it is contradicted by the rigorous analysis of data (Nielsen, 2016b; 2015d).

Artzrouni & Komlos (1985) carried out model calculations, which incorporated the assumed mechanism of Malthusian stagnation. Their contribution is important but for reasons, which were not even noticed in their publication because their results show that the mechanism of Malthusian stagnation does not work. We shall discuss this issue in one of our forthcoming publications.

The supposed characteristic features of the past human history

> Stage 1 [of the Demographic Transition Theory] presumably characterizing *most of human history*, involves high and relatively equal birth and death rates and little resulting population growth" (Guest & Almgren, 2001; p. 621. Italics added.).
>
> This stage is characterized not by changes in *average* death rates but by a *stagnation of death rates at extremely high levels* for a period of what is believed to be *thousands of years*" (Olshansky & Ault, 1986, p. 357. Italics added.).

Claims:

1. Stage 1 proposed by the Demographic Transition Theory characterised presumably most of human history.
2. During this stage there were high and relatively equal birth and death rates.
3. During this stage there was little resulting population growth.
4. This stage was not characterised by changes in the average death rates.
5. This stage was characterised by stagnation of death rates at extremely high levels.
6. This stagnation is believed to have lasted for thousands of years.

It is amazing how firmly the established knowledge is now established if so much can be so easily claimed. The declaration that Stage 1 proposed by the Demographic Transition Theory was "characterized not by changes in *average* death rates but by a *stagnation of death rates at extremely high levels* for a period of what is believed to be *thousands of years*" has to be accepted by faith and by faith alone because we can never expect to have systematic data describing death rates to check its validity. No-one has yet demonstrated the validity of the Demographic Transition Theory. No-one has yet demonstrated the existence of the first two stages of growth, let alone the existence of all stages of growth.

Examples used in support of the Demographic Transition Theory are in fact in its direct contradiction (Nielsen, 2016c). As pointed out earlier (Nielsen, 2016c), the only way to demonstrate the apparent empirical features, which seem to be in agreement with the Demographic Transition Theory, is by a suitable manipulation of data consisting in stitching together the birth and death rates data for Mauritius with the data for Sweden.

It should be also remembered that any scientific theory is acceptable only if it is consistently confirmed by empirical evidence. A single convincingly contradicting evidence questions the validity of an accepted theory. For the Demographic Transition Theory, it is the other way round. There is not a single convincing empirical evidence in support of this theory but there is overwhelming empirical evidence showing that this theory is incorrect. This theory is contradicted by birth and death rates and by the corresponding distributions describing the growth of population (Nielsen, 2016c). Furthermore, within the range of analysable data (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013) growth of population was hyperbolic (Kapitza, 2006; Kremer, 1993; Nielsen, 2016b; 2016d; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960). Stages of growth proposed by the Demographic Transition Theory did not exist.

Birth and death rates may have been high and strongly fluctuating but high and fluctuating birth and death rates do not prove the existence of a stagnant state of growth because, as mentioned earlier, growth is determined by the average *difference* between these two quantities. Furthermore, these two quantities have to behave in a very specific way to produce the stagnant state of growth. Studying just death rates *or* birth rates, or equivalently studying just the fertility rates (Lehr, 2009) cannot be used as the evidence in support of the Demographic Transition Theory. Using scraps of favourable information while ignoring contradicting evidence is strongly misleading and consequently scientifically unacceptable.

The claimed well-documented evidence

> It is well documented that the fluctuations experienced by the world's population throughout history did not have a

> regular, cyclical pattern, but were, to a large extent, brought about by randomly determined demographic crises (wars, famines, epidemics, etc.). As McKeown and others have pointed out, the main cause of these fluctuations of the past were mortality crises. There are four kinds of crises: subsistence crises, epidemic crises, combined crises (subsistence/epidemic), and finally crises from other causes, which are mainly exogenous (wars, natural or other catastrophes)
>
> Crises followed by *periods of population decline* during which the nutritional status of the population improved gave rise to fluctuations which testify to the continued existence of the 'Malthusian trap': population would not grow beyond its carrying capacity for long, and when it did, the resulting overshoot was followed by a 'crash' (i.e. the positive checks such as diseases, famines, wars, etc.) (Artzrouni & Komlos 1985, p. 24. Italics added.).

Claims:

1. There were fluctuations in the world's population throughout history.
2. These fluctuations are well documented.
3. It is well documented that these fluctuations did not have a cyclic pattern.
4. It is well documented that these fluctuations were, to a large extent, brought about by randomly determined demographic crises (wars, famines, epidemics, etc.).
5. The main cause of these fluctuations were mortality crises.
6. There are four types of crises.
7. Crises were followed by periods of population decline.
8. Population decline improved nutritional status.
9. Fluctuations testify to the continuing existence of the Malthusian trap.
10. Population was repeatedly reaching its carrying capacity.
11. Population would not grow beyond its carrying capacity for long.
12. Population growing beyond its carrying capacity was reflected in overshoots.
13. Overshoots were followed by crashes.

If all this is so well documented, where is the documentation of this well documented research? It would be interesting to see at least a few references to this important and fundamental research to see the *data* showing fluctuations "throughout history," to see a positive proof that the "the fluctuations experienced by the world's population throughout history" are *correlated* with "demographic crises (wars, famines, epidemics, etc.)," that they were "brought about by randomly determined demographic crises." It would be also interesting to see convincing evidence that population was reaching its carrying capacity, that "population would not grow beyond its carrying capacity for long," the convincing evidence of overshoots and crashes, evidence that crashes were associated with "positive checks such as diseases, famines, wars, etc." It would be interesting to see the compelling evidence of the existence of the Malthusian trap, the demonstration of frequent "periods of population decline," the compelling proof that periods of population decline caused by demographic crises were improving nutritional status. All this vital and "well documented" evidence is missing.

What is well documented is the repeated fiction stories, which have to be accepted by faith. We have many publications propagating such stories. The repeatedly related stories of fiction are by now accepted as the undisputable facts. What is well documented is a system of beliefs, doctrines, wished-for explanations, opinions, views, theories, hypotheses, conjectures and speculations, added gradually over a long time until they became the established knowledge, the "well-documented" established knowledge but the knowledge, which is contradicted by science.

In contrast, it is well documented (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013) that the growth of human population does *not* show fluctuations or random behaviour. It is well documented that the data show no signs of frequent overshoots and crashes, no signs of growth reaching its carrying capacity, no signs of the "continued existence of the 'Malthusian trap'," no evidence that the "population would not grow beyond its carrying capacity for long," and no repeated "periods of population decline." All these colourful and dramatic descriptions associated with the narrative based on the assumption of the existence of the mythical epoch of Malthusian stagnation are contradicted by data.

It is obvious, that demographic crises might have been causing occasional decline in the size of *local* populations, depending on the scale of these crises and depending on what we understand by a local crisis. Sometimes it might have been just a large death toll in a city, in a part of a country, as for instance in China (Mallory, 1926), or maybe in the whole country or even extending over a few countries. However, a large death toll does not necessarily mean a significant impact on the growth of human population. A large death toll should not be immediately interpreted as a population decline; it could have been just a slower growth over a limited time followed by a more intensified growth, as it happened after AD 1400 for the world population.

All these issues should be closely investigated by examining records of demographic catastrophes. To arrive at any reasonably supported conclusion, we would have to do some hard work. However, data which should be used for such investigations are strongly limited. We have no data showing that local demographic crises were repeatedly causing fluctuations in the growth of regional or global populations. In fact, the data show remarkably stable growth of human population, generally unaffected by demographic crises (Nielsen, 2013a; 2013c; 2016b; 2016d).

The opening paragraph in the above quotation contains two interesting and characteristic elements, the elements occurring repeatedly in the descriptions of the concept of the epoch of Malthusian stagnation: (1) it makes a highly-questionable but confident declaration about the *existence* of certain features (in this case about the existence of fluctuations) and (2) it equally confidently *explains* them while ignoring empirical evidence. The normal progression in scientific research is *first to observe* certain features and then try to *explain* them. We can also reverse the process: we can first *predict* the existence of certain features. However, to accept the prediction and the associated explanation, we would have to *demonstrate the existence* of the predicted features. This is how science works but for doctrines accepted by faith scientific process of investigation is too tedious and consequently it is readily ignored.

So in this case, we would have to show first that there were significant fluctuations in the birth and death rates and in the size of human population extending over thousands of years, and then we would also have to explain them convincingly by demonstrating that they were correlated with demographic crises. Alternatively, we would have to predict (using a suitable mathematical model) the existence of fluctuations in birth and death rates and in the size of human population and then we would have to show that our predictions are confirmed by relevant data.

We cannot prove that there were fluctuations "throughout history" in the birth and death rates because we do not have relevant data, but we can prove that there were no fluctuations "throughout history" in the *size* of human population because we have the relevant data (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013). There is nothing in the data, which calls for the explanations of fluctuations in the growth of population because there are no fluctuations. What needs to be explained is perhaps the remarkable *absence* of fluctuations, the absence of random behaviour, crashes, overshoots or "periods of population decline." What needs to be explained is why the growth of population was so remarkably stable during the past 12,000 years (Nielsen, 2016d) and why it was hyperbolic. The quoted declarations are in perfect agreement with the established knowledge but they are in conflict with science.

The supposed long-run equilibrium between population size and the food supply

Referring to three sources (Habakkuk, 1953; Kunitz, 1983; McKeown, 1983), Komlos explains:

> Malthusian positive checks (mortality crises) maintained *a long-run equilibrium between population size and the food supply*. Crises followed by periods when human nutritional status was above the level of subsistence gave rise to *cycles*. …*the cycles testify to the continued existence of the 'Malthusian population trap'*: *population could not grow beyond an upper bound imposed by the resource and capital constraints* of the economic structure in which it was imbedded. The *'escape' from this trap* occurred only when the aggregate capital stock was large enough and grew fast enough to provide additional sustenance for the population, which thereby overcame the effects of the diminishing returns that had hindered human progress during *the previous millennia*. After escaping from the Malthusian trap, population was able to grow unchecked. In historic terms, this escape corresponds to the industrial and demographic revolutions. Removal of the nutritional constraint, at least for the developed part of the world, resulted in the population explosion (Komlos, 1989, pp. 194, 195. Italics added.).

Claims:

1. There was a long-term equilibrium between population size and the food supply.
2. This equilibrium was maintained by positive checks (mortality crises).
3. Crises were followed by periods when human nutritional status was above the level of subsistence.
4. This process gave rise to cycles.
5. The cycles testify to the continued existence of the 'Malthusian population trap'.

6. Population could not grow beyond an upper bound imposed by the resource and capital constraints of the economic structure in which it was imbedded.
7. Malthusian trap was active for millennia.
8. The escape from the Malthusian trap occurred when the aggregate capital stock was large enough and grew fast enough to provide additional sustenance for the population.
9. The removal of nutritional constrains caused population explosion.

Massive amount of work would have to be done to support all these impressive declarations. We would have to study food supply over millennia and determine how they were correlated with the growth of human population. We would have to prove that there was "a long-run equilibrium between population size and the food supply." We would have to study mortality crises over millennia. We would have to establish a correlation between the growth of human population, food supply and mortality crises. We would also have to investigate upper bounds of "resource and capital constraints" and prove that over millennia the size of the population was repeatedly reaching the limits of these upper bounds.

Conducting scientific research is not easy but results have a high degree of reliability. Writing fictions stories, whose general script is already provided by the established knowledge based largely on faith is much easier, but this is not science.

It is easy to declare so much so quickly and with such a confidence, but it is harder to prove it. It is also hard to accept it, but accept we must if we want to accept the concept of the epoch of Malthusian stagnation promoted by the established knowledge.

The claimed cycles cannot possibly testify to "the continued existence of the 'Malthusian population trap'" because they did not exist in the growth of population (Nielsen, 2016b; 2016d; von Foerster, Mora & Amiot, 1960). Population growth, global and regional, was remarkably stable and unconstrained. The claim that "population could not grow beyond an upper bound imposed by the resource and capital constraints" is contradicted by the analysis of population data. This claim appears to be based on pure fantasy and on a wished-for mechanism that did not exist. There was no Malthusian trap in the growth of population.

We know nothing about any possible cycles in birth and death rates because we have no relevant data extending over a long time in the past. We do not know how large were these supposed cycles. We do not even know whether they existed. Discussions of these cycles are irrelevant because we know that cycles in birth and death rates are of little or no consequence for explaining the mechanism of growth (Nielsen, 2016c). Even if they were present they did not have any significant influence on the growth of the world population in the past 12,000 years (Nielsen, 2016d). They also had no influence on the growth of regional populations (Nielsen, 2016b). The absence of cycles in the growth of population combined with the evidence of the steadily increasing growth testify that the Malthusian trap did *not* exist. We cannot also claim that there was "'escape' from this trap" because there was no trap in the growth of population. There was also no trap in the economic growth (Nielsen, 2014; 2015a; 2016a; 2016e; 2016f; 2016g; 2016h; 2016i). Again, the quoted declarations are in perfect agreement with the established knowledge but they are in conflict with science.

The supposed fluctuation of fertility and mortality rates around zero

Discussing the first stage of the Demographic Transition Theory, Warf explains:

> Because both fertility and mortality rates are high, the *difference* between them — natural population growth — is relatively low, *fluctuating around zero*" (Warf, 2010, p. 708. Italics added.).

Claims:

1. During the first stage of the demographic transition fertility and mortality rates were high.
2. The difference between them (the natural population growth) was fluctuating around zero because they were high.

Just because fertility and mortality rates were high it does not mean that the difference between them was zero. The difference between them can fluctuate around zero even if they are low. However, this is just a minor issue.

In this quotation the "natural population growth" is identified as the *difference* between the fertility and mortality rates. It is, therefore, the rate of natural increase or the *growth rate* because, in general, migration rates are relatively small and can be neglected.

We shall recall that while the growth rate fluctuating around a constant value describes exponential growth, the growth rate "fluctuating around zero' describes the constant size of the growing entity, i.e. in our case, the constant size of the population. The claim made by Warf is contradicted by data, which show that for thousands of years the size of human population was not constant but steadily *increasing* (Nielsen, 2016b; 2016d; von Foerster, Mora & Amiot, 1960). Furthermore, the size of population was increasing hyperbolically. The "natural population growth" (growth rate) could not have been "fluctuating around zero" but it must have been increasing hyperbolically because for the hyperbolic growth, the growth rate also increases hyperbolically (Nielsen, 2016h).

The apparent roughly constant population

In line with the accepted interpretations of the first stage of the Demographic Transition Theory, Lagerlöf writes:

> The Malthusian Regime in our model is a stable situation where death and birth rates are both high, and *population roughly constant*. Moreover, mortality is highly volatile, increasing dramatically in periods of big epidemic shocks. In periods with mild shocks population expands. This worsens the impact of the next epidemic, equilibrating population back to its Malthusian state (Lagerlöf, 2003a, p. 756. Italics added.).
>
> In our model, the world can thus be stuck in a *Malthusian equilibrium* for centuries and then suddenly escape, and never contract back. As suggested by a referee, this process could possibly be interpreted in terms of wars, instead of epidemics (Lagerlöf, 2003a, p. 766. Italics added.).
>
> *Throughout human history*, *epidemics, wars and famines have shaped the growth path of population*. Such shocks to mortality are the central theme of the model set up by Lagerlöf, which endogenously generates a long phase of *stagnant population* and living standards, followed by an industrial revolution and a demographic transition (Lagerlöf, 2003b, pp. 434, 435. Italics added.).

Claims:

1. It is assumed that there was a Malthusian regime.
2. It is assumed that Malthusian regime is characterised by high birth and death rates.
3. During the Malthusian regime population is roughly constant.
4. Mortality is highly volatile.
5. Mortality increases dramatically in periods of big epidemic shocks.
6. Population expands when the mortality shocks are mild.
7. Expanding population worsens the impact of the next epidemic and equilibrates population to the Malthusian state.
8. Malthusian equilibrium lasts for centuries.
9. The process of Malthusian equilibrium can be also explained by wars instead of epidemics.
10. Throughout human history, epidemics, wars and famines have shaped the growth path of population.

11. Model based on the assumption of shocks to mortality generates a long phase of stagnant population.
12. The "long phase of stagnant population and living standards" is "followed by an industrial revolution and a demographic transition."

Here again, and quite typically, we have a series of declarations that have to be accepted by faith. However, paradoxically if not ironically, Lagerlöf was on the verge of discovering that doctrines accepted by faith were contradicted by his own model.

He has carried out an interesting and important research work but unfortunately he did not finish it: he did not compare results of his calculations with data (Maddison, 2001), which were available to him before publication of his work. He did not take the final and the most essential step. If he did, he would have discovered that the mechanism of Malthusian stagnation incorporated in his model did not produce fluctuations in the model-generated growth of population, that model-generated growth of population was not stagnant and it did not fit the relevant data. He would have found that contrary to what he claimed in his publication, his model-generated population was steadily increasing. If he cared to consult data (Maddison, 2001) he would have also found that the population reported by Maddison was also steadily increasing. We shall discuss these issues in a separate publication.

Lagerlöf presents a plot of the growth rate and calls it erroneously "Population growth" (Lagerlöf, 2003b, p. 436). He fails to take the most essential step in this type of work and to use his model-generated growth rate to calculate model-generated distribution describing the growth of population. He ignores data (Maddison, 2001) and yet his unfinished work is accepted for publication maybe because it proclaims loud and clear the doctrines of the established knowledge. Science appears to be of no importance.

Incorrect claims about the growth rate

> In our model, this leads to a *constant rate* of population growth prior to the adoption of the Solow technology. This result is consistent with population data from Michael Kremer (1993), where *the growth rate of population fluctuates around a small constant* throughout most of the Malthusian period (from 4000 B.C. to A.D. 1650) (Hansen & Prescott (2002), p. 1205. Italics added.).

Claims:

1. Growth rate of population fluctuates around small constant during the Malthusian period (i.e. prior to the adoption of Solow technology).

2. Small and roughly constant growth rate is consistent with population data from Michael Kremer (1993).

First, it appears that Hansen and Prescott might be confusing constant growth rate with constant population. It might be the same mistake as it appears to have been made by Lagerlöf (2003b). A constant (non-zero) growth rate does not produce a constant (non-zero) size of population. A constant (non-zero) growth rate produces *exponential* growth.

Second, this declaration appears to contain conflicting information. It is hard to imagine that random forces characterising the mythical Malthusian period would produce a steadily increasing exponential growth. Steadily-increasing growth suggests the presence of a dominating constant force, overruling any random forces.

Third, fluctuations in the growth rate are not readily reflected as fluctuations in the growth of population (Nielsen, 2016c). We can demonstrate it even without carrying mathematical analysis of the fluctuating growth rate. Data alone (Lehmeyer, 2004; Mauritius, 2015; Statistics Mauritius, 2014; Statistics Sweden, 1999; Wrigley & Schofield, 1981) show clearly that fluctuating growth rates do not produce significant fluctuations in the growth of population and that they have no impact on the mechanism of growth because they do not alter growth trajectories.

Fourth, we would have to show convincingly that the growth rate was indeed fluctuating around a small constant value as claimed by Hansen & Prescott (2002). There is no such proof because we do not have the data for the growth rate extending over thousands of years. However, there is a proof that the growth rate during the AD and BC eras was *not* fluctuating around a small constant value but that it was increasing hyperbolically because the growth of the population was hyperbolic (Kapitza, 2006; Kremer, 1993; Nielsen, 2016b; 2016d; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960). For the hyperbolic growth, the growth rate increases hyperbolically with time or in the direct proportion to the size of population (Nielsen, 2016h), as observed also by Kremer (1993).

Fifth, Kremer (1963) did not carry out an extensive study of the growth rate. He has presented rough calculations of this quantity using strongly varying local gradients, which do not represent the real gradient of growth. His calculations are strongly inaccurate for the BC era when individual data values are separated by large time intervals. It is scientifically unjustifiable to use such calculations and claim fluctuations around a constant value.

Sixth, for the hyperbolic growth, growth rate is small over a long time because it is also hyperbolic. Growth rate might appear to vary around a small constant but such interpretation is incorrect. Growth rate should be preferably calculated using interpolated gradients to avoid spurious effects of strongly-varying local gradients between adjacent data values. It is also useful to display growth rate using various types of displays to help in its interpretation. Using the approximate calculations of Kremer (1963) and claiming that growth rate was varying around small constant is self-misleading and scientifically unjustified.

This example illustrates that in science it is essential to carry out methodical analysis of data. In economic and demographic research this is particularly important because historical economic growth and historical growth of population were increasing hyperbolically. Hyperbolic distributions are strongly misleading and can easily lead to their misinterpretations. Furthermore, for hyperbolic distributions, the growth rate and the gradient increase in a similar fashion. The growth rate increases hyperbolically and the gradient follows the second-order hyperbolic distribution, both of them containing the same confusing features of a slow growth over a long time and a fast growth over a short time, but both increasing *monotonically* over the entire range of time. Hyperbolic growth of the GDP and population as well the monotonically-increasing growth rates and gradients cannot be divided into two or three distinctly different sections. They all have to be analysed and interpreted as a whole. The same mechanism has to be applied to the slow and to the fast growth because slow and fast growth belongs to the same, monotonically-increasing distributions.

The supposed density-dependent variations in mortality

> If population *density* increases the mortality rate rises, equilibrating population back to the Malthusian trap (Lagerlöf, 2003a, p. 765. Italics added.).

This statement has to be also accepted by faith because there is no convincing research supporting such declaration. Creative imagination appears to be taking full control in the established knowledge.

Here we have an example of an interesting *detail* added to the concept of the epoch of Malthusian stagnation, illustrating how one fantasy can lead easily to a new fantasy and how such gradual additions reinforce the established knowledge. This statement claims the dependence of mortality rate on the *density* of human population. It offers an *explanation* how the phantom Malthusian trap regulates the growth of human population. It describes some kind of a general rule that the Malthusian trap is activated when the population *density*, not its size, reaches a certain limiting value.

There is no research confirming the described mechanism; no research showing how the growth of human population depends on its *density*. Even if we could show some isolated examples of the density-dependent growth we would have to demonstrate that such mechanism applies also to regional and global populations. The best data available to us show the *time*-dependence of the size of human population and there is nothing in them to suggest any form of *density*-dependence, let alone the existence of the Malthusian trap triggered by the density of population.

This statement is yet another example of the leaps of faith, of confident declarations requiring a huge amount of work to be accepted as a reliable contribution to science. The descriptions of the epoch of Malthusian stagnation are full of such unscientific declarations. Indeed, they are made of them.

Other terms used to describe the supposed stagnant and fluctuating state of growth during this mythical epoch of Malthusian stagnation are "equilibrium trap" or "population trap" (Leibenstein, 1957; Nelson, 1956), "multiple equilibria" or "poverty trap" (Wang, 2005).

The belief in the stagnant and fluctuating growth is so strong that mathematical models are deemed successful if they can generate the desired oscillations during this mythical epoch of Malthusian stagnation, and no-one seems to care to take the next and the most essential step and to compare model calculations with population data. As long as oscillations of some kind are generated by a mathematical model, they are taken as the proof of the existence of the epoch of Malthusian stagnation. This line of reasoning shows that the primary, if not the exclusive, aim of such mathematical exercises is to translate a story into a mathematical language and when the translation is done properly, when mathematical formulae generate *any kind of oscillations*, large or small, significant or negligible, these formulae are then taken as a proof of the existence of Malthusian stagnation.

The supposed Age of Pestilence and Famine

The epoch of Malthusian stagnation is also described as the Age of Pestilence and Famine (Omran 1971; 1983; 1998).

> In this stage, the major determinants of death are the Malthusian positive checks, namely epidemics, famines and wars (Omran, 1983, p. 306; Omran, 2005, p. 737).
>
> Even if fertility approached its biologic maximum, depopulation could and did occur as a result of epidemics, wars and famines, which repeatedly pushed mortality levels to high peaks (Omran, 2005, p. 733).

> The pattern of growth [of human population] until about 1650 is cyclic (Omran, 1971, Table 4, p. 533).

Claims:

1. During the Age of Pestilence and Famine (i.e. during the epoch of Malthusian stagnation) major determinants of death are the Malthusian positive checks (epidemics, famines and wars).
2. Depopulation was occurring even when fertility was approaching its biological maximum because epidemics, wars and famines were repeatedly pushing mortality levels to high peaks.
3. Growth of population before AD 1650 was cyclic.

To justify the first claim we would have to have reliable records of the *causes of death* over thousands of years. We would then have to show convincingly that indeed the major causes of death were epidemics, famines and wars. We would also have to show that there was a clear change in the causes of death when the epoch of Malthusian stagnation ceased to exist. We cannot present such proofs because we do not have the supporting data. In principle, therefore, this claim is not scientific because we cannot check it by data. It has to be accepted by faith.

To justify the second claim, we would have to have reliable records of fertility and mortality over thousands of years. We would then have to demonstrate that fertility was approaching biological limits, that such events were coinciding with high mortality peaks and that these high mortality peaks were caused by epidemics, wars and famines. We do not have relevant data to check whether these descriptions are true. They are therefore also unscientific and they have to be accepted by faith.

The growth of population, global and regional, before AD 1650 was *not* cyclic (Nielsen, 2016b, 2016d). This statement is contradicted by data (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013).

The claimed main cause of mortality

> During the first stage, *mortality vacillated at high levels*, with infectious disease as the main cause of death plus a large proportion due to wars and famines (Robine, 2001, p. 191. Italics added.).

Claims:

1. During the first stage of demographic transitions mortality vacillated at high levels.

2. The main causes of death were infectious diseases.
3. Large proportion of death were caused by wars and famines.

We cannot prove that "mortality vacillated at high levels" because we have no relevant data for the so-called "first stage" to carry out such a study, the stage that is assumed to have lasted for thousands of years. We cannot prove that these imagined and wished-for vacillations were correlated with infectious disease, wars and famines. We cannot prove that the *main* causes of deaths were infectious diseases. We cannot prove that a *large propo*rtion of death was due to wars and famines. We do not have sufficiently extensive records of causes of death extending over thousands of years. We do not know how the causes of death were changing over time. We do not have the records to help us to distinguish between the major and minor causes. We do not know whether the main cause of death was the same over thousands of years. The concept of the epoch of Malthusian Stagnation and all these claims have to be accepted by faith.

The supposed unsustained growth of population

> The first transition phase, called the 'Age of Pestilence and Famine,' is characterized by *high and fluctuating mortality rates*, variable life expectancy with low average life span, and *periods of population growth that are not sustained* (McKeown, 2009, p. 20S. Italics added.).

Claims:

1. During the Age of Pestilence and Famine (i.e. during the hypothetical but non-existent epoch of Malthusian stagnation) mortality rates were high and fluctuating.
2. Average life span was low.
3. There were periods when the population growth was not sustained.

Mortality rates might have been high and fluctuating but we have no data extending over thousands of years to prove it. Furthermore, we would yet have to show that these hypothetical high and fluctuating mortality rates could have been responsible for creating stagnation. What we know is that strongly-fluctuating mortality rates do not change the growth of population (Lehmeyer, 2004; Mauritius, 2015; Nielsen, 2016c; Statistics Mauritius, 2014; Statistics Sweden, 1999; Wrigley & Schofield, 1981). There is also nothing in the data and in their analysis to show that "low average life span" was affecting the growth of population. As for the "periods of population growth that are not sustained" this claim is contradicted by the analysis of data (Nielsen, 2016b; 2016d).

Positive forces were supposedly balanced by negative forces

> The positive forces of growth had existed all along. However, they had been counterbalanced by the negative forces of malnutrition and disease (Komlos & Baten, 2003, p. 19).

We have no reliable empirical evidence to support this claim, no study of positive and negative forces, no study of their balancing, and no study of their influence on the growth of human population. This is not science but story-writing prompted and approved by the established knowledge.

How do we know that the so-called positive forces were balanced by forces of malnutrition and disease? They obviously were not because economic growth and the growth of population were hyperbolic and remarkably stable (Nielsen, 2016a, 2016b, 2016d). Such a strong and stable growth could have been only generated by a strong and dominating force.

Here again, authors of this declaration take an easy way out. They have made no attempt to consult data available to them at the time of the publication of their paper (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999). They have made no attempt to reconcile their interpretations with the already documented evidence of hyperbolic growth (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960). Again, this declaration is in perfect agreement with the established knowledge but is in conflict with science.

The continuing misinformation

The established knowledge is by now so strongly established that it will be difficult to change it. It continues to be supported by the scientifically-unsubstantiated claims and descriptions. It would take volumes to list and discuss all such examples and to show that these repeatedly propagated doctrines, explanations and interpretations have to be accepted by faith.

The current established knowledge based on the assumption of Malthusian stagnation followed by explosion and reinforced by many complicated explanations is similar to the established knowledge about the dynamics of celestial bodies, interpretations which were established for about two millennia before they were eventually abandoned. Describing the work of mathematicians of his time, Osiander wrote:

> With them it is as though an artist were to gather the hands, feet, head and other members from his images from divers

models, each part excellently drawn, but not related to a single body, and since they in no way match each other, the result would be monster rather than man (Copernicus, 1995).[5]

Historical economic growth and historical growth of population can be expected to be described by a simple mechanism because hyperbolic growth is simple. This issue will be discussed in a separate publication, where a simple explanation of the mechanism of hyperbolic growth will be also presented. Hyperbolic growth prevailed for at least 12,000 years for the growth of population (Nielsen, 2016d) and for hundreds of years for the economic growth (Nielsen, 2016a). The established knowledge in demography and in economic research offers complicated explanations, which have to be accepted by faith. Hopefully we shall not have to wait for two thousand years to abandon these erroneous doctrines and replace them by science.

Summary and conclusions

Established knowledge in demography and in economic research is based on a series of doctrines and explanations revolving around the concept of Malthusian stagnation and around the concept of the escape from the Malthusian trap described as explosion, takeoff, sprint or spurt. It is a system of interpretations, which have to be accepted by faith.

It is easy to understand why these concepts are so attractive because the growth of population and economic growth were increasing hyperbolically and hyperbolic growth creates an illusion of stagnation followed by explosion.

It is essential to understand that hyperbolic distributions should be analysed and interpreted *as a whole*. If we take just a few examples along the hyperbolic growth, we can easily make a mistake and arrive at incorrect conclusions. If hyperbolic distributions are already difficult to understand without their methodical analysis, linearly-modulated hyperbolic distributions (Nielsen, 2015a) describing income per capita are even more difficult to understand because they create even stronger illusion of stagnation followed by a sudden explosion. Here again, just taking a few examples along these distributions is bound to lead to

[5] This quotation comes from a letter written by Andreas Osiander, Lutheran theologian and a friend of Copernicus, a letter addressed to the chief editor, Pope Paul III. Osiander argues in favour of the mathematically simple and elegant heliocentric system as opposed to the complicated geocentric descriptions. This letter was later used as an unsigned introduction to the book *De revolutionibus orbium coelestium*, and was mistakenly attributed to Copernicus.

incorrect conclusions. These distributions have to be also analysed with care. Careful and methodical mathematical analysis of data describing historical economic growth and the growth of population is unavoidable.

Distributions describing income per capita are generated by a division of two hyperbolic distributions. The characteristic feature of this ratio is that for a long time the growth of income per capita was not just slow, as for hyperbolic distributions, but nearly constant. This feature characterises the division of any hyperbolic distributions, not just the division of the GDP and population (Nielsen, 2015a). It is a purely mathematical property, which has nothing to do with specific properties of economic growth,

The nearly constant income per capita should never be interpreted automatically as stagnation. The only way to claim stagnation for this nearly-constant income per capita is to *analyse* the GDP and population data separately and to *prove* that these distributions are not hyperbolic but stagnant.

It is incorrect to take a few values of income per capita, show that they are nearly constant and claim stagnation. If the GDP and population increase hyperbolically, then income per capita increases by following the *monotonically*-increasing linearly-modulated hyperbolic destitution and it is incorrect to try to divide such a monotonically-increasing distribution into two different sections, slow and fast. Mathematically, it is impossible to make such a division. It is impossible to identify a point or a range of points and claim them as marking the place of transition.

Even though the ratio of two hyperbolic distributions is nearly constant over a long time and nearly vertical over a short time, the transition from the nearly constant to the nearly vertical patterns occurs all the time along the entire range of such distributions. Linearly-modulated hyperbolic distributions representing income per capita should be also interpreted as a whole. The same mechanism should be applied to the nearly constant and to the nearly vertical growth, unless we can prove that the GDP and population were not following hyperbolic distributions but were stagnant.

We have presented many examples of claims revolving around the concepts of stagnation followed by explosion. We have shown why such claims are scientifically unacceptable.

The origin of the fundamental concepts of the established knowledge can be traced, perhaps not entirely correctly, to Malthus (1798). He has presented an important pioneering work but unfortunately the ensuing studies of economic growth and of the growth of population have taken a wrong turn at a certain time in the past, perhaps because relevant data were not available.

By the time the relevant data (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2001; 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999) became available, they were ignored. More recently, some of them (Maddison, 2001) were manipulated to support the established knowledge (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Earlier analyses of data (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960) showing that the growth of population was hyperbolic were also ignored. By now, the established knowledge is so well established that anything being in its conflict is methodically ignored, rejected or suppressed. This is not science.

Recent analyses of data (Nielsen, 2014; 2015a; 2016a; 2016b; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i) confirmed the earlier studies (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960) and demonstrated that the historical growth of population and the historical economic growth were hyperbolic. The established knowledge based on the scientifically-contradicted concepts of stagnation followed by explosion (takeoff or the escape from the Malthusian trap) has to be replaced by explanations based on accepting hyperbolic growth.

It is incorrect to interpret the past harsh living conditions as a proof of the existence of Malthusian stagnation. Whatever harsh living conditions might have been present in the past, their effects are generally not reflected in growth trajectories. The only known example is for the growth of global population between AD 1200 and 1400 coinciding with the convergence of *five* major demographic catastrophes (Nielsen, 2016d). However, even then, the recorded effect is small.

Negative effects of the Malthusian positive checks should be never used robotically to describe the past growth of population or the economic growth. If we want to claim that positive checks were shaping the growth of population or economic growth, we have to prove it. If we want to claim that the so-called Law of Population was shaping growth trajectories, we have to prove it. If we want to claims that demographic catastrophes were shaping the growth of population, we have to prove it. If we want to claim that Industrial Revolution was shaping growth trajectories, we have to prove it.

We cannot take shelter in the established knowledge because in this respect established knowledge is repeatedly contradicted by data. Any data we might have, should to be methodically analysed to prove the negative effects of Malthusian positive checks but whatever we would prove would be just an exception from the general and well-demonstrated pattern that the historical growth of population and historical economic growth were not only hyperbolic but that they also remarkably stable.

Interpretations based on the concepts of Malthusian stagnation and on the claims of the escape from the Malthusian trap are not only incorrect but also dangerously misleading. They suggest that after the endless epoch of stagnation we have now entered the sustained growth regime (Galor, 2005a; 2011). This hypothesis creates a sense of security. In contrast, analysis of data shows that the past growth was sustainable but now for the first time in human history it is unsustainable and insecure (Nielsen, 2015b). While in the past, economic growth and the growth of population, global and regional, were following the slowly increasing hyperbolic trajectories (Nielsen, 2014; 2015a; 2016a; 2016b; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i) indicating the unconstrained and secure growth, now the growth is at the stage of the dangerously fast increase (see Figures 1 and 2). Growth after 1950 is no longer hyperbolic but it is still increasing close to the historical hyperbolic trajectories. For the first time in human history, these growth trajectories are clearly unsustainable because such a fast increase cannot be possibly tolerated for much longer.

The established knowledge is not only in conflict with data describing the past economic growth and the growth of human population but also in conflict with the general knowledge about the current mounting problems threatening our future. The established knowledge in demography and in economic research created its own world of fiction divorced from the real world.

We have not escaped the Malthusian trap because there was no trap in the economic growth or in the growth of population. The past growth was unconstrained and sustainable as demonstrated by the undisturbed hyperbolic distributions. However, now we are in a trap. For the first time in human history we are in the trap of numerous critical problems, which threaten our global security and our survival (Nielsen, 2006). For the first time in human history our combined ecological footprint is larger than the ecological capacity and it continues to increase (WWF, 2010). For the first time in human history our growth is supported by the increasing ecological deficit.

In order to understand the past and present economic growth, erroneous interpretations revolving around the concept of Malthusian stagnation have to be abandoned and replaced by scientifically acceptable interpretations. What needs to be explained is why the past economic growth and the growth of population were hyperbolic. Why was the growth so remarkably stable? Why was it not influenced by many random forces, which might have been present? Why did the growth start to divert to slower trajectories? Why does it continue so closely to the dangerously fast hyperbolic trajectories? And the most important questions of all: How to slow down the current growth? How to control growth?

Examples presented here suggest that there is a problem not just with certain interpretations adopted and protected by the established knowledge in the demographic and economic research but with the way research is carried out in these two fields. It is not just the problem with one or two theories, such as the Demographic Transition Theory or the Unified Growth Theory, which need to be corrected or most likely replaced. It is not even just the problem with the accepted paradigm based on the concept of Malthusian stagnation, which needs to be abandoned. It is a *systemic* problem. It is a problem, with the way research is conducted in these two fields. It is a problem with creating stories and interpretations, which have to be accepted by faith. It is a problem with a selective use of data. It is a problem of ignoring contradicting evidence, such as the contradicting evidence published over 50 years ago by von Foerster, Mora and Amiot (1960). It is problem with manipulating and distorting data to fit the preconceived ideas, as it has been done repeatedly in the Unified Growth Theory and in other related publications (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). It is a problem of testing data by a theory rather than testing theory by data. It is a problem with protecting a system of doctrines, which are accepted on faith.

As outlined briefly elsewhere (Nielsen, 2016i), there are two ways of conducting research: (1) the dynamic scientific method, which is used in the self-correcting disciplines of science and (2) the stagnant method, which is used routinely in the emotional defense of doctrines accepted by faith. It is unfortunate, that as pointed out earlier (Nielsen 2013a; 2013b; 2013c; 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i), demographic and economic research appears to gravitate strongly towards the unscientific method.

The established knowledge revolving around the concept of Malthusian stagnation has to be changed because there was no stagnation in the historical economic growth and in the historical growth of population. There was also no escape from the Malthusian trap because there was no trap. This paradigm has to be changed because historical economic growth and the historical growth of population were hyperbolic. However, in order to make the demographic and economic research scientifically acceptable, the systemic problem has to be also solved. Scientific research can be based only on the well-known and generally recognised scientific rules of investigation. Anything else is not science.

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2. Method of reciprocal values and examples of its application

Introduction

What we are going to see will change the fundamental postulates in the demographic and economic research. It will change radically the way the mechanism of economic growth and of the growth of population is interpreted. Maybe the change will not come immediately because it is usually difficult to change the well-established interpretations and explanations but the change will come because this is the way science works. Incorrect interpretations are not tolerated for too long and it does not matter who are their advocates.

It might be expected that a complicated proof would be required to achieve such a radical change of direction in the economic and demographic research, that perhaps some new and complicated description of the mechanism of growth would have to be proposed. However, the proof turns out to be exceptionally simple. No complicated mathematics is required but only the way we describe data using the simplest mathematical representation: the straight line.

George Pólya, Hungarian mathematician, observed that when a proof is too simple, "youngsters" will be unimpressed (Pólya, 1981), but mathematics does not have to be complicated to be useful. He also pointed out that solving problems is a quintessential human activity and the aim is always to find *the simplest solutions*.

We are going to present here a proof so simple that it might look trivial. We are going to show how to change the confusing and complicated distributions describing the historical economic growth and the historical growth of human population into the simplest representations. We are going to show how the distributions, which suggest complicated explanations of the mechanism of growth are in fact so simple that they suggest also a simple mechanism.

Analysis of data describing the historical economic growth and the historical growth of population might look complicated but it is exceptionally simple. Anyone can do it. However, there is more to the analysis of data then just looking for their mathematical descriptions. We are going to demonstrate that this simple method of analysis makes a significant contribution to a better understanding of the mechanism of the historical growth of population and of the economic growth. It also demonstrates that there is a need to replace the traditionally used postulates based largely on impressions and conjectures by postulates based on the mathematical analysis of data.

The common problem

Hyperbolic processes appear to be causing a serious problem in the economic and demographic research. They create such a strong illusion that it deceives even the most experienced and respected researchers. The common mistake is to see them as being made of two distinctly different components, slow and fast, with a clear transition between them (Ashraf, 2009; Artzrouni & Komlos, 1985; Baldwin, Martin & Ottaviano, 2001; Becker, Cinnirella & Woessmann, 2010; Clark 2003, 2005; Currais, Rivera & Rungo 2009; Dalton, Coats & Asrabadi, 2005; Desment & Parente, 2012; Doepke, 2004; Ehrlich, 1998; Elgin, 2012; Galor 2005a, 2005b, 2007, 2008a, 2008b, 2010, 2011, 2012a, 2012b; Galor & Michalopoulos, 2012; Galor & Moav 2001, 2002; Galor & Mountford, 2003, 2006, 2008; Galor & Weil, 1999, 2000; Goodfriend & McDermott 1995; Hansen & Prescott 2002; Jones, 2001; Johnson & Brook 2011; Kelly, 2001; Khan 2008; Klasen & Nestmann 2006; Kögel & Prskawetz 2001; Komlos 1989, 2000, 2003; Komlos & Artzrouni 1990; Lagerlöf 2003a, 2003b, 2006, 2010; Lee, 2003, 2011; Mataré, 2009; McFalls, 2007; McKeown, 2009; McNeill 2000; Møller & Sharp, 2013; Mongomery, n.d.; Nelson, 1956; Omran 1971, 1983, 1986, 1998, 2005; Robine 2001; Smil 1999; Snowdon & Galor, 2008; Steinmann, Prskawetz & Feichtinger, 1998; Strulik, 1997; Tamura 2002; Thomlinson 1965; van de Kaa 2008; Voigtländer & Voth, 2005; Vollrath, 2011; Wang 2005, Warf 2010; Weisdorf 2004; Weiss 2007). The next step is then to try to explain these two perceived stages of growth and the associated but non-existent transition by proposing distinctly different mechanisms for each of these imagined components rather than seeing them as representing a *single*, *monotonically increasing* distribution governed by a *single mechanism of growth*.

This step leads progressively further away from the correct understanding of studied processes because all efforts are now concentrated on explaining the non-existing features. An increasing number of scholars are being involved. They do not analyse the relevant data but only describe their impressions created by hyperbolic illusions. The participating researchers do not question the existence of the distinctly different stages of growth or of the postulated transition – they take them for granted and concentrate their attention only on the explanation of these phantom features, proposing new mechanisms, theories and mathematical descriptions without realizing that the apparent distinctly different two stages of growth do not exist and that there is no transition but a monotonically increasing hyperbolic distribution. Their mathematical descriptions, complicated and elaborate as they might be, are not the descriptions of the studied processes but rather the descriptions of phantom impressions created by hyperbolic illusions.

The perceived two stages of growth are commonly described as stagnation and sustained growth, while the perceived but non-existent transition as an escape, sprint, sudden spurt, intensification, acceleration, explosion or by some other similar terms all emphasizing a clear and dramatic change in the pattern of growth at a certain time. Variety of forces and mechanisms are then proposed to explain the phantom stages of growth and of the associated but non-existent transition. Efforts are also made to determine the precise time of the non-existent transition, often placing it around the Industrial Revolution but sometimes around 1950, without realizing that the determination of this time is impossible because there was no unusual acceleration at any particular time or over a certain range of time.

Hyperbolic processes are prone to misinterpretations and consequently they have to be analysed with care. Fortunately, their analysis is exceptionally simple. To show how to avoid being guided by hyperbolic illusions we shall describe the simple method of their analysis and illustrate it by a few examples.

The method of reciprocal values

Hyperbolic processes can be easily analysed using the method of reciprocal values. This method is so simple that it can be explained by using just two elementary equations, and yet so powerful that it can turn around and revolutionize such fields of research as the economic growth and the growth of human population, the important fields of study because for the first time in human existence we have now reached ecological limits of our planet and the correct understanding of these two processes is essential to avoid the undesirable unsustainable developments. We have to know how these processes work and how to control them. Incorrect interpretations are potentially dangerous and cannot be tolerated. Every effort has to be made to identify and eliminate any incorrect and misleading explanations.

The first-order hyperbolic distribution is described by the following simple equation:

$$S(t) = (a_0 + a_1 t)^{-1}, \tag{1}$$

where $S(t)$ is the size of a growing entity, while a_0 and a_1 are constants. For the hyperbolic growth, $a_1 < 0$.

Example of hyperbolic growth is shown in Figure 1. It represents the growth of the world population during the AD era. We can see that hyperbolic distribution describes well the growth of population during the entire range of data.

Data have to be analysed but in general they are not. Meticulous analysis of data is particularly important in the study of hyperbolic processes because they may be strongly misleading. They easily create an illusion of stagnation followed by explosion. Unfortunately, on seldom occasions when data are used and displayed, they are displayed in a grossely distorted and self-misleading way (Ashraf, 2009; Galor 2005a, 2005b, 2007, 2008a, 2008b, 2010, 2011, 2012a, 2012b; Galor & Moav, 2002; Snowdon & Galor, 2008) as shown in Figure 2.

Figure 2 was reproduced from Galor's publication (Galor, 2005a, p. 181). His figure was based on precisely the same source of data (Maddison, 2001) as used in Figure 1 but in this distorted way they show no resemblance of this original data. Such distortions were used repeatedly during the development of the Unified Growth Theory (Galor, 2005a, 2011) making it scientifically unacceptable, incorrect and unreliable. This Figure shows incorrectly that there was a long epoch of stagnation followed by a takeoff to a fast growth.

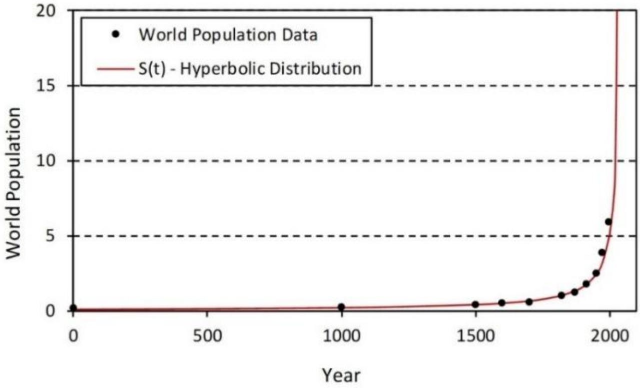

Figure 1. *Example of hyperbolic growth. Population data (Maddison, 2001) taken from the same source as used by Galor in his Unified Growth Theory (Galor, 2005a, 2011) are compared with hyperbolic distribution.*

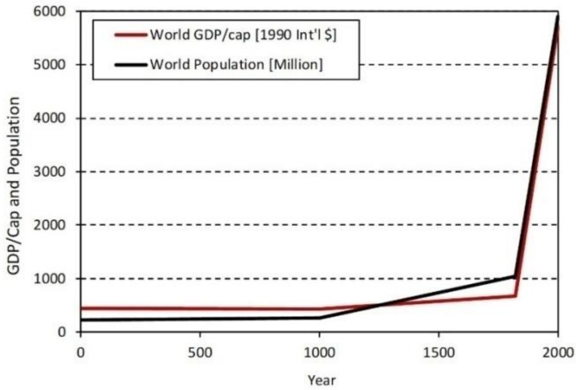

Figure 2. *Example of distorted representation of data used in academic literature (Ashraf, 2009; Galor 2005a, 2005b, 2007, 2008a, 2008b, 2010, 2011, 2012a, 2012b; Galor & Moav, 2002; Snowdon & Galor, 2008). Data presented in this figure come from precisely the same source (Maddison, 2001) as the data presented in Figure 1 but in this distorted way they bear no resemblance of the original data and they suggest incorrect interpretation of the mechanism of growth. Population is in millions and the GDP/cap in the 1990 International Geary-Khamis dollars.*

In discussions of the growth of population or of the economic growth it is easy to use some selected numbers and show that the growth was slow over a long time and fast over a short time. The slow growth is then interpreted as stagnation controlled by random forces of growth usually associated with Malthusian positive checks. The fast growth is interpreted as explosion controlled by distinctly different forces. The triggering mechanism of the supposed explosion is usually claimed to have been associated with the Industrial Revolution and Galor conveniently locates this supposed takeoff time around the time of the Industrial Revolution. Of course the growth was slow over a long time and fast over a short time because it was hyperbolic. It was not because there was stagnation followed by a takeoff or explosion leading to a new type of growth.

Hyperbolic distribution shown in Figure 1 is described by the eqn (1) with the following parameters $a_0 = 8.724$ and $a_1 = -4.267 \times 10^{-3}$. The fit to the data is remarkably good. Details of analysis are described in a separate publication (Nielsen, 2016a). They show that there was a major transition from a fast hyperbolic growth to a slow hyperbolic growth around AD 1 and that there was a minor disturbance around AD 1300. However, these details are of no concern to us in our present discussion. What is important to notice is that the growth of human population was indeed slow over a long time and fast over a short time but that these features are described remarkably well by a *single* hyperbolic distribution. These features represent nothing more than mathematical properties of hyperbolic distribution. They represent a *single* mechanism of growth.

It is important to point out that hyperbolic distribution increases monotonically. It makes no sense to divide it into two or three components and assign different mechanisms of growth to each perceived component. Hyperbolic distribution cannot and should not be divided into separate components and the best way to see it is to plot their reciprocal values $[S(t)]^{-1}$ because they convert hyperbolic distribution to a straight line:

$$[S(t)]^{-1} = a_0 + a_1 t \qquad (2)$$

Reciprocal values of hyperbolic distribution shown in Figure 1 are plotted in Figure 3. It is precisely the same distribution as shown in Figure 1 but it is presented in a different way. The confusing features such as the apparent stagnation followed by a takeoff to a fast growth increasing to infinity are replace by a clear straight line, which is easy to understand. It is obvious now that it would make no sense to divide such a straight line into distinctly different components and to claim distinctly different mechanisms of growth. It is also clear that it is impossible to identify a transition from a slow to a fast growth for hyperbolic distributions. There is no transition at any time. The transition occurs gradually over the entire range of growth. It is impossible to identify a takeoff time because there was no takeoff.

The display in Figure 3 is from AD 1000 for two reasons. (1) There is a large gap between AD 1 and 1000 so the display from AD 1000 shows better the agreement of the fitted hyperbolic distribution with data. (2) Detailed analysis of data for the AD and BC eras shows clearly that between around 500 BC and AD 500 there was a massive transition from a fast hyperbolic growth during the BC era to a significantly slower hyperbolic growth during the AD era (Nielsen, 2016a). The point at AD 1 is right in the middle of this transition and belongs to an entirely different distribution, the distribution describing the process of transition.

It should be also noticed that the point at AD 1000 in Figure 3 appears to be much further away from the fitted distribution then the point in Figure 1. The distributions are precisely the same but the display of reciprocal values magnifies the discrepancies between data and the calculated curve for small values (Nielsen, 2016b). The smaller are the values of the data and of the calculated distribution the larger is the magnification.

Reciprocal values allow for a unique identification of the first-order hyperbolic distributions because only these distributions are represented then by straight lines. This representation allows also for an easy study of departures from hyperbolic growth because deviations from a straight line are easy to notice.

Properties of growth do not change by changing the display of data but certain features, which are difficult or even impossible to recognize in one display can be easily identified in another. It is essential to remember that in the display of reciprocal values effects are reversed. Thus, for instance, a deviation to a slower trajectory will be indicated by an *upward* bending and deviation to a faster trajectory by a *downward* bending. An increasing growth is represented by a decreasing trajectory of the reciprocal values.

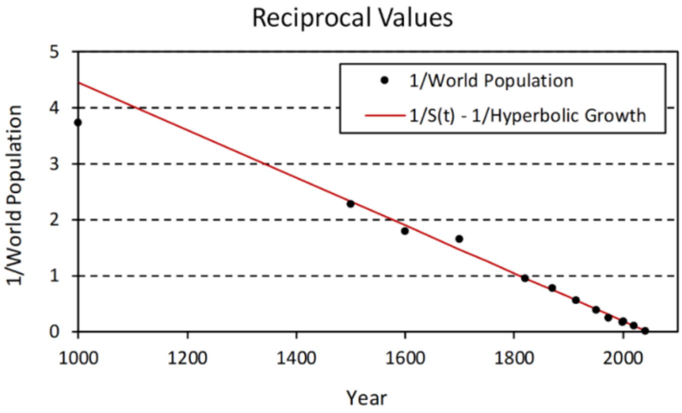

Figure 3. *Reciprocal values of the hyperbolic distribution presented in Figure 1 together with the reciprocal values of the size of population. Complicated hyperbolic distribution is now represented by a simple straight line, which is easy to understand. The takeoff around 1800 shown in Figure 2 did not happen. The straight line cannot be divided into two distinctly different components making it clear that hyperbolic distribution shown in Figure 1 is also made of a single component. The slow and the fast growth shown in Figure 1 follow the same, monotonically-increasing distribution.*

When hyperbolic growth is represented by a mathematically generated and gradually changing curve, such as shown in Figure 1, it might be clear that there was no particular time when the growth changed from being nearly horizontal to nearly vertical, but when data represented by discrete points are displayed, such a conclusion might be less obvious. The illusion becomes particularly strong when only a few strategically located points are selected (Ashraf, 2009; Galor 2005a, 2005b, 2007, 2008a, 2008b, 2010, 2011, 2012a, 2012b; Galor & Moav, 2002; Snowdon & Galor, 2008) from a significantly larger set of data as if to make the deception even more pronounced. Even if the enforcement of the perceived illusion is unintended, such crude displays of data lead readily to grossly incorrect interpretations.

However, if reciprocal values of data are displayed, their analysis is immediately made significantly simpler because if the data follow a simple, first-order hyperbolic distribution, their reciprocal values will be clearly aligned along a decreasing straight line. It is then obvious that dividing such a straight line into two sections and claiming two distinctly different regimes of growth governed by two distinctly different mechanisms simply makes no sense. It also makes no sense to try to locate a point on the decreasing straight line and claim a transition to a new trajectory because there is obviously no transition to a new trajectory on a decreasing straight line.

It should be stressed that in this representation only the first-order hyperbolic distributions describing growth will follow the decreasing straight-line trajectories. It is for this reason that this simple method is so useful in identifying the first-order hyperbolic distributions. It is a simple and yet powerful method, which can be used successfully in the analysis of data describing the historical economic growth and the growth of human population, global, regional or local, because in general they follow simple, first-order hyperbolic trajectories. Any deviations from such trajectories can be easily investigated. Higher-order hyperbolic distributions describing growth will be represented by gradually decreasing trajectories, which could be fitted using higher-order polynomial functions intercepting the horizontal axis, while the exponential growth will be represented by a decreasing exponential function.

This method might have a more general application but its specifically intended application described in this publication is to help to avoid being guided by hyperbolic illusions, the unfortunate common mistake, which often leads to seriously incorrect conclusions as we shall son demonstrate.

Going beyond the intended application, the first-order decreasing hyperbolic distributions will be represented by the increasing straight lines. Again, in this representation, any deviation from the decreasing hyperbolic distributions can be easily detected and investigated. Pareto distributions, which resemble the decreasing hyperbolic distributions, will be represented by gradually increasing functions, which in this representation might be also easier to investigate.

We shall now illustrate the application of the method of reciprocal values by using three additional examples: the growth of human population in Africa, the economic growth in Western Europe and the world economic growth.

Further examples

Growth of population in Africa

The method of reciprocal values can be used to study fine details of growth trajectories, the study which can then be used not only to improve the fit to data but also to understand the mechanism of growth. Some distributions might be made of different components, which could be difficult or even impossible to see in the direct display of data but they could be easily revealed by displaying their reciprocal values. An excellent example is the growth of human population in Africa shown in Figure 4, constructed using Maddison's data (Maddison, 2010). These Figure illustrates the added advantage of using the reciprocal values of data.

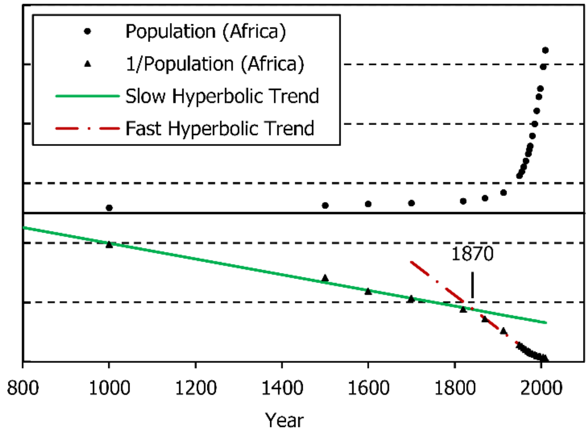

Figure 4. *Growth of human population in Africa (Maddison, 2010) illustrates how the method of reciprocal values can serve as an excellent tool in revealing hidden features of studied distributions.*

The top panel in Figure 4 contains the direct display of data for the growth of human population in Africa. The displayed shape suggests hyperbolic growth because it is slow over a long time and fast over a short time.

However, the reciprocal values of data presented in the lower panel reveal that the growth trajectory is in fact made of two major components: a slow hyperbolic distribution until around 1870 and a fast hyperbolic distribution after that year. Parameters describing the two hyperbolic components are $a_0 = 5.105 \times 10^1$, $a_1 = -2.036 \times 10^{-2}$ for the slow component and $a_0 = 1.705 \times 10^2$, $a_1 = -8.515 \times 10^{-2}$ for the fast component.

Figure 4 shows also that at a later stage, the fast hyperbolic growth started to be diverted to a slower trajectory as indicated by the upward bending of the trajectory representing the reciprocal values. Furthermore, it is now clear that the growth of population in Africa was never stagnant and that there was never a transition from stagnation to growth. The first stage of growth was hyperbolic and the transition around 1870 was a transition from hyperbolic growth to another hyperbolic growth. All these features, which are unrecognisable in the direct display of data are clearly seen in the display of the reciprocal values.

The pattern revealed by data contradicts the traditional interpretations of the mechanism growth of human population. There was no escape from the Malthusian trap because there was obviously no trap in the growth of population. The growth was slow but it was increasing monotonically with no signs of restrictions imposed by a mythical trap.

The transition from a slow to a fast hyperbolic growth in Africa occurred around the time of the Industrial Revolution but it was not a transition from a stagnant growth to a new, so-called sustained growth regime (Galor, 2005a, 2011) but from a hyperbolic growth to another but faster hyperbolic growth. It was the boosting that coincides with the intensified colonisation of Africa (Duignan & Gunn, 1973; McKay, *et al.,* 2012; Pakenham, 1992).

Contrary to the commonly accepted interpretations, this boosting in the growth of population was *not* triggered by a dramatically *decreased* intensity of Malthusian positive checks but by their dramatic *escalation*. It is clear that the accepted interpretations of the effects of Malthusian positive checks are incorrect. Their increased intensity does not lead to stagnation but to a more intensified growth (Malthus, 1798; Nielsen, 2016c). The increased intensity of Malthusian positive checks increases the mortality rate but it also increases the fertility rate with the net result of increasing the rate of natural increase or the growth rate. This correlation is also clearly demonstrated even now by the growth of population in poor countries. The poorer they are the faster is the growth of their populations. Thus, this simple analysis of data assisted by using the reciprocal values already questions the commonly accepted interpretations of the mechanism of growth of human population.

As shown in Figure 4, reciprocal values of data reveal the details of the mechanism of growth, which were impossible to identify by the direct display of data. Even if we cannot yet fully explain these details, we can already see that the growth of the populations in Africa was following a slow hyperbolic trend until around 1870. Around that year, the growth of human population in Africa experienced an unprecedented 4-fold acceleration, which diverted the growth into a significantly faster hyperbolic trajectory. The fast-hyperbolic growth continued until around 1975 when it started to be diverted to a new but slower trend.

It is this pattern of growth that we have to explain. It is for this pattern of growth that we have to propose the mechanism of growth. It is not the imaginary pattern of stagnation followed by explosion. It is not the fictitious Malthusian regime followed by the mythical takeoff from stagnation to an imagined sustained growth regime (Galor, 2005a, 2011). It is an entirely different pattern, the pattern indicated by the close analysis of data rather than by the pure fantasy. The aim of scientific investigation is not to explain figments of imagination but the evidence presented by data.

Data are essential in scientific investigations. Assisted by data we shall not be guided by the erroneous concept of stagnation but by the clear evidence of hyperbolic growth. We shall also not be guided by the erroneous concept of a takeoff from stagnation to a sustained growth regime but by the clear evidence of a transition from a hyperbolic growth to another hyperbolic growth. We shall also be guided by an observation that at a certain stage, around 1975, the long-lasting pattern of hyperbolic growth has been eventually abandoned and the growth was diverted to an entirely different trajectory.

Economic growth in Western Europe

Economic growth is measured using the Gross Domestic Product (GDP) or the GDP per capita (GDP/cap). Galor & Moav (2002) studied economic growth in Western Europe using the data of Maddison (Maddison, 2001). They have selected a few, strategically located points from a larger set of data, joined them by straight lines and concluded that there were two distinctly different regimes of growth: the "Malthusian regime" (also labelled as the "epoch of stagnation," "Malthusian era," "Malthusian epoch," "Malthusian steady-state equilibrium," "Malthusian stagnation" or "Malthusian trap") and the "sustained economic growth" (described also as the "Modern Growth Regime," "sustained economic growth" and "sustained growth regime"). Their distorted representation of Maddison's data is shown in Figure 5.

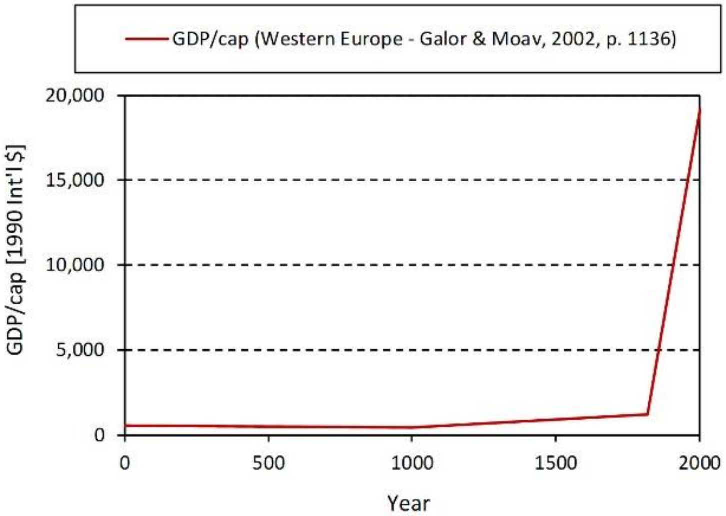

Figure 5. *A typically distorted and self-misleading representation (Galor & Moav, 2002, p. 1136) of Maddison's data (Maddison, 2001). Compare it with exactly the same data, but not distorted, presented in Figure 7.*

Referring to this crude display of data they also concluded that the Industrial Revolution had a strong impact on the economic growth causing a dramatic takeoff from stagnation to a fast growth. They made no attempt to analyse mathematically Maddison's data (Maddison, 2001) but presented a series of mathematical equations describing their imaginations, which were neither related to nor supported by the source of data they have used.

It is remarkable that data coming from *precisely the same source* as they have used contradict their claims and their interpretations of growth. Extensive analysis of the GDP/cap data, global and regional, is presented in a separate publication (Nielsen, 2016d). It is shown there that GDP/cap data follow the *monotonically increasing* trajectories. They are just the linearly modulated hyperbolic trajectories (Nielsen, 2017a), i.e. hyperbolic trajectories modulated by the linear time-dependence of the reciprocal values of the size of population. There is no stagnation and no takeoff to a distinctly different regime of growth. Both, the GDP and the population increase hyperbolically (Nielsen, 2016b, 2016e, 2016f) and thus monotonically. Consequently, their ratios increase also monotonically.

Figure 6 presents the reciprocal values of the Gross Domestic Product (GDP) for Western Europe (Maddison, 2001) in the vicinity of the supposed takeoff. The data are well aligned along a decreasing straight line, which means that they were following the simplest, first-order, hyperbolic distribution given by the eqn (1).

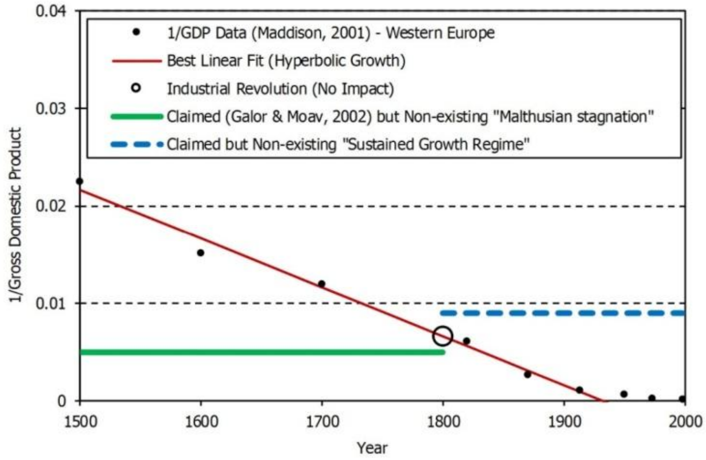

Figure 6. *Reciprocal values of data describing the Gross Domestic Product (GDP) in Western Europe (Maddison, 2001) in the vicinity of the Industrial Revolution. This is precisely the same source of data as used by Galor & Moav (2002) to construct their distorted representation shown in Figure 5. Contrary to their claim, Industrial Revolution had no effect on shaping the economic growth trajectory in Western Europe, the centre of this revolution. The two regimes of growth claimed by them also did not exist. The GDP is in billions of the 1990 International Geary-Khamis dollars.*

Industrial Revolution was between 1760 and 1840 (Floud & McCloskey, 1994), or around 1800 as shown in Figure 6. This figure demonstrates clearly and convincingly that the claimed takeoff around the time of the Industrial Revolution did not happen because the reciprocal values of the GDP data follow an undisturbed straight line trajectory representing an undisturbed hyperbolic growth. It is now clear that there was no takeoff and no escape, great or small, from the hypothetical but non-existing Malthusian trap, at least from the supposed trap in the economic growth. Maybe there were some other traps but maybe they are just figments of imagination. It is clear, however, that Industrial Revolution had absolutely no impact on shaping the economic growth trajectory in Western Europe, the centre of this revolution.

Industrial Revolution had, no doubt, many other impacts but they are not reflected in the economic growth trajectory. Their study could be important and interesting but they will not explain the growth of the GDP. The mechanism of growth was immune to the changes introduced by the Industrial Revolution. Whatever dramatic changes the Industrial Revolution might have introduced to the general style of living, to technology and even to the economic marked, these changes obviously were not shaping the economic growth trajectory.

The absence of a takeoff eliminates also the need for assuming the existence of two distinctly different regimes of growth. It obviously makes no sense to divide the straight line into two arbitrarily selected sections and claim distinctly different trajectories governed by distinctly different mechanisms of growth. What might not have been clear in the direct display of data, is now perfectly obvious if we display the reciprocal values of data. This display abolishes all elaborate theories and untidy explanations incorporating such concepts as traps, escapes, takeoffs and stagnation and replaces them by a simple interpretation of the mechanism of growth suggested by the simple equation describing hyperbolic growth. This conclusion is in agreement with the general observation that natural phenomena can be usually explained by using simple descriptions.

In Figure 7, the hyperbolic trajectory corresponding to the straight line shown in Figure 6 is extended to AD 1. The economic growth in Western Europe is well described by a simple, first-order, hyperbolic distribution. The corresponding parameters are: $a_0 = 9.697 \times 10^{-2}$ and $a_1 = -5.020 \times 10^{-5}$. The point at 1950 is not fitted by the hyperbolic trend because from the early 1900s the economic growth in Western Europe started to be diverted to a *slower* trajectory, which is again contrary to the claimed boosting or a transition from stagnation to growth. There was a transition but it was a transition from a *monotonically increasing* hyperbolic growth to a *slower* trajectory.

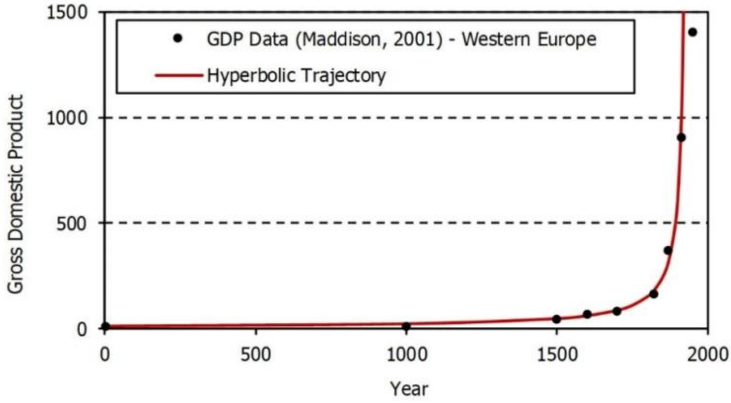

Figure 7. *Data for the Gross Domestic Product (GDP) in Western Europe (Maddison, 2001) compared with the monotonically increasing hyperbolic distribution. The GDP is in billions of the 1990 International Geary-Khamis dollars.*

We cannot claim that the growth was sustained only after the Industrial Revolution because it was sustained equally strongly during the postulated but non-existent "epoch of stagnation." Figure 6 and Figure 7 show clearly that the concept of two stages of growth is unsupported by data. When stripped of the hyperbolic illusions, the economic growth is revealed as a simple process, which can be described using just one, simple mathematical trajectory until the early 1900s when it started to be diverted to a slower, non-hyperbolic, trajectory. There is no compelling need to make this simple description complicated.

Growth of the GDP was slow in the past because it was hyperbolic. However, while being slow it was not stagnant. The growth was fast in recent years because it was hyperbolic. It followed the same undisturbed hyperbolic distribution as in the past.

We now have a completely different understanding of the economic growth in Western Europe, an important turnaround in the economic research. Rather than wasting the valuable time, energy and financial resources on trying to explain the phantom features created by hyperbolic illusions and magnified by the customary crude representation of data (Ashraf, 2009; Galor 2005a, 2005b, 2007, 2008a, 2008b, 2010, 2011, 2012a, 2012b; Galor & Moav, 2002; Snowdon & Galor, 2008) we can now focus our attention on the relevant task of trying to explain why the economic growth was so stable over such a long time and why it was hyperbolic. Rather than writing numerous articles based on impressions and publishing them in peer-reviewed scientific journals and in academic books we can now concentrate our attention on the understanding of *the science* of economic growth. In our investigations, we shall not be guided by impressions, we shall not be guided by the customary crude representations of data but by their rigorous mathematical analysis

Global economic growth

Another example of the application of the method of reciprocal values is the global economic growth. It is an important example because it questions Galor's Unified Growth Theory (Galor, 2005a, 2011) representing the culmination of his work extending over 20 years (Baum, 2011). His theory is based on an uncritical acceptance of the common interpretations, descriptions and explanations used in the economic and demographic research. In this sense, his theory offers no new insights.

The fundamental postulate of this theory is again the existence of three regimes of growth: the slow and stagnant Malthusian Regime, the short and intermediary Post-Malthusian Regime and the fast, Sustained Growth Regime. Galor also accepts that Industrial Revolution played a crucial role in the supposed dramatic takeoff from a prolonged stagnation into a rapid and sustained growth.

The welcome initiative in his theory is that he makes an attempt of using repeatedly Maddison's data (Maddison, 2001). However, he makes not even a single attempt to test his theory by the rigorous analysis of data. This is a serious mistake. The usual practice in any scientific theory is to test it by data or at least to suggest how it can be tested by data. Galor does not follow this accepted practice. He does not test his mathematical descriptions by data. Data are used repeatedly but they are never analysed. They are presented in a typically distorted way, as illustrated in Figures 2 and 5, and in this distorted way they seem to support the preconceived ideas. His work is based on prejudice and no attempt is made to check its validity.

When data are used but manipulated to confirm preconceived ideas we are not dealing with science. We also make no progress and we are not learning anything new or useful.

We shall now use exactly *the same source of data* and show that the Unified Growth Theory is scientifically unacceptable. For more extensive discussion of these issues see other publications

(Nielsen, 2016a, 2016b, 2016d, 2016e, 2016f, 2016g, 2016h, 2016i, 2016j, 2017a).

It is hard to see how much can be rescued from Galor's Unified Growth Theory. It is hard to see how many of his descriptions and explanations are based on pure and unsubstantiated speculations. His theory would have to be minutely analysed. However, its major premises are untenable. All his "mind boggling" "mysteries of the growth process" (Galor, 2005a, p 220), for instance, can be easily explained (Nielsen, 2016a, 2016d, 2016g, 2016i) – *there are no mysteries*. All his mysteries were created by his repeatedly distorted presentations of data coming from a reputable source (Maddison, 2001), the data used during the formulation of his theory but never properly analysed.

His theory certainly does not explain the mechanism of growth because it revolves around the descriptions of hyperbolic illusions. It does not even describe economic growth. His descriptions are incorrect because again they are based on the distorted presentations of data and on the unsubstantiated prejudice.

Theories come and go. Scientific integrity is not tarnished by proposing incorrect explanations and interpretations but by refusing to correct them or to reject them when they are contradicted by reliable data.

Reciprocal values of data for the world Gross Domestic Product (GDP) (Maddison, 2001) are shown in Figure 8. They follow closely a decreasing straight line, which means that the economic growth was increasing hyperbolically. It is clear that there was no takeoff of any kind, large or small, around the time of the Industrial Revolution and no repeatedly claimed great escape from the postulated but non-existing Malthusian trap. The data do not support the existence of the three regimes of growth and thus contradict the fundamental postulates of the Unified Growth Theory.

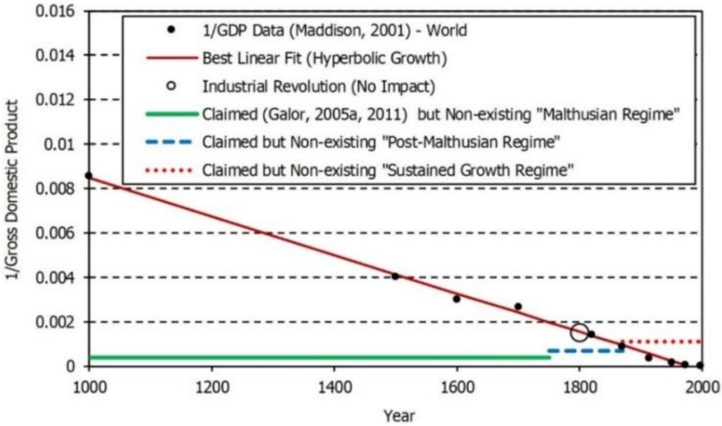

Figure 8. *Fundamental concepts of the Unified Growth Theory are contradicted by precisely the same data (Maddison, 2001), which were used (but never analysed) during its development. Reciprocal values of data follow closely a decreasing linear distribution representing a monotonically increasing hyperbolic growth. The three regimes of growth claimed by Galor (2005a, 2011) did not exist. There was no takeoff around the time of the Industrial Revolution or around any other time. The monotonically increasing hyperbolic growth remained undisturbed until the 1990s. The GDP is in billions of the 1990 International Geary-Khamis dollars.*

The last point of the data shown in Figure 8 is not fitted by the straight line, suggesting a possible diversion to a slower trajectory. This region can be studied more closely using the extended compilation of the economic growth data (Maddison, 2010). Their reciprocal values between 1700 and 2003 are shown in Figure 9 demonstrating clearly that while the Unified Growth Theory claims an unusually accelerated growth after the supposed but non-existent epoch of stagnation, the data show the opposite behaviour: a diversion to a *slower* trajectory after the earlier vigorous, well-sustained and secure economic growth. Rather than being boosted by the Industrial Revolution, the economic growth continued along the *undisturbed* hyperbolic trajectory for about one hundred years after this revolution and then started to be diverted to a *slower* trajectory.

Figure 9 illustrates again how the method of reciprocal values can unravel useful details about a studied process. Not only does it help in an unambiguous and easy identification of hyperbolic distributions but also it helps in an easy detection of deviations from such distributions. The world economic growth continues to increase but from the early 1900s it started to be diverted away from the faster accelerating historical hyperbolic trajectory to a slower trend.

The point of intersection of the reciprocal values with the horizontal axis is the point of singularity when the growth escapes to infinity. No growth can go beyond this point and any growth close to it may become unstable, unsustainable and catastrophic. Figures 8 and 9 show how close we are now to the point of the potential global economic instability and unsustainability.

Unified Growth Theory claims that after a long epoch of stagnation we have now reached an era of "sustained economic growth," the term repeated 82 times in the first detailed formulation of this theory (Galor, 2005a), the potentially misleading description because while it is true that the current economic growth is still sustained the past economic growth was not only sustained but also it was increasing along a more secure trajectory, far away from the point of singularity. Even though the growth is now diverted to a slower trajectory any further increase can be potentially dangerous.

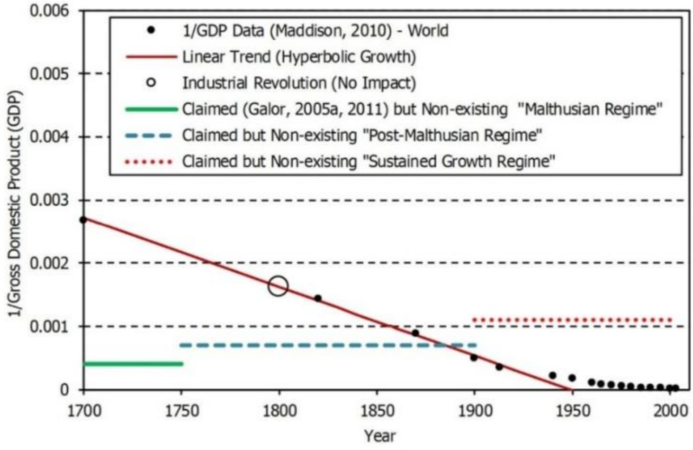

Figure 9. *Maddison's data (Maddison, 2010) show clearly that while the Unified Growth Theory (Galor, 2005a, 2011) claims a transition from stagnation to a vigorous growth, the data show the opposite behaviour: a transition from a vigorous hyperbolic growth to a slower trajectory, as indicated by the upward bending of the growth trajectory of the reciprocal values during the 1990s and 2000s. There was no stagnation and no boosting in the economic growth at any time. The claimed (Galor, 2005a, 2011) but non-existent three regimes of growth are also shown. Their existence is contradicted by data. The GDP is in billions of the 1990 International Geary-Khamis dollars.*

Reciprocal values of data show that for the first time during the AD era, and probably for the first time in human existence, we are now trapped between the already high level of economic growth and a point of no return, or equivalently between the very small reciprocal values of the GDP and zero. Any intrusion into this narrow gap has to be closely monitored. Even if the trend of the reciprocal values of the GDP data does not cross the horizontal axis any close approach to this axis could be dangerous, because it could trigger global economic instability and even a possible global economic collapse.

This simple analysis of data shows how dangerous are the generally accepted postulates presented in the Unified Growth Theory. The concept of a transition from stagnation to the so-called sustained growth regimes suggests that now for the first time in human history we can enjoy the sustained economic growth. Data, however, reveal a diametrically different pattern of growth. It was in the past that the economic growth was sustainable because it was following a stable hyperbolic distribution, which was still far away from the point of singularity. Now, however, the reciprocal values of the GDP are so close to zero, i.e. to the point when the GDP escapes to infinity, that the economic growth is no longer easily sustainable. The possibility of a serious economic instability is real. Now, the economic growth has to be closely monitored and controlled. The claim that we are now in the regime of sustained economic growth is dangerously inaccurate and misleading.

The method of reciprocal values can be also used to demonstrate that two other postulates of the Unified Growth Theory, the postulate of the differential takeoffs and the postulate of the great divergence, are contradicted by the mathematical analysis of data coming from *the same source,* which was used during the formulation of this theory (Nielsen, 2016b, 2016e, 2016i). Takeoffs never happened and consequently it makes no sense to claim that they happened at different times for developed and developing regions. The so called great divergence also never happened. Different regions are on different levels of development but they follow closely similar trajectories. They are like athletes running along similar tracks. They do not run in distinctly different directions as incorrectly claimed in the Unified Growth Theory but in the same direction.

If the economic growth continued along the historical hyperbolic trajectory it would have already reached a point of no return as indicated by the fitted straight line crossing the horizontal axis. To use the colourful description of von Foerster, Mora &

Amiot (1960), we have been saved from experiencing a doomsday in the global economic growth. However, the danger of an excessive and unsustainable growth is still not averted.

Under a suitable control, the economic growth can continue for a long time, but this is precisely the important point: from now on the economic growth has to be closely monitored and controlled because it can easily become unsustainable.

Data between 1965 and 2003 follow closely exponential trajectory. Exponential growth does not increase to infinity at a fixed time but this is hardly any consolation because eventually such a growth also becomes unsustainable.

Any other continually increasing growth can be unsustainable unless it is increasing to a certain constant asymptotic value. However, it is extremely difficult to control such a growth because the growth rate would have to finely tuned to decrease slowly to zero. A constant growth rate, even if small, would represent the undesirable exponential growth. A growth rate fluctuating around zero would be safe but our general tendency is to try to increase the growth rate or at least to keep it constant, both options leading to unsustainable economic growth.

Data describing the world economic growth (Maddison, 2001) are compared in Figure 10 with the hyperbolic trajectory calculated using the straight-line fitted to the reciprocal values shown in Figure 8. Parameters describing the historical hyperbolic growth of the world GDP are: $a_0 = 1.716 \times 10^{-2}$ and $a_1 = -8.671 \times 10^{-6}$.

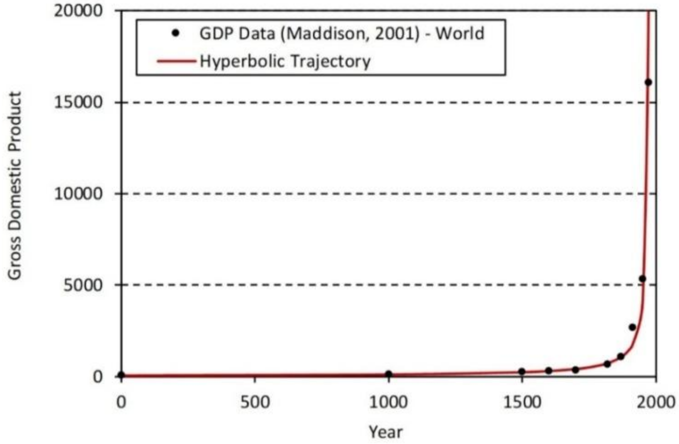

Figure 10. *The data for the world Gross Domestic Product (GDP) (Maddison, 2001) follow closely the first-order hyperbolic distribution. The claimed three regimes of growth (Galor, 2005a, 2011) did not exist. They are replaced by an uninterrupted and monotonically increasing hyperbolic growth. The GDP is in billions of the 1990 International Geary-Khamis dollars.*

Now the puzzling features of the economic growth, the features that prompted so many discussions in numerous peer-reviewed scientific journals culminating in the formulation of the Unified Growth Theory, are manifestly clear, and their explanation is surprisingly simple. Over hundreds of years, the world economic growth was slow because it was hyperbolic. Over a short time, until the early 1900s, the economic growth was fast because it was hyperbolic – it followed *the same* undisturbed hyperbolic trajectory as in the past. The apparent transition from a slow to a fast growth is just an illusion created by the hyperbolic distribution. There was no unusually accelerated transition from the slow to the fast economic growth. The acceleration was gradual over the entire range of time.

The study presented here shows how important it is to have a clear understanding of the economic growth and how the simple method of reciprocal values can assist in such studies. Application of this method can not only assist in unravelling different components of growth trajectories but also to avoid being guided by hyperbolic illusions, which are the source of numerous misinterpretations of economic growth and of the growth of population culminating in the formulation of the fundamentally flawed and strongly misleading Unified Growth Theory (Galor, 2005a, 2011).

Summary and conclusions

We have described a simple but effective method of analysis of hyperbolic distributions and we have explained its application by using the growth of the world population during the AD era. We have then demonstrated the flexibility of this method by using an example of the growth of human population in Africa. This method can be used to identify uniquely the first order hyperbolic distributions, to reveal hidden components of growth trajectories and to remove hyperbolic illusions, which are the source of numerous misinterpretations of economic growth and of the growth of population, misinterpretations prevailing over a long time in academic literature. This simple method redirects the economic and demographic research from explanations of phantom features created by hyperbolic illusions, to explanations based on the scientific analysis of data.

We have presented two other examples of analysis of data: the economic growth in Western Europe and the global economic growth. All these four examples show that the rigorous analysis of data contradicts the established knowledge in demography and in the economic research, and in particular, that it contradicts the fundamental postulates of the Unified Growth Theory (Galor, 2005a, 2011). However, what we have presented here is just a tip of an iceberg. An entirely new world is opened when we analyse more data (Nielsen, 2016a, 2016b, 2016d, 2016e, 2016f, 2016g, 2016h, 2016i, 2016j, 2016k, 2016l, 2016m, 2016n, 2016o, 2016p, 2017a, 2017b, 2017c, 2017d), the world without stagnation in the economic growth and in the growth of population, without takeoffs from the supposed stagnation to growth, the world without complicated and untidy explanations of the mechanism of growth but the elegant world where data can be described by simple mathematical distributions, the world, which opens up new vistas for the demographic and economic research.

Impressions can be strongly deceptive and persuasive. “It is clear that the earth does not move, and that it does not lie elsewhere than at the centre” declared Aristotle. Fortunately, however, in science, incorrect interpretations are sooner or later corrected.

1. *Stagnation*. Research based on impressions and reinforced by the customary crude and self-misleading representations of data (Ashraf, 2009; Galor 2005a, 2005b, 2007, 2008a, 2008b, 2010, 2011, 2012a, 2012b; Galor & Moav, 2002; Snowdon & Galor, 2008), such as shown in Figures 2 and 5, seems to confirm the generally accepted belief that there was an epoch of stagnation in the economic growth and in the growth of population. Scientific analysis of *precisely the same* (but undistorted) data demonstrates that there was no stagnation and that the economic growth and the growth of population followed *monotonically increasing hyperbolic distributions*.
2. *Takeoffs*. Research based on impressions seems to indicate that there was a transition from stagnation to growth described usually as a takeoff or explosion. Scientific analysis of precisely the same (but undistorted) data demonstrates that there was no takeoff or explosion and that economic growth and the growth of population continued to follow the monotonically increasing hyperbolic distributions. *What appears as a takeoff or explosion is in fact the natural continuation of hyperbolic growth.*
3. *Industrial Revolution.* Research based on impressions seems to indicate that Industrial Revolution played a crucial role in the economic growth and in the growth of population causing a dramatic acceleration (boosting) in the growth trajectories, described as takeoffs. Scientific analysis of precisely the same (but undistorted) data demonstrates that *Industrial Revolution had absolutely no impact on shaping growth trajectories*. Industrial Revolution can be linked to other impacts but not to shaping the population or the economic growth trajectories. This might be surprising but the evidence in data is undisputable and we have to accept it.
4. *Regimes of growth.* Research based on impressions seems to suggest that there were two or maybe even three distinctly different regimes of growth governed by distinctly different mechanisms (Galor, 2005a, 2011). Scientific analysis of precisely the same (but undistorted) data demonstrates that these two or three distinctly *different regimes of growth did not exist*. The growth was hyperbolic until recently when it started to be diverted to *slower* trajectories.
5. *Mysteries*. Research based on impressions resulted in claiming a series of "mind-boggling" and "perplexing" "mysteries of the growth process" (Galor, 2005a, pp. 177, 220). Scientific analysis of precisely the same data demonstrates that all these mysteries belong to the world of fiction created by a good

dose of fantasy guided by the misleading impressions and reinforced by the customarily distorted presentations of data (Ashraf, 2009; Galor 2005a, 2005b, 2007, 2008a, 2008b, 2010, 2011, 2012a, 2012b; Galor & Moav, 2002; Snowdon & Galor, 2008) such as shown in Figures 2 and 5. Science is supported by a methodical analysis of data. There are no mysteries when precisely the same data are properly analysed.

In particular, the mystery of the great divergence is explained: there was no great divergence (Nielsen, 2016i). Various regions are on different levels of economic growth but they all follow closely similar trajectories. Their economic growth did not diverge into distinctly different trajectories as incorrectly suggested by the crude representations of data.

The mystery of the supposed sudden spike in the growth rate of income per capita has been explained: there was no sudden spike (Nielsen, 2016g). The growth rate of income per capita followed a monotonically increasing trajectory, which is readily represented by a mathematical distribution derived using hyperbolic growth for the growth of the GDP and for the growth of population.

The mystery of the puzzling features of income per capita has been explained (Nielsen, 2017a). The distribution representing income per capita is nothing more than just a linearly modulated hyperbolic distribution. It reflects nothing more than the purely mathematical property of dividing two hyperbolic distributions.

Other questions listed by Galor as representing the mysteries of the growth process can be easily answered. They refer to features that do not exist, features based on impressions reinforced by ineffectual handling of empirical evidence. They are in the same category as the question "Why does the sun revolve around the earth?"

6. *Mechanism*. Research based on impressions leads to proposing numerous complicated mechanisms of growth. Scientific analysis of data shows that the mechanism of growth is exceptionally *simple* (Nielsen, 2016p), which is hardly surprising because hyperbolic distributions are described by an exceptionally simple equation [see eqn (1)].
7. *Unified Growth Theory.* Research based on impressions prompted the development of a Unified Growth Theory (Galor, 2005a, 2011). Mathematical analysis shows that the fundamental postulates of this theory are *contradicted by the same data,* which were used during its development. Galor could have saved 20 years of his life and could have directed

his academic skills to developing a useful theory if he did what any scientist is supposed to do: if he based his deductions and explanations on a scientific analysis of data. He had access to excellent data but he did not analyse them. He was guided by preconceived ideas and he supported them by distorted presentations of data.

The analysis data suggests new lines of research. Thus, for instance, the relevant question is not why the historical economic growth was so unstable in the past or what caused the perceived transition from supposed stagnation to growth but *why the economic growth was so remarkably stable* in the past. The same question applies to the growth of population but it was already answered (Nielsen, 2016c, 2017d). The growth of population was remarkably stable because of the combination of the generally low impacts of demographic catastrophes (at least on the global and regional scales) and the high level of human resilience expressed in the efficient process of regeneration (Malthus, 1798; Nielsen, 2016c). If we accept that there is a close relationship between the growth of population and the economic growth, then the question about the stability of the historical economic growth has been also already answered. However, it is possible that some new insights could be still added to this explanation.

The relevant question is not why the Industrial Revolution and the unprecedented technological development boosted the economic growth because they did not. The relevant question is *why the Industrial Revolution and the unprecedented technological development did not boost the economic growth*. Why these apparently strong technological and socio-economic forces had no impact on shaping the economic growth trajectories.

The relevant question is not why the economic growth increased so fast in modern time, because we have shown that this fast increase was just the natural continuation of the monotonically increasing hyperbolic growth until in recent years it started to be diverted to a *slower* but still fast-increasing trajectory. The relevant question is *why the economic growth was diverted to a slower trajectory*. What new force or forces were so strong that they were able to overpower the historically strong force of growth. Another relevant question is also whether this new trajectory is likely to develop into a historically preferable and potentially catastrophic, hyperbolic growth. Furthermore, the relevant question is how to control the current fast economic growth. The same question applies also to the growth of population but it was at least partly answered in the study of the effects of Malthusian positive checks

(Nielsen, 2016c). The primary if not exclusive way of controlling the growth of human population is to improve the living conditions in developing countries.

The method of reciprocal values is so simple that it can be used by anyone and it is, therefore, expected that it will be of interest to many scientists who look for a simple method of analysis of empirical evidence, a method that does not involve any complicated mathematical formulae, any intricate mathematical algorisms or the use of powerful computers but a simple display of data and a remarkably simple fitting procedure. We have demonstrated that even a simple mathematical method can have a dramatic influence on scientific research.

It is essential to understand that by claiming that there was no stagnation in the economic growth or in the growth of population we are not claiming that there was no stagnation in the standard of living. We are only claiming that the two processes were decoupled. We might, if we insist, describe the past general living conditions as primitive or even stagnant, but there is no evidence that they were shaping the trajectories describing the growth of population or the economic growth.

It is also essential to understand that by claiming that there was no takeoff in the economic growth or in the growth of population we are not claiming that there was no takeoff in the technological development, or generally in the intellectual progress and in the dramatic changes in human experience and in living conditions. We are only claiming that these possible takeoffs had no impact on changing the economic growth trajectories or the trajectories describing the growth of population. There were no takeoffs in any of these two processes. Industrial Revolution can be linked with many changes in human living experience but all these changes had no impact on changing the economic or demographic growth trajectories.

There is no reason why scientific evidence presented here and in other related publications (Nielsen, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h, 2016i, 2016j, 2016k, 2016l, 2016m, 2016n, 2016o, 2016p, 2017a, 2017b, 2017c, 2017d) should not be accepted by the scientific community. The only alternative option is to reject data but this would be no longer science.

Even Galor and his associates accept the same data and use them in their research. Their unfortunate mistake was only in choosing to support their investigations by the grossly distorted and self-misleading representations of data (Ashraf, 2009; Galor 2005a, 2005b, 2007, 2008a, 2008b, 2010, 2011, 2012a, 2012b;

Galor & Moav, 2002; Snowdon & Galor, 2008). Consequently, the only way to reject scientific evidence and to accept the doctrines of stagnation and takeoffs, and all other associated erroneous explanations of the dynamics of the economic growth and of the growth of population, is to accept data but distort them in such a way as to make them to conform with preconceived ideas, but then again it is not science.

Evidence in data is overwhelming and leaves no room for accepting incorrect interpretations. In order to have progress in the demographic and economic research, incorrect interpretations of growth have to be abandoned and a new paradigm has to be developed. There is no other, scientifically justified, way. A serious mistake in scientific investigations is not in stumbling and in making mistakes but in refusing to learn from them and to correct them.

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3. Unified Growth Theory contradicted by the analysis of the Gross Domestic Product

Introduction

The latest publication of excellent data by the world-renown economist (Maddison, 2001; 2010) offers an unprecedented opportunity to study the mechanism of the historical economic growth. Earlier study (Nielsen, 2014), based on these data, indicated that historical economic growth can be described using hyperbolic distributions in much the same way as the growth of human population (von Foerster, Mora & Amiot, 1960). Unlike exponential growth, which is more familiar and which is easier to understand, hyperbolic distributions are strongly deceptive because they appear to be made of two distinctly different components, slow and fast, joined perhaps by a certain transition component. This illusion is so strong that even the most experienced researchers can be easily deceived particularly if their research is based on a limited body of data, as it was in the past. Fortunately, Maddison's data solve this problem, and fortunately also their analysis is trivially simple because, as pointed out earlier (Nielsen, 2014), hyperbolic distributions can be easily identified and analysed using the reciprocal values of data.

Hyperbolic distribution describing *growth* is represented by a *reciprocal* of a linear function:

$$S(t) = \frac{1}{a - kt}, \tag{1}$$

where $S(t)$ is the size of the growing entity, in our case the Gross Domestic Product (GDP), while a and k are *positive* constants.

The reciprocal of such hyperbolic growth, $1/S(t)$, is represented by a *decreasing linear* function:

$$\frac{1}{S(t)} = a - kt. \tag{2}$$

Hyperbolic *distributions* should not be confused with hyperbolic *functions* ($\sinh(t)$, $\cosh(t)$, etc). Furthermore, *reciprocal* functions should not be confused with *inverse* functions. Thus, for instance, for the expression given by the eqn (1) the objective of finding the inverse function would be to calculate time *t* for a given size $S(t)$. The roles of the dependent and independent variables would be reversed. For the reciprocal function, the objective is to convert eqn (1) into eqn (2). The roles of dependent and independent variables are not changed.

Reciprocal values help in an easy and generally unique identification of hyperbolic growth because in this representation hyperbolic growth is given by a decreasing straight line. Apart from serving as an alternative way to analyse data, reciprocal values allow also for the investigation of even small deviations from hyperbolic distributions because deviations from a straight line can be easily noticed.

Reciprocal values allow also for an easy identification of different components of growth. This property can be used, in comparing empirical information with theoretical interpretations (Galor, 2005; 2011), which are based on the assumption of the existence of different components of growth.

When comparing mathematically-calculated distributions with the reciprocal values of data, we have to remember that the sensitivity of the reciprocal values to small deviations increases with the decreasing size S of the growing entity.

Suppose we have two values of S at a given time: S_1 and S_2, representing, for instance, the empirical and calculated values. It is clear that

$$\Delta\left(\frac{1}{S}\right) = -\frac{\Delta S}{S_1 S_2}, \tag{3}$$

where $\Delta(1/S)$ is the difference between two inverse values and ΔS is the difference between S values.

For a given $|\Delta S|$, $|\Delta(1/S)|$ increases rapidly with the decreasing S_1 and S_2 values. The separation of small values of data from calculated distributions are magnified. Similar magnifications, though less pronounced, are also shown in the semilogarithmic displays of data. We shall use both displays to examine the quality of fits to the data.

It should be noted that the *decreasing* reciprocal values describe *growth*, while a deviation to *larger* reciprocal values describes decline. Consequently, a diversion to a faster trajectory will be indicated by a downward bending of a trajectory of the reciprocal values, away from an earlier observed trajectory, while the diversion to a slower trajectory will be indicated by an upward bending.

The data describing the historical economic growth (Maddison, 2001; 2010) do not allow for a detailed analysis below AD 1500 because there are two large gaps in the data: between AD 1 and 1000 and between AD 1000 and 1500. The best sets of data are from AD 1500. However, the compilation prepared by Magnuson appears to be the best and themost reliable source of the historical economic growth data.

Throughout the analysis presented here, the values of the Gross Domestic Product (GDP) will be expressed in billions of the 1990 International Geary-Khamis dollars. All diagrams are presented in the Appendix

Theories play an important role in scientific research because they crystallise interpretations of studied phenomena. However, theories have to be always tested by data. In science it is important to look for data confirming theoretical explanations but it is even more important to discover contradicting evidence, because data confirming a theory confirm only what we already know but contradicting evidence may lead to new discoveries.

Currently, the best and the most complete theory describing the mechanism of the historical economic growth is the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012). One of the fundamental postulates of this theory is the postulate of the existence of three regimes of growth governed by three distinctly different mechanisms: (1) the Malthusian regime of stagnation, (2) the post-Malthusian regime, and (3) the sustained-growth regime.

According to Galor (2005; 2008; 2011; 2012), Malthusian regime of stagnation was between 100,000 BC and AD 1750 for developed regions and between 100,000 BC and AD 1900 for less-developed regions. The claimed starting time appears to be based entirely on conjecture because Maddison's data are terminated at

AD 1 and even they contain significant gaps below AD 1500. The post-Malthusian regime was supposedly between AD 1750 and 1850 for developed regions and from 1900 for less-developed regions. The sustained-growth regime was supposed to have commenced around 1850 for developed regions.

Unified Growth Theory (Galor, 2005; 2008; 2011; 2012) can be tested in many ways but the easiest way to test itis to look for the dramatic takeoffs from stagnation to growth. These takeoffs are described as a "remarkable" or "stunning" escape from Malthusian trap (Galor, 2005, pp. 177, 220). It is a signature, which cannot be missed.

This change in the pattern of growth is described as "the sudden take-off from stagnation to growth" (Galor, 2005, pp. 177, 220, 277) or as a "sudden spurt" (Galor, 2005, 177, 220). According to Galor, for developed regions, the end of the Malthusian regime of stagnation coincides with the Industrial Revolution. "The take-off of developed regions from the Malthusian Regime was associated with the Industrial Revolution" (Galor, 2005, p. 185). Indeed, the Industrial Revolution is considered to have been "the prime engine of economic growth" (Galor, 2005, p. 212).

This signature is characterised by three features: (1) it should be a prominent change in the pattern of growth, (2) it should be a transition from stagnation to growth and (3) it should occur at the time predicted by the theory. For developed regions, the postulated takeoffs should occur around AD 1750, or around the time of the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994). For less-developed regions, they should occur around 1900. The added advantage of using this simple test is that there are no significant gaps in the data around the time of the postulated takeoffs and consequently the stagnation and the expected prominent transitions from stagnation to growth should be easily identifiable.

A transition from growth to growth is not a signature of the postulated takeoff from stagnation to growth. Thus, a transition is from hyperbolic growth to another hyperbolic growth or to some other steadily-increasing trajectory is not a signature of the sudden takeoff from stagnation to growth. Likewise, a transition at a distinctly different time is not a confirmation of the theoretical expectations.

World economic growth

Results of mathematical analysis of the world economic growth are presented in Figures 1-3. Reciprocal values of historical data can be fitted using a straight line (representing hyperbolic growth) between AD 1000 and 1955. From around 1955, the world economic growth started to be diverted to a slower trajectory as indicated by the *upward* bending of the reciprocal values. This section is magnified in Figure 2. Global economic growth is now approximately exponential (Nielsen, 2014; 2015a).

Hyperbolic fit to the world GDP data (Maddison, 2010) is shown in Figure 3. The fit is remarkably good. The point at AD 1 is 77% away from the fitted curve. We would need more data between AD 1 and 1000 to decide whether such a difference is of any significance but it could reflect a pattern similar to the pattern observed for the growth of human population (Nielsen, 2016).Hyperbolic economic growth of the historical GDP has been uniquely identified by the straight-line fitting the reciprocal values of data.

Parameters describing hyperbolic trajectory fitting the data between AD 1000 and 1955 are: $a = 1.684 \times 10^{-2}$ and $k = 8.539 \times 10^{-6}$. Its singularity is at $t = 1972$. However, from around 1955, the world economic growth started to be diverted to a slower trajectory bypassing the singularity by 17 years (see Table 1).

The search for a takeoff in the world economic growth produced negative results. The data reveal a different pattern of growth than claimed by the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012). The theory claims a long period of stagnation followed by a sudden takeoff. The data show a stable hyperbolic growth followed by a diversion to a slower trajectory.

The data also demonstrate that the Industrial Revolution had no impact on changing the economic growth trajectory. These results might not be surprising because the world economic growth is represented by the economic growth in developed and less-developed regions. However, even then, it would be hard to expect that the data would follow such a remarkably stable and specific trajectory. We would expect some distortions reflecting takeoffs around the time of the Industrial Revolution for developed regions and takeoffs around 1900 for less-developed regions. We see no signs of such distortions; no signs of the presence of such takeoffs.

The straight-line representing the reciprocal values of the GDP data shown in Figure 1 follows the data closely until 1955. There was no boosting in the economic growth, no unusual acceleration at *any time* between AD 1000 and 1955. The world economic growth was increasing monotonically before and after the Industrial Revolution as shown by either a steadily increasing hyperbolic distribution in Figure 3 or by the steadily-decreasing straight line (representing hyperbolic distribution) shown in Figure

1. Which point on a straight line should be selected to mark a boundary between different patterns of growth? How can we claim different patterns of growth on a straight line if the straight line shows clearly only one pattern? There was no takeoff in the world economic growth at any time, let alone around the time of the Industrial Revolution or around 1900.

Economic growth may have been slow over a long time but it was not stagnant. The growth was hyperbolic, and the characteristic feature of hyperbolic growth is a slow growth over a long time and a fast growth over a short time. Hyperbolic growth increases monotonically and it is *impossible* to locate a place marking a transition from a slow to fast growth because *such a transition does not exist.*

Hyperbolic growth of the world economy is in harmony with the hyperbolic growth of the world population (Nielsen, 2016; von Foerster, Mora & Amiot, 1960). In both cases, the growth was indeed slow over a long time and fast over a short time. In both cases the growth creates an illusion of stagnation followed by a sudden takeoff. However, in both cases the growth was hyperbolic. There was no stagnation and no sudden takeoff. Furthermore, in both cases the growth started to be diverted, relatively recently, to slower trajectories.

Western Europe

The growth of the GDP in Western Europe is shown in Figures 4-6. Western Europe is represented by the total of 30 countries: Austria, Belgium, Denmark, Finland, France, Germany, Italy, The Netherlands, Norway, Sweden, Switzerland, United Kingdom, Greece, Portugal, Spain and by14 small, but unspecified countries. Ireland is missing in this list because it was included only from 1921.

The best hyperbolic fit to the data is between AD 1500 and 1900. Parameters for this distribution are $a = 9.859 \times 10^{-2}$ and $k = 5.112 \times 10^{-5}$. The point of singularity is at $t = 1929$. Between 1900 and 1910, economic growth started to be diverted to a slower, but still fast-increasing, trajectory bypassing the singularity by 29 years (see Table 1).

The most complete set of data for Western Europe is for Denmark, France, the Netherlands and Sweden. They are analysed separately and results are presented in Figures 7 and 8. According to Maddison (2010), these four countries accounted for 34% of the total GDP of the 30 countries of Western Europe in 2008.

Parameters describing the historical hyperbolic growth of the GDP in these four countries are: $a = 3.821 \times 10^{-1}$ and $k = 1.986 \times 10^{-4}$. The point of singularity is at $t = 1923$. From around 1875 economic growth in Denmark, France, the Netherlands and Sweden was diverted to a slower trajectory, bypassing the singularity by 48 years.

The quality of the hyperbolic fit to the data is virtually the same as for the total of the 30 countries but now the fitted curve passes also through the AD 1 point. However, it still does not reproduce the point at AD 1000. This point is only 41% below the fitted hyperbolic distribution.

The historical growth of the GDP in Western Europe was definitely hyperbolic from AD 1500 to 1900 but there is also a good indication that it might have been hyperbolic from AD 1 (see Figures7 and 8). Even if we make allowance for this uncertainty, the search for a sudden takeoff around the expected time, i.e. around the time of the Industrial Revolution, produced negative results for the 30 countries of Western Europe and for the four (Denmark, France, the Netherlands and Sweden) characterised by the most complete sets of data.

The claim of a stunning or remarkable takeoff is contradicted by data. There was no takeoff of any kind and at any time, stunning or less stunning, remarkable or less remarkable, sudden or gradual; none at all. The Industrial Revolution, the supposed "prime engine of economic growth" (Galor, 2005a, p. 212), made no impression on changing the economic growth trajectory in regions where this engine should have been working most efficiently. Industrial Revolution brought many other important changes but, surprisingly perhaps, did not change the economic growth trajectory in the countries closest to this monumental development.

Eastern Europe

Systematic data for Eastern Europe are available only for seven countries: Albania, Bulgaria, Czechoslovakia, Hungry, Poland, Rumania and Yugoslavia. For other countries there are no data until 1990. The analysis of the historical data for Eastern Europe is summarised in Figures 9-11.

The best hyperbolic fit to the data is between AD 1000 and 1890. Hyperbolic parameters are: $a = 7.749 \times 10^{-1}$ and $k = 4.048 \times 10^{-4}$. The point of singularity is at $t = 1915$. From around 1890, economic growth in Eastern Europe was diverted to a slower trajectory, bypassing the singularity by 25 years.

There was no stagnation and no takeoff at any time. Industrial Revolution had no impact on changing the economic growth trajectory in the countries of Eastern Europe.

Former USSR

The analysis of the data for the countries of the former USSR is presented in Figures 12-14. The hyperbolic fit to the data is between AD 1 and 1870. Parameters fitting the data are: $a = 6.547 \times 10^{-1}$ and $k = 3.452 \times 10^{-4}$. The point of singularity is at $t = 1897$. From around 1870, or maybe even a little earlier (shortly after the Industrial Revolution) economic growth in the Former USSR was diverted to a slower trajectory, bypassing the singularity by at least 27 years.

There was no stagnation and no takeoff *at any time*. Industrial Revolution had no impact on changing the economic growth trajectory in the countries of former USSR.

Asia

Analysis of the historical economic growth in Asia is summarised in Figures 15-17. The best hyperbolic fit is between AD 1000 and 1950. Parameters fitting the data are: $a = 2.303 \times 10^{-2}$ and $k = 1.129 \times 10^{-5}$. The point of singularity is at $t = 2040$.

Asia is made primarily of less-developed countries (BBC, 2014; Pereira, 2011) and consequently, according to the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012), economic growth in this region should have been characterised by stagnation until around 1900, the year marking the supposed stunning escape from Malthusian trap, the escape manifested by the postulated dramatic takeoff. The data and their analysis show that there was no stagnation, at least from AD 1000 and no expected takeoff. The data reveal a steadily increasing hyperbolic growth until around 1950. From around that year economic growth *was* diverted to a fast trajectory. This boosting can be seen clearly in Figures 16 and 17 and it occurred close to the time of the postulated takeoff from stagnation to growth. However, it was *not* a transition from stagnation to growth but from hyperbolic growth to a slightly faster trajectory of a different kind. It is, therefore, not the takeoff postulated in the Unified Growth Theory. Furthermore, it was only a temporary boosting, which is now returning to the original hyperbolic trajectory and, as indicated by the reciprocal values of the data, this new growth is likely to be slower than the original trajectory. Thus, it is a boosting of a completely different kind. It would be interesting to explain it but we cannot be helped by Unified Growth Theory because it discusses mechanisms, which are repeatedly contradicted by data. This transition is not even recognised in this theory

Reciprocal values of data presented in Figure 16 show that the economic growth became temporarily *slower* at the time overlapping the time of the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994), because while the point in 1820 is still located on the straight line, representing hyperbolic growth, the point in 1870 is above this line. The deceleration in the economic growth occurred sometime between 1820 and 1870.

This brief deceleration was followed by a transient growth between 1870 and 1940, which appears to have been also hyperbolic but a little faster than the earlier hyperbolic growth. This transition occurred earlier than the postulated takeoff around 1900 and it was not a transition from stagnation to growth but a transition from hyperbolic growth to hyperbolic growth. Furthermore, it was also a minor transition, which could be hardly noticed in the direct display of data shown in Figure 17. In summary, therefore, the examination of data for the economic growth in Asia demonstrates that the postulated takeoff (Galor, 2005; 2008; 2011; 2012) never happened. There was no stagnation and no sudden dramatic escape to a new and rapid growth.

Africa

Results of the analysis of the economic growth in the 57 African countries are presented in Figures 18-20. Reciprocal values of the GDP data, presented in Figures 18 and 19, show clearly that the economic growth was following *two* hyperbolic distributions. At first it was a slow hyperbolic growth between AD 1 and 1820 characterised by parameters $a = 1.244\times10^{-1}$ and $k = 5.030\times10^{-5}$ and by the singularity at $t = 2473$. Then, around 1820, this slow hyperbolic growth was replaced by a significantly faster hyperbolic growth characterised by parameters $a = 4.192\times10^{-1}$ and $k = 2.126\times10^{-4}$ and by the singularity at $t = 1972$. Defined by the parameter *k*, this new growth was 4.2 times faster than the earlier hyperbolic growth. From around 1950, this fast hyperbolic growth was diverted to a slower, non-hyperbolic trajectory, bypassing singularity by 22 years.

Africa is also made of less-developed countries (BBC, 2014; Pereira, 2011) so according to the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012) it should have experienced stagnation in the economic growth until around 1900 followed by a clear takeoff around that year. These expectations are contradicted by the economic growth data because (1) economic growth was not stagnant but hyperbolic until 1950, (2) there was no takeoff from stagnation to growth around 1900 or around any other time and (3) shortly after the expected time of the takeoff, economic growth in Africa started to be diverted to a slower trajectory.

Acceleration in the economic growth in Africa occurred around 1820, but it was not a transition from stagnation to growth but *from growth to growth*. Even more specifically, it was a transition from the hyperbolic growth to another hyperbolic growth. It was also acceleration at a wrong time, not around 1900 but around the time of the Industrial Revolution. This acceleration can be explained by noticing that it appears to coincide with the intensified colonisation of Africa (Duignan & Gunn, 1973; McKay, *et al.* 2012; Pakenham, 1992). The fast increasing GDP after 1820 was not reflecting the rapidly improving living conditions of African population brought about by the beneficial changes caused by the Industrial Revolution but the rapidly increasing wealth of new settlers and their countries of origin at the expense of the deploring living conditions of the native populations.

The search for the takeoff from stagnation to growth, claimed by the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012), produced negative results. The data show also that there was no stagnation in the economic growth over the entire range of time, from AD 1 to the present time.

Latin America

Results of the analysis of the economic growth in Latin America are presented in Figures 21 - 23. Data for Latin America are difficult to analyse because there was a significant decline in the economic growth between AD 1500 and 1600 but they also appear to follow two distinctly different hyperbolic trajectories. However, the identification of the first trajectory is not as clear as for Africa. The identification of the second hyperbolic trajectory is more convincing. Tentative conclusion is that the economic growth in Latin America was following a slow hyperbolic distribution between AD 1 and 1500 and a fast hyperbolic distribution between AD 1600 and around 1870.

The tentatively assigned slow hyperbolic growth between AD 1 and 1500 is characterised by parameters $a = 4.421 \times 10^{-2}$ and $k = 2.093 \times 10^{-5}$. Its singularity is at $t = 2113$. The better determined fast hyperbolic growth between AD 1600 and 1870 is characterised by parameters $a = 1.570 \times 10^{-1}$ and $k = 8.224 \times 10^{-5}$. Its singularity is at $t = 1910$. Defined by the parameter *k*, this growth was 3.9 times faster than the earlier hyperbolic growth. From around 1870, this fast hyperbolic growth started to be diverted to a slower trajectory bypassing the singularity by 40 years. The transition from the earlier apparent hyperbolic growth to a new and rapid hyperbolic growth, which occurred between around AD 1500 and 1600 appears to coincide with commencement of the Spanish conquest (Teeple, 2002).

Latin America is also made of less-developed countries (BBC, 2014; Pereira, 2011) so again, according to the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012), the economic growth in this regions should have been stagnant until around 1900 and fast-increasing from around that year. This pattern of growth is not confirmed by data. The data show a diametrically different pattern: (1) there is no convincing evidence of the existence of stagnation over the entire range of time between AD 1 and 1870 but there is a sufficiently convincing indication of the hyperbolic growth particularly between AD 1600 and 1870, (2) there was no takeoff from stagnation to growth at any time, and (3) around the time of the postulated takeoff in 1900 there was a diversion to a slower trajectory in 1870.

Even if the identification of the hyperbolic growth between AD 1 and 1500 is questioned, the overall pattern of growth in Latin America is similar to the pattern in Africa: a slow hyperbolic growth is followed by a fast hyperbolic growth. However, in any case, there is no convincing evidence that the growth was ever stagnant. On the contrary, there is sufficiently convincing evidence that the growth was never stagnant. It was clearly not stagnant between AD 1600 and 1870.

There was also no takeoff, dramatic or modest, from stagnation to growth around the expected time of 1900, *first* because the growth before that year was not stagnant but hyperbolic and *second* because around the time of the expected remarkable takeoff, the economic growth started to be diverted to a slower trajectory. The search for the postulated takeoff produced negative results.

Summary and conclusions

Results of mathematical analysis of the historical economic growth are presented in Table 1. The listed parameters *a* and *k* are for the fitted hyperbolic distributions. The last column shows the results of the search for the takeoffs from stagnation to growth claimed by the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012).

Table 1. *Summary of the mathematical analysis of the historical economic growth*

Region/Countries	*a*	*k*	Hyperbolic Range	Singularity	Proximity	Takeoff
World	1.684×10^{-2}	8.539×10^{-6}	1000 – 1955	1972	17	X
Western Europe	9.859×10^{-2}	5.112×10^{-5}	1500 – 1900	1929	29	X
Western Europe (4)	3.821×10^{-1}	1.986×10^{-4}	1 – 1875	1923	48	X
Eastern Europe	7.749×10^{-1}	4.048×10^{-4}	1000 – 1890	1915	25	X
Former USSR	6.547×10^{-1}	3.452×10^{-4}	1 – 1870	1897	27	X
Asia	2.303×10^{-2}	1.129×10^{-5}	1000 – 1950	2040	90	X
Africa	1.244×10^{-1}	5.030×10^{-5}	1 – 1820	2473		
			1820 – 1950	1972	22	X
	4.192×10^{-1}	2.126×10^{-4}				
Latin America	4.421×10^{-2}	2.093×10^{-5}	1 – 1500	2113		
			1600 – 1870	1910	40	X
	1.570×10^{-1}	8.224×10^{-5}				

Notes: *a* and *k* – Hyperbolic growth parameters [see eqn (1)]. *Hyperbolic Range* - The empirically-confirmed range of time when the economic growth can be described using hyperbolic distributions. *Singularity* - The time of the escape to infinity for a given hyperbolic distribution. *Proximity* - Proximity (in years) of the singularity at the time when the economic growth departed from the hyperbolic growth to a new trajectory. *Western Europe (4)* - Four countries of Western Europe: Denmark, France, the Netherlands and Sweden. *X* - No takeoff. The takeoff from stagnation to growth claimed by the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012) never happened.

This analysis demonstrates that the natural tendency for the historical economic growth was to increase hyperbolically. In general, there is a remarkably good agreement between the data and the calculated hyperbolic distributions.

Unlike the more familiar exponential distributions, which are easier to understand because they show more readily a gradually increasing growth, hyperbolic distributions appear to be made of two or maybe even three components: a slow component, a fast component and perhaps even a transition component located between the apparent slow and fast components. The illusion is so strong that even the most experienced researchers can be deceived particularly if they have no access to good sets of data, which was in the past. Now, however, excellent data are available (Maddison, 2001; 2010) and we can use them not only to check the earlier interpretations of economic growth but also to expand the scope of the economic research.

The postulate of the existence of the epoch of Malthusian stagnation is suggested by a slow economic growth over a long time but this slow growth is just a part of the hyperbolic growth, which is convincingly identified using reciprocal values. Hyperbolic distributions create also the illusion of a sudden takeoff but this feature is also a part of the hyperbolic growth. Hyperbolic growth *is* slow over a long time and fast over a short time but the slow and fast growth are the integral features of the same monotonically increasing distribution, which is easier to understand by using the reciprocal values of the growing entity (Nielsen, 2014). In such displays, the illusion of distinctly different components disappears because hyperbolic growth is then represented by a decreasing straight line, which is easy to understand. It then becomes obvious that hyperbolic distribution cannot be divided into distinctly different sections governed by different mechanism because it makes no sense to divide a straight line into arbitrarily chosen sections and claiming different mechanism to such arbitrarily-selected section. It is also then clear that it is *impossible* to pinpoint the transition from a slow to a fast growth. Which point on a straight line should we select to identify such a transition? The transition does not happen at any specific time but gradually over the whole range of time.

Our search for the postulated takeoffs from stagnation to growth (Galor, 2005; 2008; 2011; 2012) produced negative results: *there were no takeoffs*. Galor's elaborate discussion revolving around his postulated three regimes of growth and the postulated takeoff from stagnation to growth are irrelevant because there was no takeoff in the growth of the GDP and in the growth of income per capita (GDP/cap) (Nielsen, 2015b). In science, just one contradicting evidence in data is sufficient to show that a theory advocating the contradicted postulate or postulates has to be either rejected or revised to bring it in the agreement with empirical evidence. In the case of the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012), the postulated takeoff from stagnation to growth is contradicted repeatedly by the economic growth in Western Europe, Eastern Europe, former USSR, Asia, Africa, and Latin America as well as by the world economic growth.

The data and their analysis suggest new lines of research of economic growth. They suggest that our attention should not be directed towards explaining the mechanism of stagnation and of the sudden takeoffs from stagnation to growth because these features are contradicted by data. What needs to be explained is why the historical economic growth was hyperbolic and why relatively recently it was diverted to a slower trajectory. Maddison published excellent data describing not only economic growth but also the growth of human population and these data can be used effectively in trying to explain the historical economic growth.

Appendix

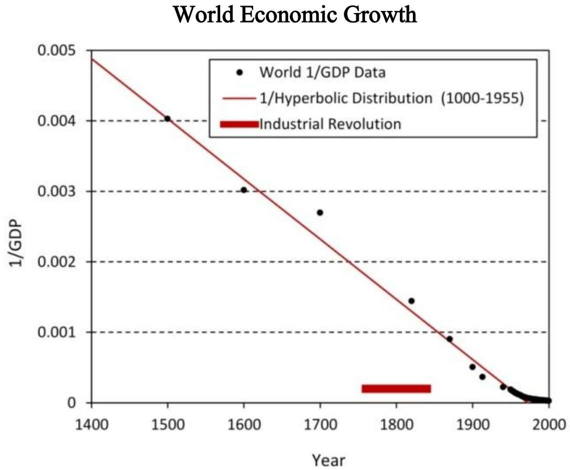

Figure 1. *Reciprocal values of the GDP data (Maddison, 2010) are fitted using straight line between AD 1000 and 1955 representing hyperbolic growth. There was no stagnation and no takeoff from stagnation to growth, claimed by the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012). Industrial Revolution had no impact on changing the economic growth trajectory. From around 1955, the economic growth started to be diverted to a slower trajectory.*

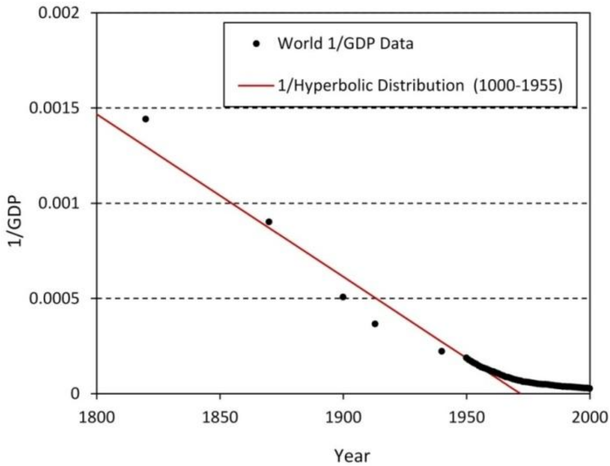

Figure 2. *Reciprocal values of the GDP data (Maddison, 2010) showing the diversion of the economic growth to a slower trajectory from around 1955, as indicated by the upward bending. The current global economic growth is approximately exponential (Nielsen, 2014; 2015a).*

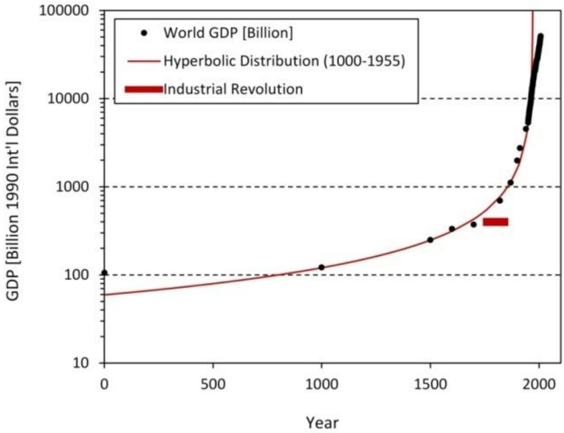

Figure 3. *World GDP data (Maddison, 2010) fitted using hyperbolic distribution. The point at AD 1 is 77% higher than the calculated distribution. There was no stagnation and no takeoff from stagnation to growth, both features incorrectly claimed by the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012). Industrial Revolution had no impact on changing the economic growth trajectory. From around 1955, the world economic growth started to be diverted to a slower but still fast-increasing trajectory, which is now approximately exponential (Nielsen, 2014; 2015a).*

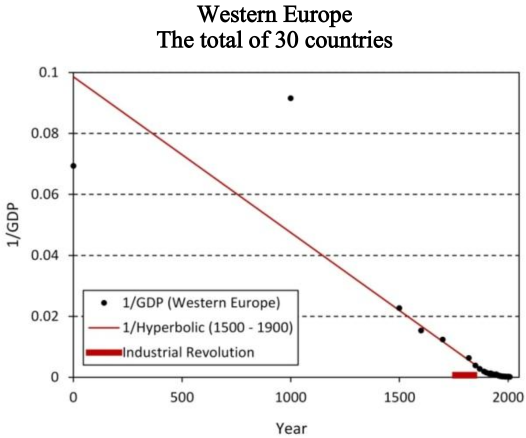

Figure 4. *Reciprocal values of the GDP data (Maddison, 2010) for Western Europe are compared with the hyperbolic distribution represented by the decreasing straight line. The growth was hyperbolic from at least AD 1500 and 1900. There was no takeoff from stagnation to growth. Industrial Revolution had no impact on changing the economic growth trajectory in Western Europe, the centre of this revolution. On the contrary, from around 1900, shortly after the Industrial Revolution, the economic growth in Western Europe started to be diverted to a slower trajectory as indicated by the upward bending of the trajectory representing the reciprocal values of data.*

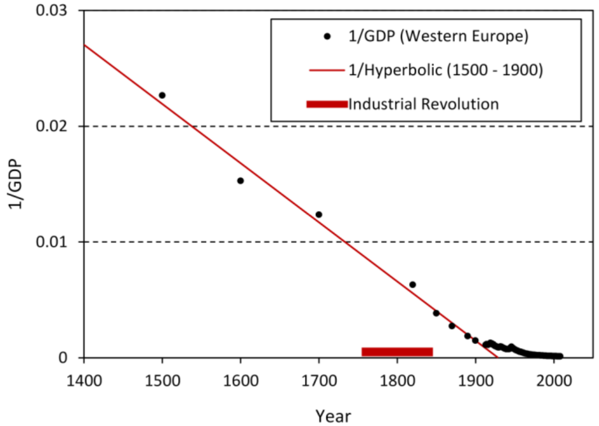

Figure 5. *Reciprocal values of the GDP data (Maddison, 2010) for Western Europe between AD 1500 and 2008 showing a diversion to a slower trajectory from around 1900. There was no takeoff from stagnation to growth, claimed incorrectly by the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012). Industrial Revolution had absolutely no impact on changing the economic growth trajectory in Western Europe, the centre of this revolution,*

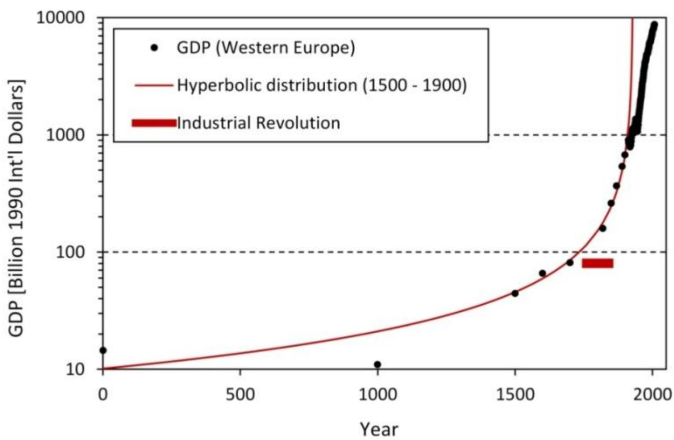

Figure 6. *Economic growth in Western Europe. The GDP data (Maddison, 2010) are compared with hyperbolic distribution. The growth was hyperbolic from at least AD 1500 to around 1900. The point at AD 1 is 42% higher than for the calculated distribution and 48% lower at AD 1000. There was no takeoff from stagnation to growth, claimed incorrectly by the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012). Industrial Revolution had no impact on changing the economic growth trajectory in Western Europe, the centre of this revolution. From around 1900, economic growth in Western Europe started to be diverted to a slower trajectory.*

Denmark, France, Netherlands and Sweden

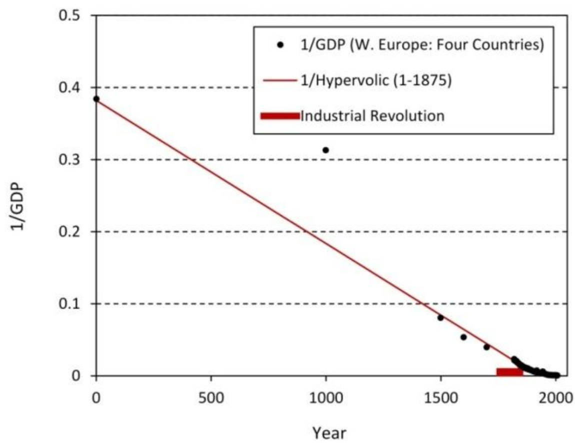

Figure 7. *Reciprocal values of the GDP data (Maddison, 2010) describing economic growth in four countries of Western Europe (Denmark, France, Netherlands and Sweden) compared with the straight line representing hyperbolic growth fitting the data between AD 1 and 1875. From around 1875, or shortly after the Industrial Revolution, economic growth in these four countries started to be diverted to a slower trajectory. Industrial Revolution did not boost economic growth. There was no takeoff from stagnation to growth.*

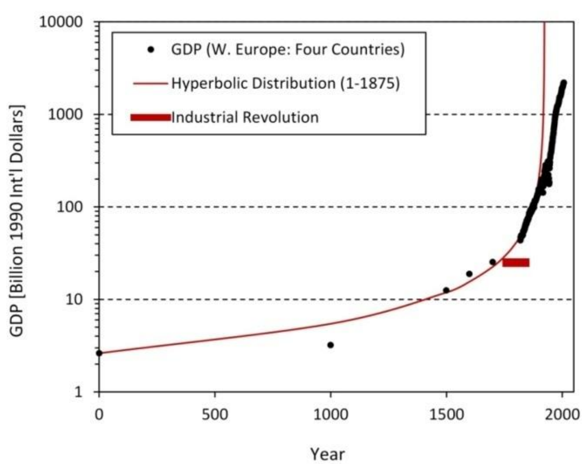

Figure 8. *Economic growth in Denmark, France, Netherlands and Sweden. The data (Maddison, 2010) are compared with hyperbolic distribution. The point at AD 1000 is 41% lower than for the calculated distribution. From around 1875, the economic growth started to be diverted to a slower trajectory. There was no takeoff from stagnation to growth.*

Eastern Europe

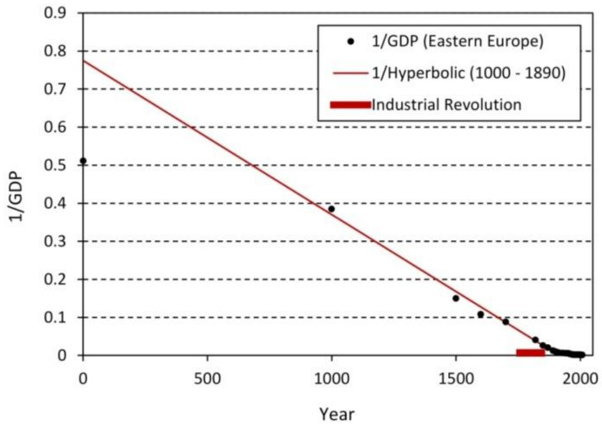

Figure 9. *Reciprocal values of the GDP data (Maddison, 2010) for Eastern Europe are compared with the hyperbolic distribution represented by the decreasing straight line. Economic growth was hyperbolic from at least AD 1000. The takeoff from stagnation to growth never happened. Industrial Revolution did not boost the economic growth in Eastern Europe.*

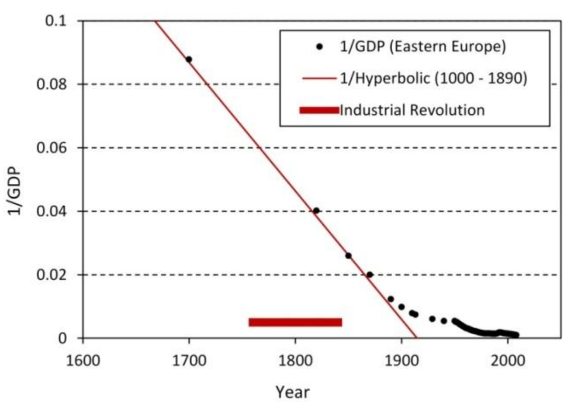

Figure 10. *Reciprocal values of the GDP data (Maddison, 2010) for Eastern Europe showing that from around 1890, shortly after the Industrial Revolution, the economic growth started to be diverted to a slower trajectory. There was no takeoff from stagnation to growth. Industrial Revolution did not boost the economic growth in Eastern Europe. Hyperbolic growth around that time remained undisturbed.*

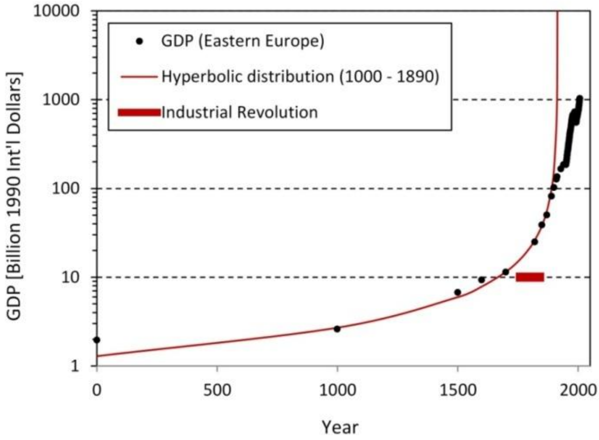

Figure 11. *Economic growth in Eastern Europe. GDP data (Maddison, 2010) are compared with the best hyperbolic fit. The point at AD 1 is 51% higher than for the calculated distribution. From around 1890, shortly after the Industrial Revolution, economic growth started to be diverted to a slower trajectory. Industrial Revolution did not boost the economic growth in Eastern Europe. Contrary to the Unified Growth Theory (Galor, 2005; 2008; 2011; 2012), there was no stagnation and no takeoff from stagnation to growth.*

Former USSR

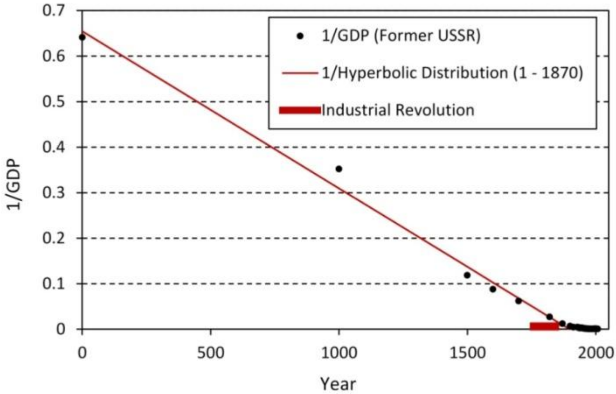

Figure 12. *Reciprocal values of the GDP data (Maddison, 2010) for the former USSR compared with the hyperbolic distribution represented by the decreasing straight line. Data indicate that the economic growth was hyperbolic from AD 1 to 1870. Industrial Revolution did not boost the economic growth. There was no stagnation and no takeoff from stagnation to growth. Shortly after the Industrial Revolution, the economic growth in Eastern Europe started to be diverted to a slower trajectory. Unified Growth Theory (Galor, 2005; 2008; 2011; 2012) is contradicted by the economic growth data.*

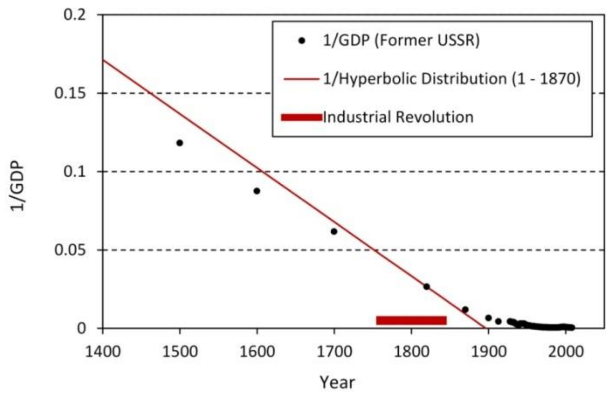

Figure 13. *Reciprocal values of the GDP data (Maddison, 2010) for the former USSR showing that from around 1870, shortly after the Industrial Revolution, economic growth started to be diverted to a slower trajectory.*

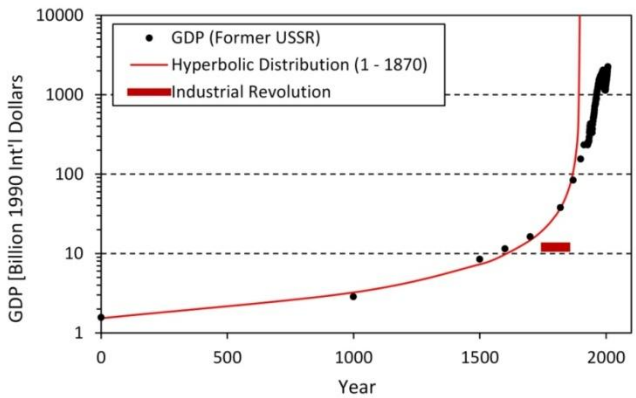

Figure 14. *Economic growth in the former USSR. GDP data (Maddison, 2010) are compared with the best hyperbolic fit. The growth was hyperbolic from AD 1 to 1870. From around 1870, shortly after the Industrial Revolution, economic growth started to be diverted to a slower trajectory. Epoch of stagnation did not exist. Industrial Revolution did not boost the economic growth. There was no takeoff from stagnation to growth because there was no stagnation but a steadily-increasing growth. Unified Growth Theory (Galor, 2005; 2008; 2011; 2012) is contradicted by the economic growth data.*

Asia

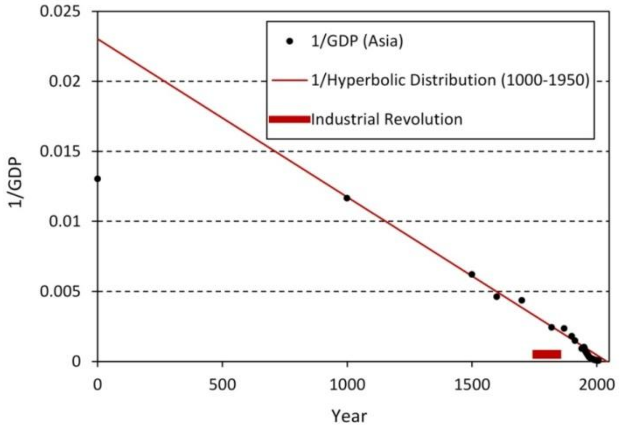

Figure 15. *Reciprocal values of the GDP data (Maddison, 2010) for Asia compared with the hyperbolic distribution represented by the decreasing straight line. Economic growth was hyperbolic from at least AD 1000. There was no expected transition from stagnation to growth*

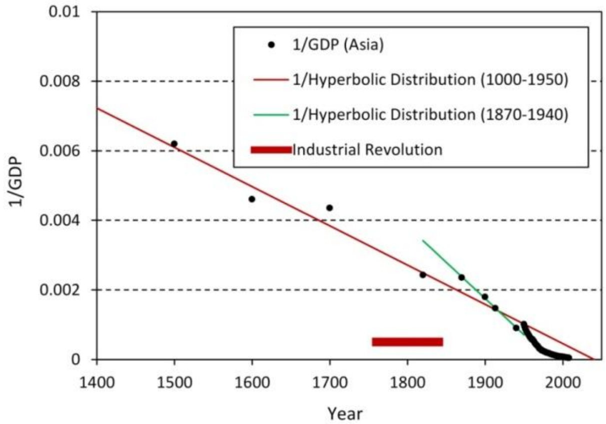

Figure 16. *Reciprocal values of the GDP data (Maddison, 2010) for Asia. The data show a minor deceleration of growth towards the end of the time of the Industrial Revolution followed by a slightly faster hyperbolic growth between 1870 and 1940. The expected takeoff from stagnation to growth around 1900 (Galor; 2005; 2008; 2011; 2012) did not happen. The data show a small boosting around 1950 but it was not a transition from stagnation to growth. The search for the postulated takeoff (Galor; 2005; 2008; 2011; 2012) produced negative results.*

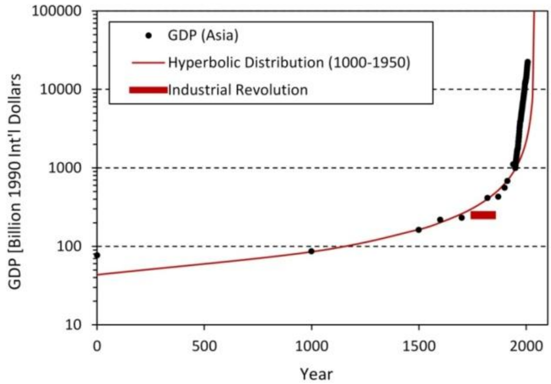

Figure 17. *Economic growth in Asia. The data (Maddison, 2010) are compared with the hyperbolic distribution. The point at AD 1 is 76% higher than the calculated value. The data show a minor boosting around 1950 but it was not a transition from stagnation to growth but from the hyperbolic growth to a slightly faster trajectory, which is now coming closer to the earlier hyperbolic trajectory. The boosting was not only small but also it did not last long. The search for the postulated takeoff from stagnation to growth (Galor; 2005; 2008; 2011; 2012) produced negative results.*

Africa

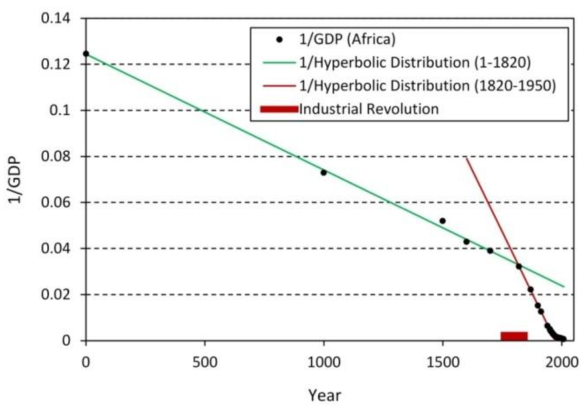

Figure 18. *Reciprocal values of the GDP data (Maddison, 2010) for Africa compared with hyperbolic distributions represented by the decreasing straight lines. There was no stagnation in the economic growth. Economic growth was increasing hyperbolically between AD 1 and around 1820 and again from 1820 to around 1950. The expected takeoff from stagnation to growth (Galor; 2005; 2008; 2011; 2012) never happened. The acceleration around 1820 was not a transition from stagnation to growth but transition from growth to growth. It also occurred earlier than expected (in 1820 rather than around 1900). Furthermore, close to the postulated takeoff, economic growth started to be diverted to a slower trajectory. The search for the takeoff from stagnation to growth around 1900 produced negative results. Unified Growth Theory (Galor; 2005; 2008; 2011; 2012) is contradicted by data.*

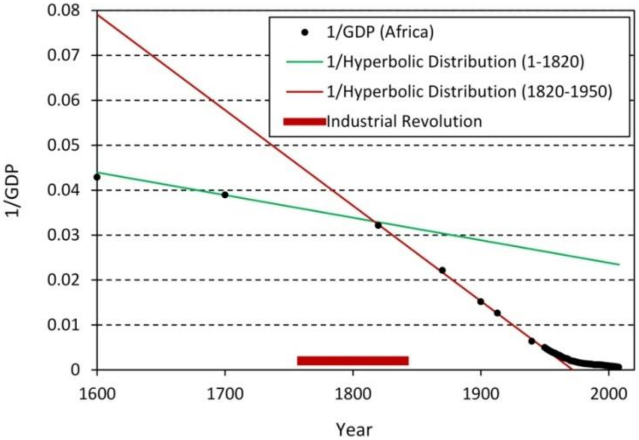

Figure 19. *Reciprocal values of the GDP data (Maddison, 2010) for Africa showing that from around 1950 economic growth started to be diverted to a slower trajectory. There was no takeoff around 1900, not even from growth to growth.*

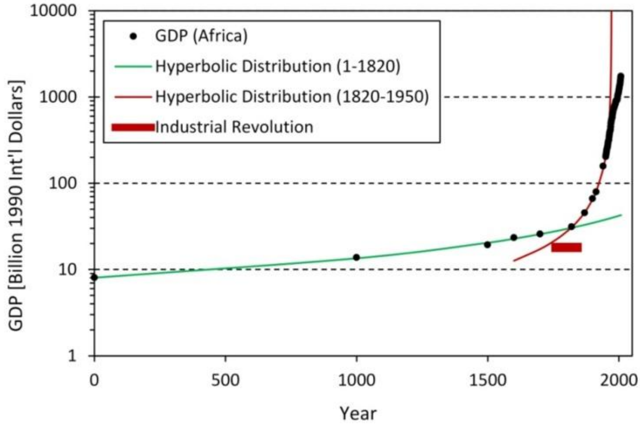

Figure 20. *Economic growth in Africa. Data (Maddison, 2010) are compared with hyperbolic distributions. The claimed takeoff from stagnation to growth (Galor; 2005; 2008; 2011; 2012) never happened because there was no stagnation. Furthermore, the transition from hyperbolic growth to hyperbolic growth occurred earlier (around 1820) than the postulated takeoff from stagnation to growth (around 1900). From around 1950, close the claimed but non-existing takeoff from stagnation to growth, economic growth started to be diverted to a slower trajectory. Unified Growth Theory (Galor; 2005; 2008; 2011; 2012) is contradicted by data.*

Latin America

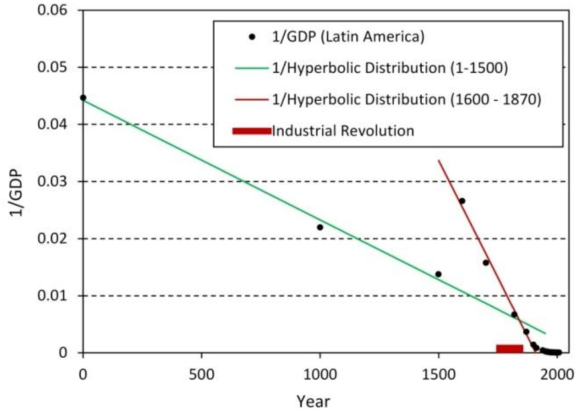

Figure 21. *Reciprocal values of the GDP data (Maddison, 2010) for Latin America are compared with hyperbolic distributions represented by the decreasing straight lines. The pattern of growth in Latin America is similar to the pattern of growth in Africa. The expected takeoff from stagnation to growth around 1900 (Galor; 2005; 2008; 2011; 2012) did not happen, because there was no stagnation and because, from around 1870, economic growth started to be diverted to a slower trajectory.*

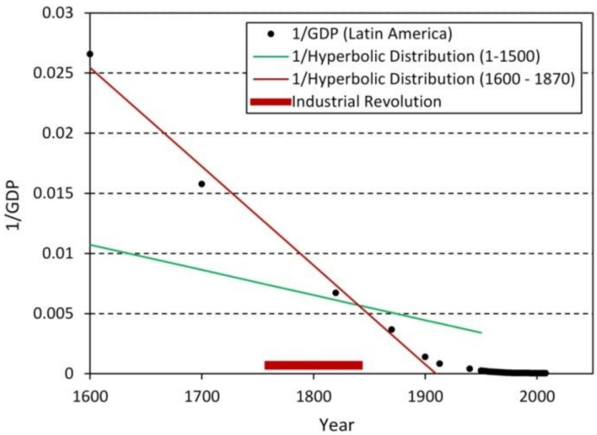

Figure 22. *Reciprocal values of the GDP data (Maddison, 2010) for Latin America showing that from around 1870, i.e. close to the time of the expected takeoff (around 1900) from stagnation to growth (Galor; 2005; 2008; 2011; 2012) economic growth started to be diverted to a slower trajectory. The data show also that the takeoff from stagnation to growth could not have happened because there was no stagnation.*

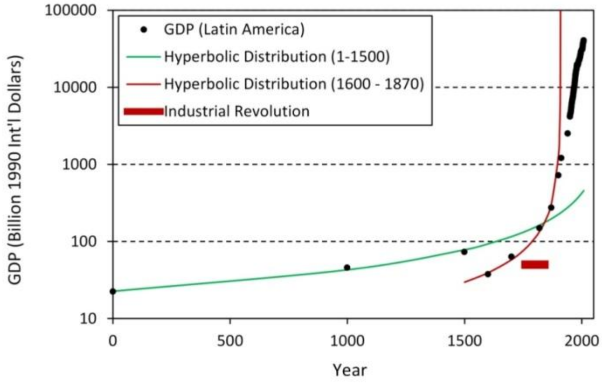

Figure 23. *Economic growth in Latin America. Economic growth data (Maddison, 2010) are compared with hyperbolic distributions. Unified Growth Theory (Galor; 2005; 2008; 2011; 2012) is contradicted by data. Economic growth was not stagnant before the postulated takeoff from stagnation to growth (around 1900) but hyperbolic. The growth was also stable and hyperbolic around the time of the Industrial Revolution in the Western world. The transition from stagnation to growth could not have happened because there was no stagnation. Furthermore, from around 1870, i.e. from around the time of the postulated takeoff, economic growth started to be diverted to a slower trajectory. The search for the takeoff from stagnation to growth produced negative results.*

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4. Unified Growth Theory contradicted by the absence of differential takeoffs in the Gross Domestic Product

Introduction

One of the fundamental postulates of the Unified Growth Theory (Galor, 2005a, 2011) is the postulate of takeoffs from stagnation to growth. This feature is supposed to mark a boundary between the ages-long epoch of Malthusian stagnation and a new epoch of a rapid economic growth. An easy way to test the Unified Growth Theory is to look for such postulated takeoffs because they should be easily identifiable. The added advantage of using this test is that it also checks the validity of yet another postulate of this theory, the postulate of the differential takeoffs.

In our analysis we shall use the excellent data published by the world-renown economist (Maddison, 2010). The data presented in this compilation are virtually the same as in his earlier compilation (Maddison, 2001), which Galor was using during the formulation of his Unified Growth Theory. The difference between the two compilations is that the new set of data was extended to the 21st century. These extended data are not essential for testing the Unified Growth Theory but they help in demonstrating the latest transitions from the historical hyperbolic growth to slower trajectories. Unfortunately, Galor did not analyse Maddison's data. His interpretations of the mechanism of economic growth are based on strongly questionable quotations of isolated numbers, on the unfortunate simplistic and self-misleading examination of data and on the habitual use of grossly distorted diagrams (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010;

2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

Historical economic growth and the growth of human population can be described using hyperbolic distributions (Nielsen, 2014; 2015a; 2015b; 2015c; 2015d; 2016a; 2016b; 2016c; 2016d; von Foerster, Mora & Amiot, 1960) Unlike the better-known exponential growth, which is easier to understand, hyperbolic distributions are strongly deceptive because they appear to be made of two distinctly different components, slow and fast, joined perhaps by a certain transition component. This illusion is so strong that even the most experienced researchers can be easily deceived particularly if their research is based on a limited body of data, as it was in the past. Fortunately, Maddison's data solve this problem, and fortunately also their analysis is trivially simple because, as pointed out earlier (Nielsen, 2014), hyperbolic distributions can be easily identified and analysed using the reciprocal values of data. Consequently, if in the past, researchers were basing their conclusions on the strongly-limited sets of data and imagined that there was a prolonged epoch of stagnation followed by sudden takeoffs in various geographical regions, now there is no excuse to continue with such interpretations because we have excellent sets of data, which lead to the entirely different conclusions. It is, therefore surprising, if not disappointing, that Galor, who had access to these excellent data and even used them during the formulation of his theory, did not analyse them properly but followed the traditional and incorrect interpretations of the historical economic growth.

Theories play an important role in scientific research because they crystallise interpretations of studied phenomena. However, theories have to be always tested by data. In science it is important to look for data confirming theoretical explanations but it is even more important to discover contradicting evidence, because data confirming a theory confirm only what we already know but contradicting evidence may lead to new discoveries.

According to Galor, historical economic growth can be divided into three distinctly-different stages governed by three distinctly different mechanisms: (1) the Malthusian regime of stagnation, (2) the post-Malthusian regime, and (3) the sustained-growth regime. We have already demonstrated that this postulate of the three regimes of economic growth is contradicted by the data for Western Europe, Eastern Europe, Asia, countries of former USSR, Africa and Latin America (Nielsen, 2015a; 2015b; 2015c; 2015d; 2016b), ironically by the same data which were used but never analysed by Galor.

This fundamental postulate of the three regimes of growth is used repeatedly throughout the narrative of the Unified Growth Theory and serves as the essential support for the discussed interpretations and explanations. Without this corner stone the whole theory becomes unsupported.

According to Galor, "The take-off of developed regions from the Malthusian Regime was associated with the Industrial Revolution and occurred at the beginning of the 19th century, whereas the take-off of less developed regions occurred towards the beginning of the 20th century and was delayed in some countries well into the 20th century" (Galor, 2005a). Even more precisely (Galor, 2008a; 2012a), Malthusian regime of stagnation was supposed to have been between 100,000 BC and AD 1750 for developed regions and between 100,000 BC and AD 1900 for less-developed regions. The post-Malthusian regime was supposedly between AD 1750 and 1850 for developed regions and from 1900 for less-developed regions. The sustained-growth regime was supposed to have commenced around 1850 for developed regions.

The claimed starting time of the Malthusian regime appears to be based entirely on conjecture because Maddison's data are terminated at AD 1 and then even contain significant gaps below AD 1500. The claimed date of 100,000 BC is also hanging in the middle of nowhere because the origin of *Homo sapiens* is usually placed around 200,000 BC. However, Weaver, Roseman & Stringer (2008) have pointed out that the divergence of the lineages of modern humans and Neanderthals might have occurred around 309,000 BC or even 433,000 BC.

We have no mathematically analysable data over such a long time so any claim of the existence of Malthusian stagnation in the economic growth in the distant past is based on questionable conjectures. However, we have mathematically-analysable data describing the growth of the population from 10,000 BC and they show that the growth of the population was not stagnant but hyperbolic not only during the AD era, as pointed out over 50 years ago by von Foerster, Mora & Amiot (1960) but also during the BC era (Nielsen, 2016c).

Hyperbolic growth was slow in the past but it was not stagnant. Slow hyperbolic growth should never be interpreted as stagnant because if we want to interpret the slow hyperbolic growth as stagnant, and governed by the usually assumed multitude of random forces, we should use precisely the same mechanism to explain the fast hyperbolic growth. It is impossible to divide the monotonically-increasing hyperbolic distributions into slow and fast components (Nielsen, 2014). Hyperbolic distributions have to be interpreted as a whole and the same mechanism has to be applied to the apparent slow growth and to the apparent fast growth. There is no clearly defined transition between the apparent slow and the apparent fast growth.

The supposed transition at the end of the postulated regime of Malthusian stagnation for various regions and countries is described by Galor as "the sudden take-off from stagnation to growth" (Galor, 2005a, pp. 177, 220, 277), as a "sudden spurt" (Galor, 2005a, 177, 220) or as "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220). It is a signature, which cannot be missed.

For developed regions, this signature is supposed to have coincided with the onset of the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994). Indeed, Industrial Revolution is considered to have been "the prime engine of economic growth" (Galor, 2005a, p. 212).

The signature of the takeoffs is characterised by three features: (1) it should be a prominent change in the pattern of growth, (2) it should be a transition from stagnation to growth and (3) it should occur at the time claimed by the theory. For developed regions, the postulated takeoffs should occur around AD 1750. For less-developed regions, they should occur around 1900.

A transition from growth to growth is *not* a signature of the postulated takeoff from stagnation to growth. Thus, for instance, a transition from hyperbolic growth to another hyperbolic growth is not a signature of the sudden takeoff from stagnation to growth. Likewise, a transition at a distinctly different time is not a confirmation of the theoretical expectations.

We shall now demonstrate that the postulated takeoffs never happened and consequently that the concept of the differential takeoffs is contradicted by data, because in the absence of takeoffs it makes no sense to claim that they occurred at different times for different regions. In the future we shall also demonstrate that "The mind-boggling phenomenon of the Great Divergence" (Galor, 2005a, p. 220) is mind-boggling only because it is hard to understand how anyone familiar with mathematics could be puzzled by such an artificially-created structure. If hyperbolic distributions are not properly analysed they can be used to generate such phantom and totally meaningless features. Scientific analysis of Maddison's data opens a new outlook on the interpretation of the historical economic growth.

Throughout the analysis presented here, the values of the Gross Domestic Product (GDP) will be expressed in billions of the 1990 International Geary-Khamis dollars. Parameters describing the fitted distributions were determined by the mathematical analysis (Nielsen, 2016b) of Maddison's data (Maddison, 2010).

World economic growth

Results of mathematical analysis of the world economic growth are presented in Figure 1. If the Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) is correct, we should see clear signs of *two* takeoffs: around 1750 for developed regions and around 1900 for less-developed regions. We see none of them.

The data and their analysis are in the direct contradiction of this theory. They show that the economic growth was remarkably stable and that the claimed or wished-for takeoffs never happened. The absence of the two claimed takeoffs is strikingly conspicuous. Galor's claim of the "spectacular" or "stunning" escapes from Malthusian trap (Galor, 2005a, pp. 177, 220) is spectacularly and stunningly contradicted by the analysis of the economic-growth data, the same data, which he used, but never properly analysed, during the formulation of his theory.

The absence of the takeoffs has been also demonstrated for the income per capita data (GDP/cap) for the world economic growth (Nielsen, 2015e). In science, such single demonstration would have been sufficient to show that the Unified Growth Theory needs to be revised to bring it in agreement with data, however, when closely analysed this theory is found to be repeatedly contradicted by data (Nielsen, 2015a; 2015b; 2015c; 2015d; 2015e; 2016a; 2016b).

Hyperbolic growth of the world economy is in harmony with the hyperbolic growth of the world population (Nielsen, 2016c; von Foerster, Mora & Amiot, 1960). In both cases, the growth was indeed slow over a long time and fast over a short time. In both cases the growth creates an illusion of stagnation followed by a sudden takeoff. However, in both cases the growth was hyperbolic. There was no stagnation and no sudden takeoff. Furthermore, in both cases the growth started to be diverted, relatively recently, to slower trajectories.

Western Europe

The growth of the GDP in Western Europe is shown in Figure2. Results of analysis show that there was no takeoff from stagnation to growth because (1) there was no stagnation and (2) because the economic growth, which is described well by the hyperbolic trajectory, was stable during the time of the supposed takeoff. The takeoff simply did not happen.

The claim of the stunning or remarkable takeoff is contradicted by data. There was no takeoff of any kind, stunning or less stunning, remarkable or less remarkable, sudden or gradual; none at all. The Industrial Revolution, the supposed "prime engine of economic growth" (Galor, 2005a, p. 212), made no impression on changing the economic growth trajectory in the region where this engine should have been working most efficiently. Industrial Revolution brought many other important changes but, surprisingly perhaps, did not change the economic growth trajectory in the countries closest to this monumental development.

Eastern Europe

The analysis of the historical data for Eastern Europe is summarised in Figure 3. There was no stagnation and no takeoff at any time. Industrial Revolution had no impact on changing the economic growth trajectory in the countries of Eastern Europe.

Former USSR

The analysis of the data for the countries of the former USSR is presented in Figure 4. There was no stagnation and no takeoff at any time. Industrial Revolution had no impact on changing the economic growth trajectory in the countries of former USSR.

Asia

Analysis of the historical economic growth in Asia is summarised in Figure 5. Asia is made primarily of less-developed countries (BBC, 2014; Pereira, 2011) and consequently, according to the Unified Growth Theory (Galor, 2005a, 2008a; 2011; 2012a), economic growth in this region should have been stagnant until around 1900, the year marking the supposed stunning escape from Malthusian trap, the escape manifested by the postulated dramatic take off.

The data and their analysis show that there was no stagnation and no claimed takeoff from stagnation to growth. The data reveal a steadily increasing and stable hyperbolic growth until around 1950. From around that year, economic growth *was* diverted to a slightly faster trajectory. This boosting occurred close to the time of the postulated takeoff from stagnation to growth. However, it was *not* a transition from stagnation to growth but from growth to growth.

It should be noted that this temporary boosting is now returning to the original hyperbolic trajectory and is likely to move to the other side. It is already following a slower trajectory, because its gradient is smaller than the gradient of the historical trajectory. It would be interesting to explore and explain the mechanism of this boosting but we shall not find its explanation in the Unified Growth Theory. This theory does not even notice this feature.

Africa

Results of the analysis for Africa are presented in Figure 6. Africa is also made of less-developed countries (BBC, 2014; Pereira, 2011) so according to the Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) it should have experienced stagnation in the economic growth until around 1900 followed by a clear takeoff from stagnation to growth around that year. These expectations are contradicted by the economic growth data because (1) economic growth was not stagnant but hyperbolic (Nielsen, 2015d; 2016b), (2) there was no takeoff from stagnation to growth around 1900 or around any other time (3) shortly after the expected time of the takeoff, economic growth in Africa started to be diverted to a slower trajectory.

As discusses elsewhere (Nielsen, 2015d; 2016b), there was an acceleration in the economic growth in Africa around 1820. However, this acceleration occurred significantly earlier than the expected takeoff around 1900 and it was not a transition from stagnation to growth but from growth to growth. Even more specifically, it was a transition from the hyperbolic growth to another hyperbolic growth. This acceleration can be explained by noticing that it appears to coincide with the intensified colonisation of Africa (Duignan & Gunn, 1973; McKay, Hill, Buckler, Ebrey, Beck, Crowston, & Wiesner-Hanks, 2012; Pakenham, 1992). The fast increasing GDP after 1820 was not reflecting the rapidly improving living conditions of the African population brought about by the beneficial changes caused by the Industrial Revolution but the rapidly increasing wealth of new settlers and their countries of origin at the expense of the deploring living conditions of the native populations.

The takeoff from stagnation to growth, claimed by the Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a), did not happen in the region where it should have been prominently present. Economic growth was always stable in Africa (Nielsen, 2015d; 2016b) and now it is being diverted to a slower trajectory. Escape from the Malthusian trap never happened because there was no trap. Economic growth was never stagnant in Africa but hyperbolic.

Latin America

Results of the analysis of the economic growth in Latin America are presented in Figure 7. Latin America is also made of less-developed countries (BBC, 2014; Pereira, 2011) so again, according to the Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a), economic growth in this region should have been stagnant until around 1900 and fast-increasing from around that year. This pattern of growth is stunningly contradicted by data, the same data, which were used, but never properly analysed, during the formulation of this theory. At the time of the claimed "stunning" and "remarkable" escape from Malthusian trap (Galor, 2005a, pp. 177, 220) economic growth in Latin America was already diverted to a *slower* trajectory.

Summary and conclusions

Results of the mathematical analysis of Maddison's data (Maddison, 2010) show convincingly that takeoffs from stagnation to growth, claimed repeatedly in the Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) never happened. The growth of the GDP was not stagnant but hyperbolic and, in general, remarkably stable.

It is essential to understand that claims about the existence of the epoch of Malthusian stagnation in the economic growth or in the growth of human population are not supported by the scientifically-analysable data. They are based on conjectures and impressions and they introduce the unwelcome and undesirable ballast in the economic and demographic research, directing them into unproductive channels, which move the economic and demographic research away from science and develop them into a fiction, because in the absence of scientifically analysable data the concepts of stagnation and of the dramatic escape from the mythical Malthusian trap are supported by creative writing.

A clear way of demonstrating that the doctrine of Malthusian stagnation and its effects on the economic growth or on the growth of human population is incorrect is by demonstrating the absence of the takeoffs from the supposed stagnation to growth. As demonstrated here, such takeoffs did not exist in the economic growth. They also did not exist in the growth of human population (Nielsen, 2016c; 2016d). Demographic Transition Theory, the only theory used by demographers to explain the historical growth of human population, also claims the existence of Malthusian stagnation followed by a dramatic takeoff from stagnation to growth but this theory is repeatedly contradicted by data (Nielsen, 2016e).

Slow economic growth or the growth of human population is routinely interpreted as stagnation but such interpretations are incorrect because the slow growth is an integral part of the hyperbolic growth, which cannot be divided into slow and fast components (Nielsen, 2014) and which has to be interpreted as a whole by using the same mechanism for the whole distribution. We already know that the growth of human population during the AD and BC eras was not stagnant but hyperbolic from at least 10,000 BC (Nielsen, 2016c; von Foerster, Mora & Amiot, 1960). We do not have mathematically-analysable data for the economic growth over such a long time, but the data we have (Maddison, 2010) show conclusively that during the time described by these data, economic growth was also hyperbolic and consequently that it was not stagnant. Furthermore, we have also proven that Galor's concept of the existence of the three regimes of growth is contradicted by the analysis of the economic growth in Western Europe, Eastern Europe, Asia, countries of the former USSR, Africa and Latin America (Nielsen, 2015a; 2015b; 2015c; 2015d; 2016a).

There is no scientific support for the concept of Malthusian stagnation and for the dramatic escape from the Malthusian trap, which is supposed to have been manifested in the dramatic takeoffs. Mathematically analysable data describing economic growth and the growth of human population show repeatedly and consistently that takeoffs from stagnation to growth never happened because there was no stagnation. Mathematically analysable data show repeatedly and consistently that the economic growth and the growth of human population were hyperbolic. Concepts of prolonged stagnation followed by a "remarkable" or "stunning" escape from Malthusian trap (Galor, 2005a, pp. 177, 220) are repeatedly and consistently contradicted by data.

In science, such overwhelming evidence would have been more than sufficient to show that the theory is unacceptable and that it should be either thoroughly revised or rejected and replaced by a more suitable theory, a theory based on a scientific analysis of data, a reliable theory, which could be used in the economic growth research. In its present form, Unified Growth Theory is neither reliable nor useful. In fact it is strongly misleading.

Our analysis of Maddison's data (Maddison, 2010) shows not only that the concept of Malthusian regime of stagnation followed by dramatic escapes from Malthusian trap is incorrect but also that the concept of the differential takeoffs is incorrect because we cannot have differential takeoffs without takeoffs.

Unified Growth Theory is riddled with questionable claims and interpretations. In due time, we shall demonstrate that this theory is contradicted by regional GDP/cap data in much the same way as it is contradicted by the global data (Nielsen, 2015e). We shall show that this theory is contradicted by the economic growth in the UK, the centre of the Industrial Revolution where the Unified Growth Theory should have the strongest support. It can be also shown that this theory is contradicted by the economic growth in other individual countries.

We shall demonstrate that the postulate of the great divergence is also based on the incorrect interpretation of the mathematical properties of hyperbolic distributions. Furthermore, we shall demonstrate that Galor's repeated interpretation of growth rates of income per capita is incorrect.

In its present form, Unified Growth Theory is unacceptable. In order to improve it, it would be necessary to examine it closely to determine not only how much of it is based on the incorrect interpretation of data but also how much is just a pure fantasy. However, the best solution would probably be to replace it by a new theory.

Close analysis of Maddison's data (Maddison, 2010) opens new and fascinating avenues for the economic research. Rather than devoting time and financial resources on explaining features based on impressions and conjectures, we can focus our attention of explaining the features confirmed by the scientific analysis of data. In particular, the relevant and still unanswered questions are why the historical economic growth was hyperbolic, what mechanism should we use to explain this type of growth and why, relatively recently, the economic growth, global and regional, has been diverted to generally slower trajectories. Even the temporarily slightly boosted economic growth in Asia appears to be also a part of the generally-observed diversions to slower trajectories.

Appendix

World Economic Growth

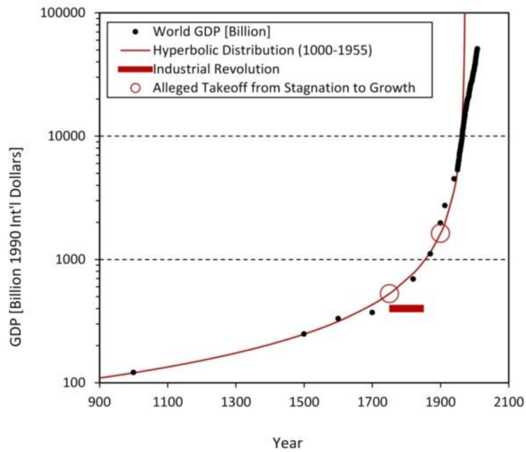

Figure 1. *No takeoffs from stagnation to growth. Two postulated takeoffs are indicated (Galor, 2005a; 2008a; 2011; 2012a): for developed regions around 1750 and for less-developed regions around 1900. The world economic growth was not stagnant but hyperbolic and it was remarkably stable. Industrial Revolution, "the prime engine of economic growth" (Galor, 2005a, p. 212), had no impact on changing the economic growth trajectory. Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) is contradicted by data.*

Western Europe

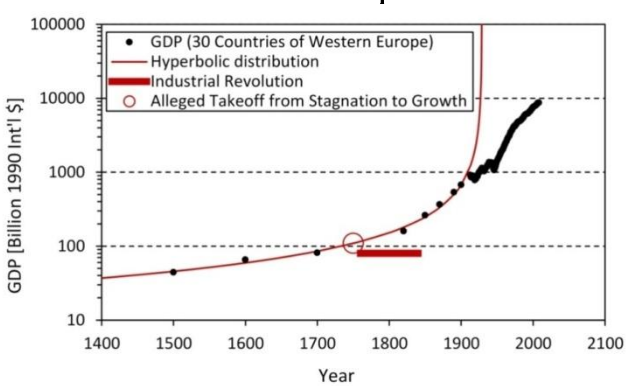

Figure 2. *No takeoff from stagnation to growth. Economic growth in Western Europe was not stagnant but hyperbolic and it was remarkably stable. Industrial Revolution, "the prime engine of economic growth" (Galor, 2005a, p. 212), had no impact on changing the economic growth trajectory where this "engine" should have worked most efficiently. Unified Growth Theory (Galor, 2005a, 2008a, 2011, 2012a) is contradicted by data.*

Eastern Europe

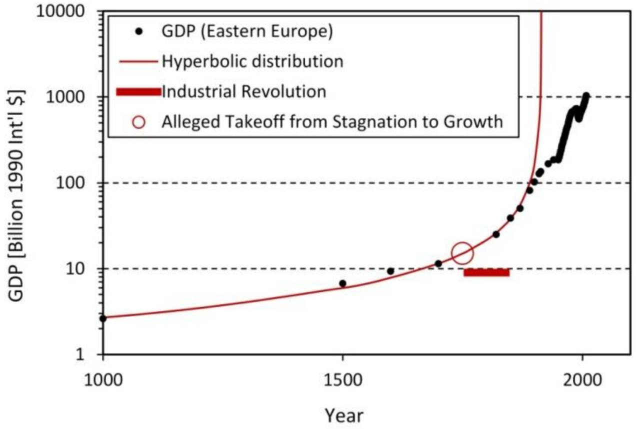

Figure 3. *No takeoff from stagnation to growth. Economic growth in Eastern Europe was not stagnant but hyperbolic and it was remarkably stable. Industrial Revolution, "the prime engine of economic growth" (Galor, 2005a, p. 212), had no impact on changing the economic growth trajectory. Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) is contradicted by data.*

Former USSR

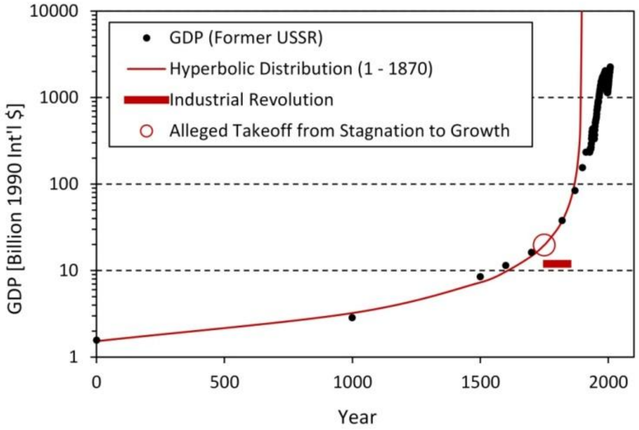

Figure 4. *No takeoff from stagnation to growth. Economic growth in the former USSR was not stagnant but hyperbolic and it was remarkably stable. Industrial Revolution, "the prime engine of economic growth" (Galor, 2005a, p. 212), had no impact on changing the economic growth trajectory. Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) is contradicted by data.*

Asia

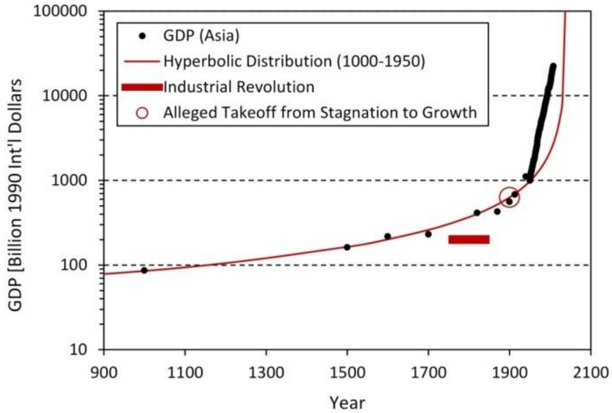

Figure 5. *No takeoff from stagnation to growth. Economic growth in Asia was not stagnant but hyperbolic before the supposed takeoff and it was remarkably stable. The minor boosting after the supposed takeoff was not a transition from stagnation to growth but a transition from growth to growth. It was similar to the commonly-observed transitions to slower trajectories but in this case it was preceded by a minor and temporary boosting. Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) is contradicted by data.*

Africa

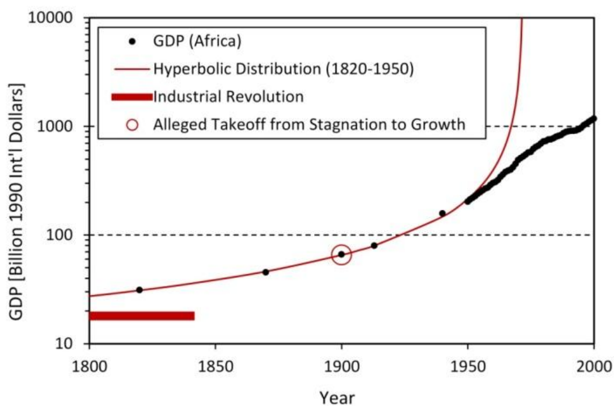

Figure 6. *No takeoff from stagnation to growth. Economic growth in Africa was not stagnant but hyperbolic. Unified Growth Theory (Galor, 2005a, 2008a, 2011, 2012a) is contradicted by data. Shortly after the supposed dramatic but non-existent escape from the postulated Malthusian trap, economic growth in Africa started to be diverted to a slower trajectory.*

Latin America

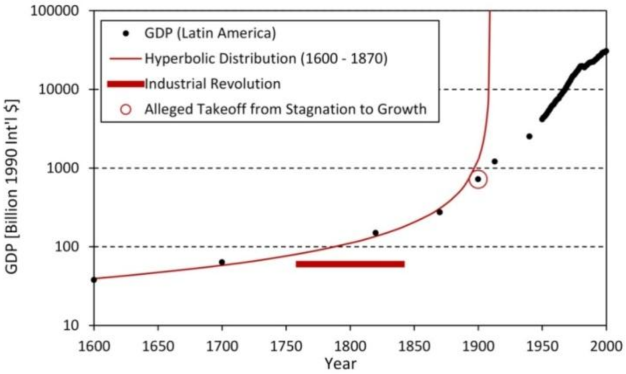

Figure 7. *No takeoff from stagnation to growth. Economic growth in Latin America was not stagnant but hyperbolic. At the time of the supposed takeoff, economic growth in Latin America was already following a slower trajectory. The supposed takeoff is replaced by a slower growth. The "spectacular" or "stunning" escapes from Malthusian trap (Galor, 2005a, pp. 177, 220) never happened because there was no stagnation and no trap. Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) is contradicted by data.*

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5. Unified Growth Theory contradicted by the mathematical analysis of the historical growth of human population

Introduction

Historical economic growth can be studied using the Gross Domestic Product (GDP). However, to understand the time dependence of the income per capita (GDP/cap) it is necessary to understand not only the economic growth, expressed in terms of the GDP, but also the growth of human population. We have already analysed the GDP data (Nielsen, 2016a; 2016b; 2016d). Now, we shall analyse the growth of human population using the same source of data (Maddison, 2010). The aim of these studies is to investigate the validity of the Unified Growth Theory (Galor, 2005a; 2011) and to understand the mechanism of growth as revealed by data because the correct interpretation of the historical growth can help in the correct interpretation of the current economic growth and of the growth of population.

Our earlier analysis (Nielsen, 2016a; 2016b; 2016d) demonstrated that the historical economic growth, regional and global, was hyperbolic. Thus, if unchecked, the natural tendency for the economic growth is to follow hyperbolic distributions. This type of spontaneous growth is undesirable because hyperbolic distributions escape to infinity at a fixed time. To avoid such a rapid and potentially catastrophic increase economic growth has to be closely monitored and controlled.

Analysis published over 50 years ago (von Foerster, Mora & Amiot, 1960) demonstrated that the growth of the world population was also hyperbolic during the AD era. The follow-up analysis (Nielsen, 2016c) demonstrated that the growth of the world population was hyperbolic not only during the AD era but also during the BC era, for the total of around 12,000 years. This particular analysis identified two demographic transitions in the past growth of the population: between 500 BC and AD 500 and between AD 1200 and 1400. However, these transitions were of a different kind than the transitions assumed routinely in demographic research. They were not transitions from stagnation to growth but from growth to growth, or more precisely, from hyperbolic growth to hyperbolic growth. The first transition was from a fast hyperbolic growth during the BC era to a significantly slower hyperbolic growth during the AD era. During this transition, the size of human population reached a maximum around AD 1 and after reaching a minimum between AD 400 and 500 it resumed it slower hyperbolic growth during the AD era. However, the starting size of global population in AD 500 was significantly larger than in 10,000 BC and the slower hyperbolic growth increased rapidly to reach a large size of the population in only about 2000 years. During this first demographic transition, the growth rate *decreased* from 0.252% in 500 BC to 0.066% in AD 500. The second transition was hardly noticeable but it resulted in a change from a slow hyperbolic trajectory to a slightly faster hyperbolic trajectory. During this transition, after a short delay in the growth of the population, the growth rate *increased only marginally* from 0.123% in AD 1200 to 0.157% in AD 1400. Currently the growth of the world population experiences a third demographic transition to a yet unknown trajectory.

Unified Growth Theory

The latest and the most elaborate theory describing economic growth is the Unified Growth Theory (Galor, 2005a; 2011). It is not a theory, which is widely accepted by economist and used in their research. In fact, the opposite seems to be true. However, we are using this theory *as an example* for two reasons. *First*, it is a theory, which is firmly based on traditional but erroneous assumptions about the historical economic growth and about the historical growth of human population. Our primary aim here, the same aim we had in earlier publications (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e), is not just to test the validity of this theory or the validity of a similar Demographic Transition Theory (see Nielsen, 2016e and references therein) but to test the validity of the *fundamental postulates* used in the economic and demographic research. *Second*, Unified Growth Theory appears to be the only theory where Maddison's data (Maddison, 2001) were used systematically but unfortunately they were never analysed. They were manipulated and distorted to support preconceived ideas. Now, *precisely the same data* can be used to show that the preconceived ideas used and promoted in this theory are incorrect (Nielsen, 2014; 2015a; 2016a; 2016b; 2016d). Here we have a difference between a study based on the manipulation of data and a study based on the rigorous analysis of data.

In the last years of his life, Magnusson, the world-renown economist, published excellent data describing not only the economic growth as expressed by the Gross Domestic Product (GDP) but also the growth of human population, global, regional and national (Magnuson, 2001; 2010). These data are a treasure trove, which can be used in the economic and demographic research. In particular, they can be used to test the fundamental postulates supporting these two fields of research. Galor used the earlier compilation of these data (Magnuson, 2001) but any of them can be used to test the fundamental postulates supporting economic and demographic research, and in particular to test the validity of the Unified Growth Theory.

Unfortunately, this theory and its fundamental postulates are based on the habitually distorted and self-misleading presentations of data (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). This counterproductive approach to research was used to promote such scientifically-unsupported concepts as the concept of the three regimes of growth (Malthusian regime of stagnation, post-Malthusian regime and sustained-growth regime), the concept of sudden takeoffs from stagnation to growth, the concept of differential timing of takeoffs and the concept of the great divergence. An example of the strongly deceptive and misleading diagrams used in the Unified Growth Theory and in other related publications is shown in Figure 1. (All diagrams are presented in the Appendix.)

Hyperbolic distributions do not have to be distorted to be confusing. They are already sufficiently confusing and it is easy to make mistakes with their interpretations. Hyperbolic distributions have to be carefully and methodically analysed and fortunately their analysis becomes trivial when using the reciprocal values of data (Nielsen, 2014). Displays, such as presented in Figure 1, which is based on a figure presented by Galor (2005a, p. 181), are self-misleading and they inevitably lead to incorrect conclusions.

The correct and accurate display of Maddison's data (Maddison, 2001), precisely the same data as used but never scientifically analysed during the formulation of the Unified Growth Theory (Galor, 2005a, 2011), is presented in Figure 2. Analysis of these data reveals that they follow *monotonically-*

increasing distributions, which are impossible to divide into distinctly-different regimes of growth governed by distinctly-different mechanisms (Nielsen, 2014; 2015a). There was no stagnation and no transition from stagnation to growth. There was no escape from the Malthusian Trap because there was no trap in the economic growth.

Whether expressed by using the GDP or GDP/cap, economic growth was slow over a long time and fast over a short time but it was monotonically increasing all the time. What appears as stagnation was a part of the monotonically-increasing distribution, and what appears as a sudden takeoff was the natural continuation of the same monotonically-increasing distribution.

Attempts to determine the time of the perceived transition from slow to fast growth are bound to be unsuccessful because there was no transition (Nielsen, 2014; 2015a). The growth of the GDP is described by hyperbolic distributions (Nielsen, 2014; 2016a) and the growth of the GDP/cap by the linearly-modulated hyperbolic distributions (Nielsen; 2015a).

One of the fundamental postulates of the Unified Growth Theory is the postulate of the existence of three regimes of growth governed by three distinctly different mechanisms: (1) the Malthusian regime of stagnation, (2) the post-Malthusian regime, and (3) the sustained-growth regime. This postulate applies not only to the growth of the GDP but also to the growth of human population because Galor discusses the growth of income per capita, (GDP/cap), which is made of two components: the growth of the GDP and the growth of population.

According to Galor (2005a; 2008a; 2011; 2012a), Malthusian regime of stagnation was between 100,000 BC and AD 1750 for developed regions and between 100,000 BC and AD 1900 for less-developed regions. The claimed starting time appears to be based entirely on conjecture because Maddison's data are terminated at AD 1 and even they contain significant gaps below AD 1500. The post-Malthusian regime was supposedly between AD 1750 and 1850 for developed regions and from 1900 for less-developed regions. The sustained-growth regime was supposed to have commenced around 1850 for developed regions.

Unified Growth Theory (Galor, 2005a; 2008a; 2011; 2012a) can be tested in many ways but the easiest way to test it is to look for the dramatic takeoffs from stagnation to growth. These takeoffs are described as a "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a: pp. 177, 220). It is a signature, which cannot be missed.

This change in the pattern of growth is described as "the sudden take-off from stagnation to growth" (Galor, 2005a: pp. 177, 220, 277) or as a "sudden spurt" (Galor, 2005a: 177, 220). According to Galor, for developed regions, the end of the Malthusian regime of stagnation coincides with the Industrial Revolution. "The take-off of developed regions from the Malthusian Regime was associated with the Industrial Revolution" (Galor, 2005a: p. 185). Indeed, the Industrial Revolution is considered to have been "the prime engine of economic growth" (Galor, 2005a: p. 212).

The signature of takeoffs is characterised by three features: (1) it should be a prominent change in the pattern of growth, (2) it should be a transition from stagnation to growth and (3) it should occur at the time predicted by the theory. For developed regions, the postulated takeoffs should occur around AD 1750, or around the time of the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994). For less-developed regions, they should occur around 1900. The added advantage of using this simple test is that there are no significant gaps in the data around the time of the postulated takeoffs and consequently the stagnation and the expected prominent transitions from stagnation to growth should be easily identifiable.

A transition from growth to growth is not a signature of the postulated takeoff from stagnation to growth. Thus, for instance, a transition from hyperbolic growth to another hyperbolic growth or to some other steadily-increasing trajectory is not a signature of the sudden takeoff from stagnation to growth. Likewise, a transition at a distinctly different time is not a confirmation of the theoretical expectations.

The takeoffs claimed by Galor are in the income per capita (GDP/cap), which means that there should be takeoffs from stagnation to growth in at least one of these components (in the GDP or in the population or in both of them) at a specific time (Galor, 2008a; 2012a). We have already demonstrated that the Unified Growth Theory is contradicted by the GDP data describing the world economic growth as well as the economic growth in Western Europe, Eastern Europe, countries of the former USSR, Asia, Africa and Latin America (Nielsen, 2016a; 2016b; 2016d). We have also demonstrated that the Unified Growth Theory is contradicted by the data describing the growth of the world income per capita (Nielsen, 2015a). Our next step now is to extend our analysis to the growth of population and thus to extend our study of income per capita, not only global but also regional.

We have already demonstrated that there were no takeoffs in the growth of the GDP. Consequently, to confirm the Unified Growth

Theory we would have to show not only that there were takeoffs from stagnation to growth in the growth of the population but also that these takeoffs occurred at the specific time claimed by Galor (2008a; 2012a), around AD 1750 for developed regions (Western Europe, Eastern Europe and the former USSR) and at around AD 1900 for less developed regions (Asia, Africa and Latin America). We shall now demonstrate that there were no such takeoffs. Thus we shall demonstrate implicitly that there were no takeoffs in the income per capita, which means that Galor's postulate of the differential timing in takeoffs is also contradicted by data, because we cannot have differential timing in takeoffs without takeoffs.

Essentials of the mathematical analysis

Hyperbolic distribution describing growth is represented by a reciprocal of a linear function:

$$S(t) = \frac{1}{a - kt}, \tag{1}$$

where $S(t)$ is the size of the growing entity, in our case the population, while a and k are positive constants.

As pointed out earlier (Nielsen, 2014), hyperbolic distributions are confusing because they create an illusion of being made of two components, slow and fast, with perhaps even a third component in the middle. It is easy to make a mistake with their interpretations. Fortunately, these distributions are easy to analyse by using the reciprocal values of data, $1/S(t)$:

$$\frac{1}{S(t)} = a - kt. \tag{2}$$

In this representation, data follow a decreasing straight line, which obviously cannot be divided into two or three distinctly different components.

Reciprocal values help in an easy and generally unique identification of hyperbolic growth. Apart from serving as an alternative way to analyse data, reciprocal values allow also for the investigation of even small deviations from hyperbolic distributions because deviations from a straight line can be easily noticed.

The illusion of different components also disappears when using semi logarithmic scales of reference. Both types of displays help in an easy identification of disagreements between data and fitted curves for small values of data and we shall use both of these displays.

Growth of the world population

Results of mathematical analysis of the world population are presented in Figures 3 and 4. Reciprocal values of historical data identify uniquely hyperbolic distribution between AD 1000 and around 1950 because they follow a decreasing straight line. From around 1950, the growth of the world population started to be diverted to a slower trajectory but first it was slightly boosted. The boosting was small (it is hardly noticeable in the displayed diagrams) and it did not last long.

Hyperbolic fit to the world population data (Maddison, 2010) is shown in Figure 4. The fit is remarkably good. The point at AD 1 is 75% higher than the fitted curve. This discrepancy is in perfect agreement with the analysis of the growth of the world population over the past 12,000 years (Nielsen, 2016c), which demonstrated a maximum around that year.

Parameters describing hyperbolic trajectory fitting the data between AD 1000 and 1950 are: $a = 7.739 \times 10^{0}$ and $k = 3.765 \times 10^{-3}$. Its singularity is at $t = 2056$. However, from around 1950, the growth of the world population started to be diverted to a slower trajectory bypassing the singularity by a safe margin of 106 years. This diversion was first manifested in a minor and short-lasting boosting of the growth of the world population.

The data are in disagreement with the Unified Growth Theory (Galor, 2005a; 2011), which erroneously claims stagnation and takeoffs from stagnation to growth. There was no stagnation but monotonically-increasing hyperbolic distribution. There were also no takeoffs from stagnation to growth around AD 1750 for developed regions and around AD 1900 for less-developed regions because there was no stagnation and because hyperbolic growth continued undisturbed. If there were such takeoffs in the respective regions they must have been too weak to change the growth of global population because the growth trajectory was remarkably stable during these supposed takeoffs. Furthermore, our analysis shows that the Industrial Revolution had no impact on the growth trajectory. Unified Growth Theory is yet again demonstrably contradicted by data.

With the absence of takeoffs in the growth of the population and with the earlier demonstrated absence of takeoffs in the growth of the GDP (Nielsen, 2016a), this analysis shows that there were no takeoffs in the income per capita (GDP/cap) distribution confirming our previous analysis based on the earlier compilation of Maddison's data (Maddison, 2001). Unified Growth Theory (Galor, 2005a; 2011) is contradicted by data, which were used but never analysed during the formulation of this theory.

Western Europe

Growth of population in Western Europe is shown in Figures 5 and 6. Western Europe is represented by the total of 30 countries: Austria, Belgium, Denmark, Finland, France, Germany, Italy, The Netherlands, Norway, Sweden, Switzerland, United Kingdom, Greece, Portugal, Spain and by 14 small, but unspecified countries. Ireland is missing in this list because it was included only from 1921.

The straight line fitting the reciprocal values of data, shown in Figure 5, identifies uniquely hyperbolic distribution between AD 1000 and around 1915. Parameters describing hyperbolic growth in Western Europe are: $a = 7.542 \times 10^{1}$ and $k = 3.749 \times 10^{-2}$. The point of singularity is at $t = 2012$. From around 1915, the growth of population in Western Europe started to be diverted to a slower, but still fast-increasing, trajectory bypassing the singularity by a safe margin of 97 years. The size of the population in AD 1 is 89% larger than for the fitted hyperbolic distribution. This discrepancy is probably reflecting the maximum in the growth of the world population around that year (Nielsen, 2016c).

Figures 5 and 6 show that hyperbolic growth between AD 1000 and 1915 remained undisturbed. Industrial Revolution had absolutely no impact on changing the hyperbolic growth trajectory in the region where the effects of this revolution should be most prominent. There was no takeoff from stagnation to growth at the postulated time (Galor, 2008a; 2012a) because there was no stagnation in the growth of population. There was even no transition to a faster hyperbolic trajectory.

With the absence of the takeoff in the growth of the population in Western Europe and with the earlier demonstrated absence of the takeoff in the growth of the GDP (Nielsen, 2016a), this analysis shows that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory (Galor, 2005a, 2011) is contradicted by data, which were used but never analysed during the formulation of this theory.

Eastern Europe

Results of analysis of the growth of population in Eastern Europe are summarized in Figures 7 and 8. Reciprocal values of data shown in Figure 7 identify uniquely hyperbolic distribution between AD 1000 and around 1935. From that year, the growth of population started to be diverted to a slower trajectory.

Hyperbolic parameters are: $a = 3.055 \times 10^2$ and $k = 1.525 \times 10^{-1}$. The point of singularity is at $t = 2003$. Figures 7 and 8 demonstrate that the Industrial Revolution had no impact on the trajectory of the growth of population in Eastern Europe and that there was no takeoff from stagnation to growth at the postulated time (Galor, 2008a; 2012a) because there was no stagnation but hyperbolic growth. There was even no takeoff to a faster hyperbolic growth. The size of the population in AD 1 was 45% higher than the calculated curve reflecting probably the maximum in the growth of the world population around that year (Nielsen, 2016c).

With the absence of the takeoff in the growth of population in Eastern Europe and with the earlier demonstrated absence of the takeoff in the growth of the GDP (Nielsen, 2016a), this analysis shows that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory (Galor, 2005a; 2011) is contradicted by data, which were used but never analysed during the formulation of this theory.

Former USSR

The analysis of data for countries of the former USSR is presented in Figures 9 and 10. Reciprocal values shown in Figure 9 identify uniquely hyperbolic distribution between AD 1 and around 1920. Hyperbolic fit to the data is between AD 1 and 1870. Parameters fitting the data are: $a = 2.618 \times 10^2$ and $k = 1.333 \times 10^{-1}$. The singularity is at $t = 1965$. From around 1920, the growth of population in the former USSR started to be diverted to a slower trajectory, bypassing the singularity by around 45 years.

Figures 9 and 10 show that the Industrial Revolution had no impact on shaping the growth of human population in countries of the former USSR. There was also no takeoff from stagnation to growth around the postulated time Galor (2008a; 2012a) or around any other time because the growth was not stagnant but hyperbolic. There was even no transition to a faster hyperbolic trajectory but there was a transition to a slower, non-hyperbolic growth around 1920.

With the absence of the takeoff in the growth of the population in countries of the former USSR and with the earlier demonstrated absence of the takeoff in the growth of the GDP (Nielsen, 2016a), this analysis shows that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory (Galor, 2005a; 2011) is contradicted by data, which were used but never analysed during the formulation of this theory.

Asia

Analysis of the growth of human population in Asia (including Japan) is summarised in Figures 11 and 12. Reciprocal values presented in Figure 11 identify uniquely hyperbolic distribution between AD 1000 and around 1920. Parameters describing this distribution are: $a = 1.068 \times 10^{1}$ and $k = 4.999 \times 10^{-3}$. The point of singularity is at $t = 2135$.

Asia is made primarily of less-developed countries (BBC, 2014; Pereira, 2011) and consequently, according to Galor (2008a; 2012a), the growth of human population in Asia should have been characterised by stagnation until around 1900, the year marking the supposed stunning escape from the Malthusian trap, the supposed escape manifested by the postulated dramatic takeoff. (The population of Japan before AD 1900 was on average less than 4% of the total population of Asia.) The data and their analysis show that there was no stagnation, at least from AD 1000 and there was also no expected takeoff.

The data reveal a steadily increasing hyperbolic growth until around 1920. From around that year the growth of population was diverted to a faster trajectory. This boosting can be seen clearly in Figures 11 and 12 and it occurred close to the time of the postulated takeoff from stagnation to growth. However, it was not a transition from stagnation to growth but from hyperbolic growth to a slightly faster trajectory of a different kind. It is, therefore, not the takeoff postulated by Galor. Furthermore, it was only a temporary boosting, which is now returning to the original hyperbolic trajectory and, as indicated by the reciprocal values of data, this new trend is likely to be slower than the original trajectory.

With the absence of the postulated takeoff in the growth of population in Asia and with the earlier demonstrated absence of the takeoff in the growth of the GDP (Nielsen, 2016a), this analysis shows that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory (Galor, 2005a;

2011) is contradicted by data, which were used but never analysed during the formulation of this theory.

Africa

Results of analysis of the growth of human population in 57 African countries are presented in Figures 13 and 14. Reciprocal values identify uniquely *two* hyperbolic trajectories: AD 1-1840 and AD 1840-1980. At first it was a slow hyperbolic growth characterised by parameters $a = 5.794\times10^{1}$ and $k = 2.473\times10^{-2}$ and by the singularity at $t = 2343$. Then, around 1840, this slow hyperbolic growth was replaced by a significantly faster hyperbolic growth characterised by parameters $a = 1.571\times10^{2}$ and $k = 7.834\times10^{-2}$ and by singularity at $t = 2006$. Defined by the parameter *k*, this new growth was 3.2 times faster than the earlier hyperbolic growth. From around 1980, this fast hyperbolic growth was diverted to a slower, non-hyperbolic trajectory, bypassing singularity by 26 years.

Africa is also made of less-developed countries (BBC, 2014; Pereira, 2011) so according to Galor (2008a; 2012a) it should have experienced stagnation until around 1900 followed by a clear takeoff around that year. These expectations are contradicted by data because (1) the growth of population was not stagnant but hyperbolic until around 1980 and (2) because there was no takeoff from stagnation to growth around 1900 or around any other time. In fact, around that time hyperbolic growth continued unaffected in contradiction of the wished-for interpretations.

The acceleration in the growth of human population in Africa occurred around 1840, but it was not a transition from stagnation to growth but from growth to growth. Even more precisely, it was a transition from hyperbolic growth to another hyperbolic growth.

Africa is the only region where the commencement of the rapid growth of population coincides with the Industrial Revolution but it also the region, which demonstrates that the usually-claimed effects of Industrial Revolution are contradicted by empirical evidence. According to the generally accepted interpretation, Industrial Revolution improved medical care and introduced many other beneficial effects, which were supposed to have caused population explosion. Data for Africa demonstrate that this wished-for mechanism does not work because the growth of population on this continent coincides with the rapidly-deteriorating living conditions of native populations brought about by the intensified colonisation (Duignan & Gunn, 1973; McKay, Hill, Buckler, Ebrey, Beck, Crowston, & Wiesner-Hanks, 2012; Pakenham, 1992).

This fast growth of population in Africa, which commenced around the time when living conditions started to deteriorate rapidly, is easy to explain by noticing that the growth rate of population is *directly proportional to the level of deprivation* (Nielsen, 2013), the process, which is diametrically opposite to the usually claimed influence of the Industrial Revolution. It appears, therefore, that it is not the improved living conditions but the increased level of deprivation that have a stimulating effect on the growth of population.

With the absence of the takeoff around AD 1900 in Africa and with the earlier demonstrated absence of the takeoff in the growth of the GDP (Nielsen, 2016a), this analysis shows that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory (Galor, 2005a; 2011) is contradicted by data, which were used but never analysed during the formulation of this theory.

Latin America

Results of analysis of population growth in Latin America are presented in Figures 15 and 16. Data for Latin America are difficult to analyse because there was a significant decline in the growth of population between AD 1500 and 1600 but they also appear to follow two distinctly different hyperbolic trajectories, which can be easily identified using the reciprocal values of data (see Figure 15). However, the identification of the first trajectory is not as clear as for Africa. The identification of the second hyperbolic trajectory is more convincing. Tentative conclusion is that the growth of population in Latin America was following a slow hyperbolic distribution between AD 1 and 1500 and a fast hyperbolic distribution between AD 1600 and around 1900.

The tentatively assigned slow hyperbolic growth between AD 1 and 1500 is characterised by parameters $a = 1.765 \times 10^{2}$ and $k = 8.242 \times 10^{-2}$. Its singularity is at $t = 2142$. The better determined fast hyperbolic growth between AD 1600 and 1900 is characterised by parameters $a = 6.561 \times 10^{2}$ and $k = 3.371 \times 10^{-1}$. Its singularity is at $t = 1947$. Defined by the parameter *k*, this growth was 4.1 times faster than the earlier hyperbolic growth. From around 1900, this fast hyperbolic growth started to be diverted to a slower trajectory bypassing the singularity by 47 years. The transition from the earlier apparent hyperbolic growth to a new and rapid hyperbolic growth, which occurred between around AD 1500 and 1600 appears to coincide with the commencement of the Spanish conquest (Bethell, 1984) and with the rapidly-deteriorating living conditions. The mechanism of this fast growth of population in Latin America appears to be the same as the mechanism of the fast growth in Africa, which commenced around AD 1840.

Latin America is also made of less-developed countries (BBC, 2014; Pereira, 2011) so again, according to Galor (2008a; 2012a), the growth of human population in this regions should have been stagnant until around 1900 and fast-increasing from around that year. This pattern of growth is contradicted by data. The data show a diametrically different pattern: (1) there is no convincing evidence of the existence of stagnation over the entire range of time between AD 1 and 1900 (there are no signs of Malthusian oscillations) but there is a sufficiently convincing evidence of hyperbolic growth particularly between AD 1600 and 1900; (2) there was no takeoff from stagnation to growth at any time; and (3) at the time of the postulated takeoff in 1900 the growth of population started to be diverted to a slower trajectory. The wished-for takeoff is replaced by a slower growth. However, even if we had a takeoff around that time it would have been a takeoff of a different kind, not a takeoff from stagnation to growth as required by the Unified Growth Theory (Galor, 2005a; 2011) but a takeoff from growth to growth.

With the absence of the takeoff in the growth of population in Latin America and with the earlier demonstrated absence of the takeoff in the growth of the GDP (Nielsen, 2016a), this analysis shows that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory (Galor, 2005a; 2011) is contradicted by data, which were used but never analysed during the formulation of this theory.

Summary and conclusions

Results of mathematical analysis of the historical growth of human population are summarised in Table 1. The listed parameters, a and k, are for the fitted hyperbolic distributions.

This analysis demonstrates that the natural tendency for the historical growth of human population was to increase hyperbolically. In general, there is a remarkably good agreement between the data and the calculated hyperbolic distributions.

Unlike the more familiar exponential distributions, which are easier to understand because they show more readily a gradually increasing growth, hyperbolic distributions appear to be made of two or maybe even three components: a slow component, a fast component and perhaps even a transition component located between the apparent slow and fast components. The illusion is so strong that even the most experienced researchers can be deceived particularly if they have no access to good sets of data, which was in the past. Now, however, excellent data are available (Maddison, 2001; 2010) and we can use them to check the earlier interpretations of economic growth and of the growth of human population.

Table 1. *Summary of the mathematical analysis of the historical growth of population*

Region/Countries	a	k	Hyperbolic Range	Singularity	Proximity	Takeoff
World	7.739×10^{0}	3.765×10^{-3}	1000 – 1950	2056	106	X
Western Europe	7.542×10^{1}	3.749×10^{-2}	1000 – 1915	2012	97	X
Eastern Europe	3.055×10^{2}	1.525×10^{-1}	1000 – 1935	2003	68	X
Former USSR	2.618×10^{2}	1.333×10^{-1}	1 – 1920	1965	45	X
Asia	1.068×10^{1}	4.999×10^{-3}	1000 – 1920	2135	215	X
Africa	5.794×10^{1}	2.473×10^{-2}	1 – 1840	2343		
	1.571×10^{2}	7.834×10^{-2}	1840 – 1980	2006	26	X
Latin America	1.765×10^{2}	8.242×10^{-2}	1 – 1500	2142		
	6.561×10^{2}	3.371×10^{-1}	1600 – 1900	1947	47	X

Notes: a and k – Hyperbolic growth parameters [see eqn (1)]. *Hyperbolic Range* - The empirically-confirmed range of time when the growth of population can be described using hyperbolic distributions. *Singularity* - The time of the escape to infinity for a given hyperbolic distribution. *Proximity* - Proximity (in years) of the singularity at the time when the growth of population departed from the hyperbolic growth to a new trajectory. *X* - No takeoff from stagnation to growth. Takeoffs claimed by the Unified Growth Theory (Galor, 2005a, 2008a, 2011, 2012a) did not happen because there was no stagnation in the growth of population. The growth was monotonically hyperbolic.

The postulate of the existence of the epoch of Malthusian stagnation is suggested by a slow growth over a long time but this slow growth is just a part of the hyperbolic growth, which can be convincingly identified using reciprocal values. Hyperbolic distributions create also the illusion of a sudden takeoff but this feature is also a part of hyperbolic growth.

Hyperbolic growth is slow over a long time and fast over a short time but the slow and fast growth are the integral features of the same *monotonically increasing* distribution, which is easier to understand by using the reciprocal values of the growing entity (Nielsen, 2014). In such displays, the illusion of distinctly different components disappears because hyperbolic growth is then represented by a decreasing straight line, which is easy to understand. It then becomes obvious that hyperbolic distribution cannot be divided into distinctly different sections governed by different mechanism because it makes no sense to divide a straight line into arbitrarily chosen sections and claim different mechanism for such arbitrarily-selected section. It is then also clear that it is impossible to determine the transition from a slow to fast growth. Which point on a straight line should we select to identify such a transition? The transition does not happen at any specific time but gradually over the whole range of time.

Our analysis shows that the Industrial Revolution had generally no impact on the growth of human population. The only boosting of growth, which coincided with the Industrial Revolution was in Africa but this boosting appears to have not been caused by the usually assumed beneficial effects of the Industrial Revolution but by the rapidly deteriorating living conditions associated with the colonisation of Africa. Our analysis also shows that the postulated takeoffs from stagnation to growth (Galor, 2005; 2008a; 2011; 2012a) never happened because there was no stagnation in the growth of population. We have shown earlier (Nielsen, 2016a) that there were no takeoffs in the growth of the GDP, global or regional. The demonstrated now absence of takeoffs in the growth of population shows that the claimed by Galor takeoffs in the income per capita (GDP/cap) did not exist.

Galor describes the imaginary and non-existing features, which have nothing to do with the economic growth or with the growth of human population, features which were conjured from such habitually distorted displays as shown in Figure 1, interpretations based on impressions, which were never checked by the scientific analysis of data. They describe a world of fiction. All his explanations of the mechanism of economic growth based on these and other imaginary features are not only irrelevant but also misleading.

Galor's Unified Growth Theory is fundamentally incorrect and is repeatedly contradicted by data (Nielsen, 2014; 2015a; 2016a; 2016b; 2016d), ironically by the same data, which were used but never analysed during the formulation of this theory. The evidence contradicting the fundamental postulates of the Unfired Growth Theory is overwhelming and further evidence will be presented in forthcoming publications. This evidence questions not only the fundamental postulates of the Unified Growth Theory but also many similar postulates used traditionally in economic and demographic research, postulates which are based largely on impressions and conjectures but postulates, which are repeatedly contradicted by the analysis of Maddison's data (Maddison, 2001; 2010) as well as by other related research (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975; see also Nielsen, 2016c; 2016e and references therein).

In science, just one contradicting evidence is sufficient to show that contradicted postulates need to be closely examined and revised. Unified Growth Theory is scientifically unacceptable and so are also many traditional interpretations of the historical economic growth and of the growth of human population.

Data and their analysis suggest new lines of research. There is no need to waste time to discuss and explain the mechanism of stagnation and takeoffs from stagnation to growth because these features are contradicted by data. What needs to be explained is why the historical economic growth and the growth of human population were hyperbolic and why relatively recently they were diverted to slower but still fast-increasing trajectories. There is also a need to find out how to control these fast-increasing trajectories.

Unified Growth Theory is not only spurious but also dangerously misleading. It claims erroneously that after a long epoch of stagnation in the economic growth we have now entered a sustained-growth regime. This concept suggests a prosperous and secure future. However, mathematical analysis of data shows that the past economic growth was stable and sustainable but now it increases alarmingly fast (Nielsen, 2015b; 2016a). The false sense of security is replaced by the realisation of the urgent need to control and regulate economic growth and by the generally know need to control the growth of population.

Appendix

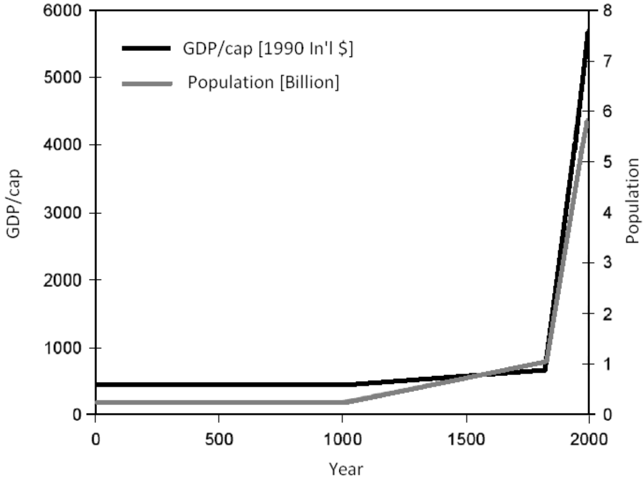

Figure 1. *Example of the ubiquitous, grossly-distorted and self-misleading diagrams used to create the Unified Growth Theory (Galor, 2005a, 2011). Madison's data (Maddison, 2001) were used during the formulation of this theory but they were never analysed. Such state-of-the-art was used to construct a system of scientifically-unsupported concepts, interpretations and explanations.*

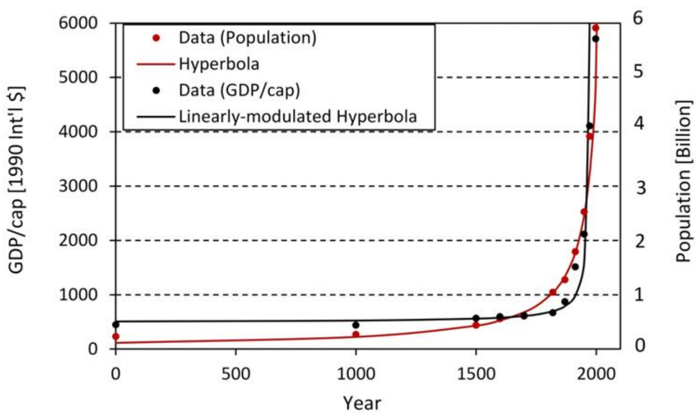

Figure 2. *The same data (Maddison, 2001) as used in Figure 1 but now displayed accurately and analysed. They follow monotonically-increasing distributions, which cannot be divided into distinctively-different components (Nielsen, 2014, 2015a).*

World Population

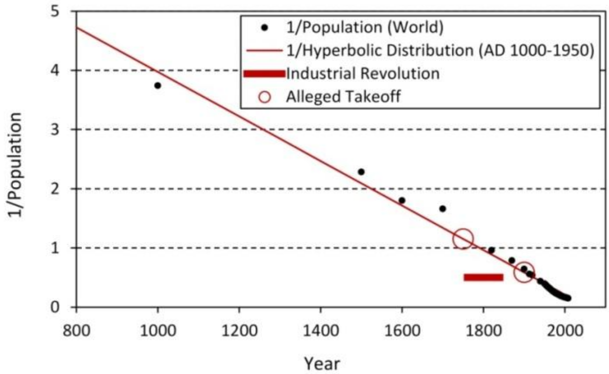

Figure 3. *Reciprocal values of the world population data (Maddison, 2010) identify uniquely hyperbolic distribution between AD 1000 and around 1950 because they follow a decreasing straight line. From around 1950, the growth of population started to be diverted to a new trajectory. Industrial Revolution had no impact on changing the growth trajectory. There were also no takeoffs from stagnation to growth around the postulated times for developed and less-developed regions (Galor, 2008a, 2012a). This analysis and the absence of takeoffs in the corresponding GDP distribution (Nielsen, 2016a) show that there were no takeoffs in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

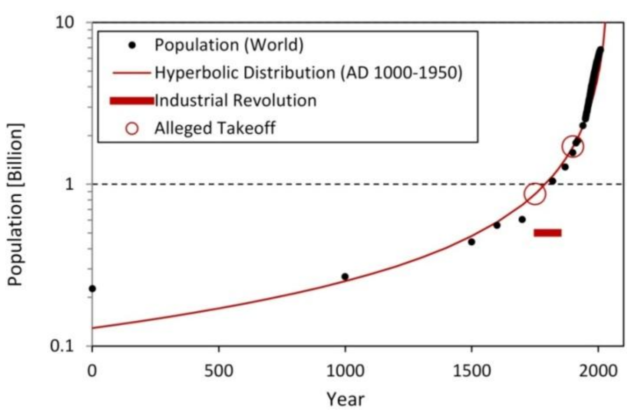

Figure 4. *Growth of the world population. Data of Maddison (2010) are compared with hyperbolic distribution. The point at AD 1 is 75% higher than the fitted curve because there was a maximum in the growth of the world population around that time (Nielsen, 2016c). Industrial Revolution had no impact on the growth of population. There were no takeoffs from stagnation to growth around the postulated time (Galor, 2008a, 2012a) for developed and less-developed regions. This analysis and the absence of takeoffs in the corresponding GDP distribution (Nielsen, 2016a) show that there were no takeoffs in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

Western Europe

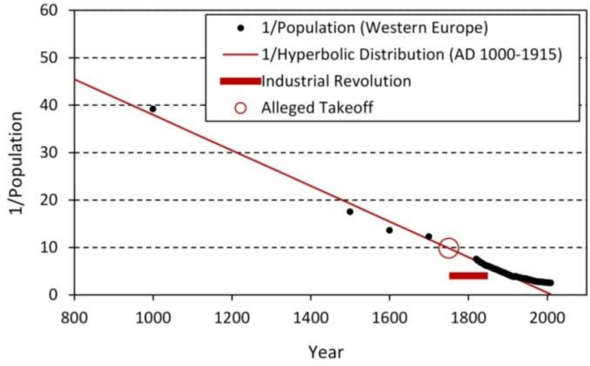

Figure 5. *Reciprocal values of population data for Western Europe (Maddison, 2010) identify uniquely hyperbolic distribution between AD 1000 and around 1915 because they follow a decreasing straight line. From around 1915, the growth of population started to be diverted to a slower trajectory. Industrial Revolution had no impact on changing the growth trajectory in the region where its influence should have been most pronounced. There was also no takeoff from stagnation to growth around the postulated time (Galor, 2008a, 2012a). This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

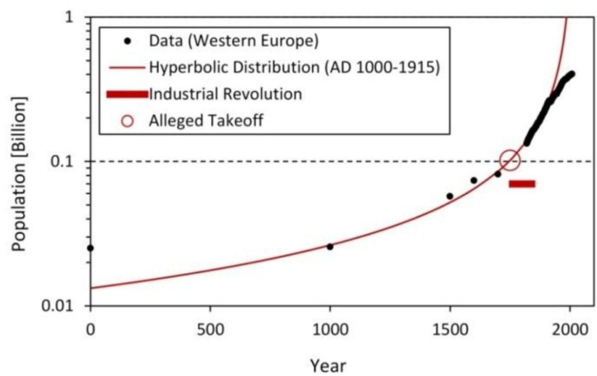

Figure 6. *Growth of human population in Western Europe. Data of Maddison (2010) are compared with hyperbolic distribution. The point at AD 1 is 89% higher than the fitted curve. This discrepancy might be reflecting the maximum in the growth of the world population (Nielsen, 2016c). Industrial Revolution had no impact on the growth of population in Western Europe where the effects of this revolution should have been most prominent. There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a, 2012a). This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

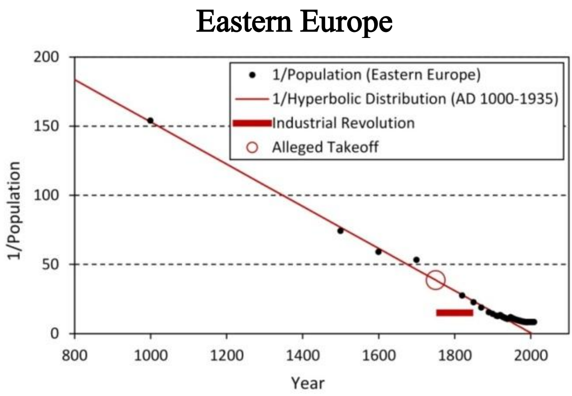

Figure 7. *Reciprocal values of population data for Eastern Europe (Maddison, 2010) identify uniquely hyperbolic distribution between AD 1000 and around 1935 because they follow a decreasing straight line. From around 1935, hyperbolic growth started to be diverted to a slower trajectory. Industrial Revolution had no impact on changing the growth trajectory in Eastern Europe. There was also no takeoff from stagnation to growth around the postulated time (Galor, 2008a, 2012a). This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

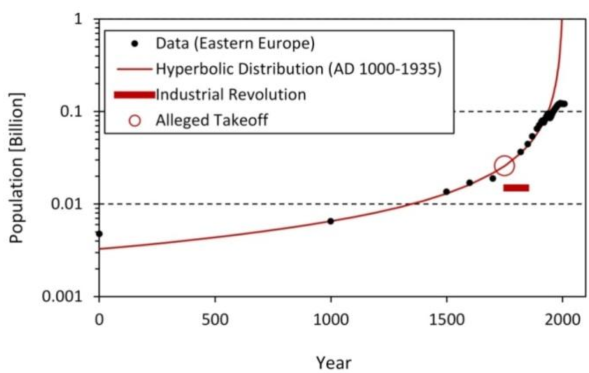

Figure 8. *Growth of human population in Eastern Europe. Data of Maddison (2010) are compared with hyperbolic distribution. The point at AD 1 is 45% higher than the fitted curve. This discrepancy might be reflecting the maximum in the growth of the world population (Nielsen, 2016c) around that time. Industrial Revolution had no impact on the growth of population in Eastern Europe. There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a). This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

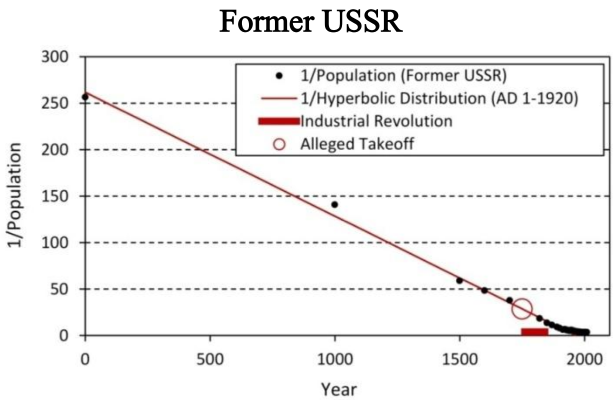

Figure 9. *Reciprocal values of population data for the former USSR (Maddison, 2010) identify uniquely hyperbolic distribution between AD 1 and 1920 because they follow closely the decreasing straight line. From around 1920 the growth started to be diverted to a slower trajectory. Industrial Revolution had no impact on changing the growth trajectory. There was also no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a) or around any other time because there was no stagnation. There was even no transition to a faster hyperbolic growth. This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

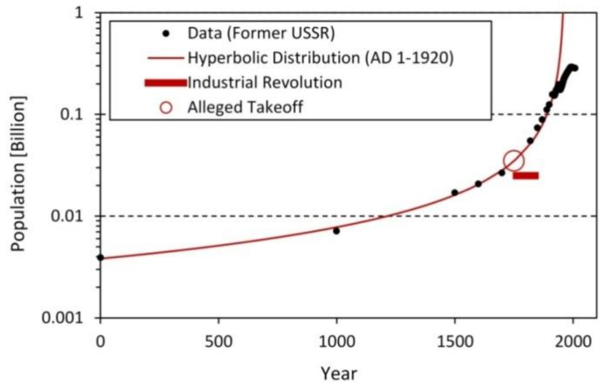

Figure 10. *Growth of human population in countries of the former USSR. Data of Maddison (2010) are compared with hyperbolic distribution. Industrial Revolution had no impact on the growth of population. There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a) or around any other time because there was no stagnation. This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

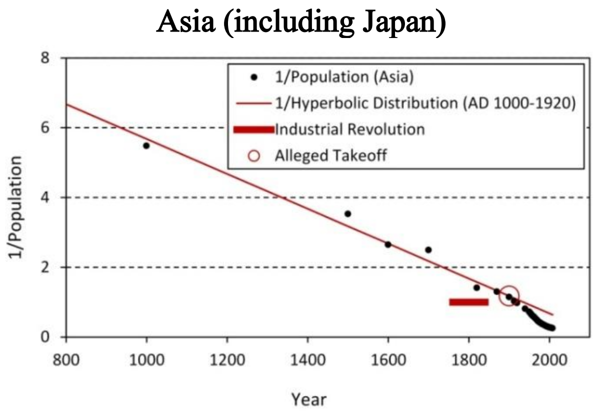

Figure 11. *Reciprocal values of population data for Asia (Maddison, 2010) identify uniquely hyperbolic distribution between AD 1 and 1920 because they follow closely the decreasing straight line. From around 1920, the growth started to be diverted to a temporary faster trajectory. There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a) because there was no stagnation. The temporary boosting around 1920 appears to be a part of the commonly observed transition from the historical hyperbolic growth to a slower trajectory. This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

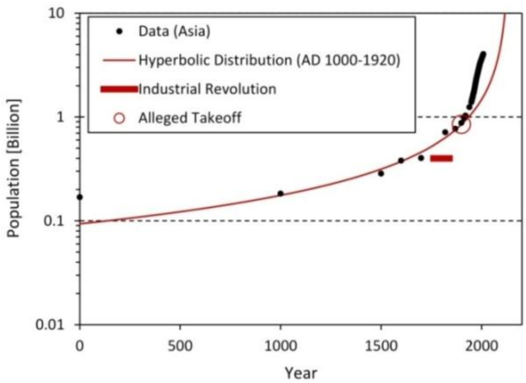

Figure 12. *Growth of human population in Asia. Data of Maddison (2010) are compared with the hyperbolic distribution. There was no stagnation but a hyperbolic growth between at least AD 1000 and 1920. The size of the population at AD 1 is 80% higher than the fitted hyperbolic distribution, reflecting probably the maximum in the growth of the world population around that year (Nielsen, 2016c). There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a) because there was no stagnation before the temporary boosting from around 1920. This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet* again by the same data which were used but not analysed during the formulation of this theory.

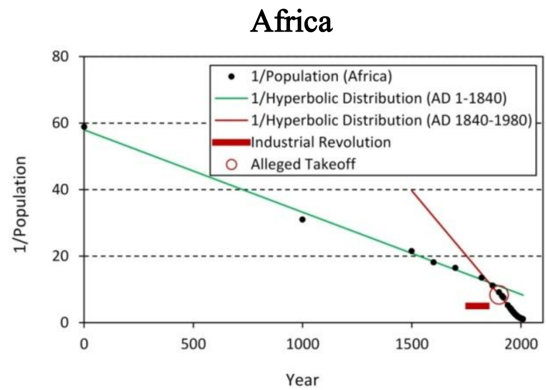

Figure 13. *Reciprocal values of population data for Africa (Maddison, 2010) identify uniquely two hyperbolic distributions: AD 1-1840 and AD 1840-1980 because they follow closely the decreasing straight lines. From around 1980 the growth started to be diverted to a slower trajectory. There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a) because there was no stagnation. However, there was a transition around AD 1840 from a slow to a fast hyperbolic trajectory. This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

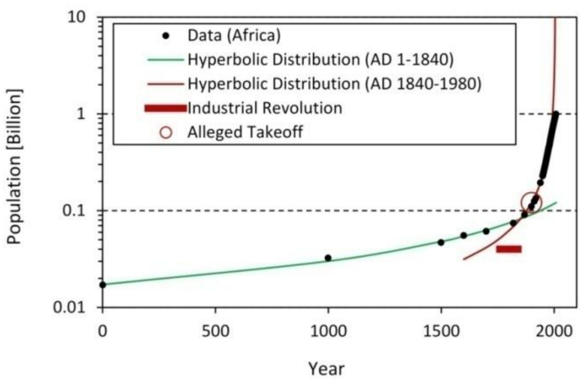

Figure 14. *Growth of human population in Africa. Data of Maddison (2010) are compared with two hyperbolic distributions, AD 1-1840 and AD 1840-1980. There was no stagnation but a hyperbolic growth. There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a) because there was no stagnation. The fast hyperbolic growth, continued undisturbed until 1980 when it started to be diverted to a slower trajectory. Around 1840, there was a transition from a slow to a fast hyperbolic trajectory. This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

Latin America

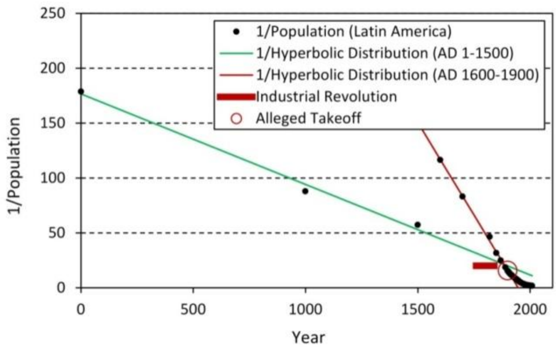

Figure 15. *Reciprocal values of population data for Latin America (Maddison, 2010) identify two hyperbolic distributions: AD 1-1500 and AD 1600-1900 because they follow closely the decreasing straight lines. From around 1900 the growth started to be diverted to a slower trajectory. There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a) but there was a transition around the postulated takeoff to a slower trajectory. Data replace Galor's takeoff by a transition to a slower trajectory. This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

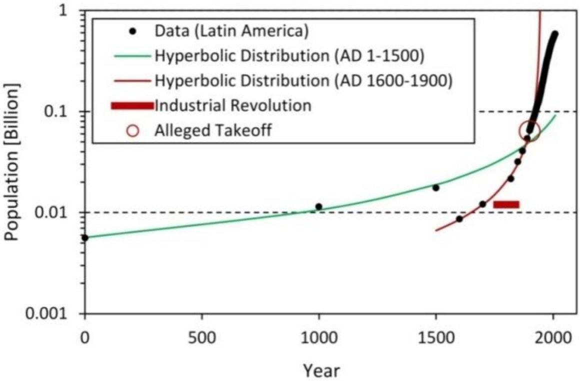

Figure 16. *Growth of human population in Latin America. Data of Maddison (2010) are compared with two hyperbolic distributions, AD 1-1500 and AD 1600-1900. There was no stagnation but a hyperbolic growth. There was no takeoff from stagnation to growth around the postulated time (Galor, 2008a; 2012a) because there was no stagnation. The fast hyperbolic growth continued undisturbed until 1900 when it started to be diverted to a slower trajectory. Data replace Galor's takeoff by a transition to a slower trajectory. This analysis and the absence of the takeoff in the corresponding GDP distribution (Nielsen, 2016a) show that there was no takeoff in the income per capita (GDP/cap) distribution. Unified Growth Theory is contradicted yet again by the same data which were used but not analysed during the formulation of this theory.*

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6. Unified Growth Theory contradicted by the absence of the three regimes of growth

Introduction

There is no science without data but there is also no science without scientific analysis of data. We can have excellent data but if we do not analyse them properly we are likely to draw incorrect conclusions. A perfect example is the Unified Growth Theory (Galor, 2005a; 2011). Excellent data (Maddison, 2001) were available and even used during its formulation but they were never properly analysed. Now, it can be easily demonstrated that the fundamental postulates of this theory are repeatedly contradicted by data, making it fundamentally incorrect and, consequently, unacceptable.

Many attractive theories and explanations can be formulated but if they are not based firmly on the rigorous analysis of data they are only, at best, just interesting stories. They may contain elements of truth but folklores of many cultures are full of such stories and they also contain elements of truth. Fantasy and leaps of faith might be inspiring and productive even in scientific research but they have to be soon tested by the scientific process of investigation.

However, if one leap of faith is followed by another, if one fantasy creates another, then we no longer deal with science but with fiction. It is then easy to loose scientific perspective and defend emotionally the widely-accepted dogmas, based on faith.

Any theory that cannot be checked by data is unscientific even if it is based on scientifically attractive ideas. Such a theory has to be put aside until it can be checked by relevant data. Even if a theory is confirmed by many sets of data it can be still challenged by a single set of contradicting data. Any theory contradicted by just one set of good data has to be either revised or rejected. Any research, any intellectual activity, which ignores these fundamental principles of scientific investigation is unscientific even if it is intellectually stimulating and attractive.

In science it is important to look for data confirming theoretical explanations but it is even more important to discover contradicting evidence, because data confirming a theory confirm only what we already know but contradicting evidence may lead to new discoveries.

If scientific analysis of data is found to be in agreement with a proposed theory, this theory may then be considered to be supported by data and its explanations of studied phenomena may be then accepted. However, if just one set of data is found to be in contradiction with this theory, then this theory can no longer be accepted in its original form. It has to be then either modified to bring it in agreement with data, or rejected if such modification is impossible. There is no scientific gain in accepting such a theory. On the contrary, its continuing acceptance is detrimental to science.

When an incorrect theory is rejected we can then look for a better explanation of studied phenomena. There are no sentimental values in scientific research and no emotional attachments, and any scientist should be prepared to have his or her theories challenged by science.

Unified Growth Theory

Currently, the most complete theory of the historical economic growth is the Unified Growth Theory (Galor, 2005a, 2011). It follows closely the traditional interpretations of economic growth. One of its fundamental postulates is the existence of the three regimes of growth. It claims that the historical economic growth in various countries and regions can be divided into *three distinctly different regimes of growth governed by distinctly different mechanisms*. We shall show that these three regimes did not exist.

The supposed regimes are:

1. The regime of Malthusian stagnation. According to Galor, and indeed according to the currently accepted interpretations, this regime "characterized most of human history" (Galor, 2005a, p. 178). Economic growth was supposedly in the endless state of stagnation described as the Malthusian trap or "the Malthusian steady-state equilibrium" (e.g. Galor, 2005a. pp. 236, 237, 244). Galor claims that this epoch of stagnation commenced in 100,000 BC (Galor 2008a; 2012a) and was terminated around AD 1750, or

around the time of the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994), in developed regions and around AD1900 in less-developed regions.

The beginning of this regime in 100,000 BC is highly speculative because Maddison's data do not extend to the BC era. Furthermore, the emergence of *Homo Sapiens* is usually claimed to have been around 200,000 BC or maybe even earlier (Weaver, Roseman, & Stringer, 2008). We simply do not know about the economic growth in such a distant past because we do not have relevant data. Judging by the available evidence (Nielsen, 2016a; 2016b;2016c), the growth was probably hyperbolic but whatever we might want to suggest will be based on speculations. However, we do not have to go so far back in time to test the Unified Growth Theory because the postulate of the existence of the three regimes of growth cannot be even tested using the economic growth data for the BC era. Even if such data were available they would be inapplicable for this purpose because the existence of the three regimes of growth is not claimed for the BC era but only for the AD era. The data we need to use are the data of Maddison (2001; 2010) because they cover the time when the supposed three regimes were supposed to have existed.

2. The post-Malthusian regime. According to Galor (2008a; 2012a), this regime was between AD 1750 and 1870 for developed regions but it commenced a little later, around AD 1900, for less-developed regions. Thus, the supposed escape from the Malthusian trap and the commencement of the fast economic growth occurred around the onset of the Industrial Revolution for developed regions and a little later for less-developed regions.

3. The sustained-growth regime. According to Galor (2008a; 2012a), this regime commenced around AD 1870 for developed regions.

The general idea of this interpretation of the historical economic growth is that after the endless epoch of "the Malthusian steady-state equilibrium," humans were finally able to break through the impenetrable barrier of stagnation, escape the Malthusian trap and enter into a new era of sustained and rapid economic growth. This is not only incorrect but also dangerous concept because the data describing the historical economic growth (Maddison, 2001; 2010) present a diametrically opposite interpretation. The economic growth was sustained and secure in the past (Nielsen, 2016a) but now it entered a stage of the insecure future (Nielsen, 2015a).

We shall now demonstrate that *Golor's concept of the three regimes of growth is contradicted by the economic growth data* (Maddison, 2010). We shall show that his three regimes of growth have no correlation with data and no positive connection with the real world. Within the range of the mathematically-analysable data, there was no stagnation and no transition to a fast economic growth, described as the sustained-growth regime or the modern-growth regime. We shall show that during this supposed new, fast-increasing and sustained-growth regime, economic growth started to be diverted from the fast-increasing historical hyperbolic trajectories to *slower* trajectories.

Historical economic growth, global and regional, was so well sustained that it followed stable hyperbolic trajectories. However, such trajectories escape to infinity at a fixed time and any growth, which follows them, has to be, at a certain stage, diverted to a slower trajectory. Economic growth, global and regional, is now diverted to slower trajectories. However, the momentum gained during the sustained historical growth keeps on propelling the economic growth along trajectories, which are still increasing too fast to feel comfortable about their future.

Galor's Unified Growth Theory is *not* based on the scientific analysis of data. He had access to the excellent set of data (Maddison, 2001) but he did not analyse them. Now, precisely the same data can be used to show that his theory is fundamentally incorrect.

Regrettably, Unified Growth Theory is based on impressions created by the customary disfigured presentation of data (Ashraf, 2009; Galor, 2005a; 2005b; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Example of such distorted presentation of data is shown in Figure 1. This way of handling data is a perfect prescription for drawing incorrect conclusions.

In science, data are treated with respect because the primary aim of science is to discover the truth, and for this purpose there is nothing as reliable as good sets of data. Not all data can be accepted but we have to have good reasons for rejecting data. If reasons for rejecting data are unacceptable, then reasons for rejecting data have to be rejected.

Many attractive theories and explanations may be formulated but they all have to pass the test of data. Without such a test, they are just stories, which might or might not be true.

Galor's predecessors might be excused for believing in the existence of Malthusian stagnation and in the dramatic impact of the Industrial Revolution on changing the economic growth trajectories because they were using strongly limited information. They had no access to the excellent source of data published by the world-renown economist (Maddison, 2001). Galor not only had access to these data but he also used them repeatedly during the formulation of his theory but unfortunately he distorted them so much that they were creating an impression of being in agreement with his postulates.

In our discussion we shall use the latest data describing economic growth (Maddison, 2010). This publication contains some additional information but any of Maddison's compilations, the compilation used by Galor or this new compilation, can be used to demonstrate that the Unified Growth Theory is contradicted by data. The advantage of using the new compilation (Maddison, 2010) is that it helps to understand the recent transitions to slower trajectories because the earlier compilation was extended to include the data for the 21stcentury.

Method of analysis and related issues

We shall use two ways of displaying data: (1) semilogarithmic display of the GDP data and (2) the display of their reciprocal values, 1/GDP. These two types of display are suitable for studying data varying over a large range of values. The GDP values will be expressed in billions of 1990 International Geary-Khamis dollars.

Hyperbolic distributions, which describe the historical economic growth (Nielsen, 2016a), are represented by the simple mathematical formula:

$$S(t) = (a - kt)^{-1} \tag{1}$$

where, in our case, $S(t)$ is the GDP while a and k are positive constants.

The reciprocal values of hyperbolic distributions are represented by straight lines:

$$\frac{1}{S(t)} = a - kt \tag{2}$$

In general, hyperbolic growth can be uniquely identified by the decreasing straight line of the reciprocal values of the size of the growing entity in much the same way as the exponential growth can be identified by their logarithm. Reciprocal values of data can also help in identifying easily any deviations from hyperbolic trend because deviations from a straight line are easy to notice.

In using the reciprocal values it should be remembered that a deviation to a slower trajectory is indicated by an *upward* bending away from the previous linear trend while deviations to faster trajectories are indicated by *downward* bending. In particular, any form of boosting or takeoff, repeatedly claimed by Galor for global and regional economic growth, should be indicted by a clear change in the *downward* direction of the reciprocal values.

If the straight line fitting the reciprocal values of data remains undisturbed, it shows that there was no diversion to a faster or slower trajectory. In particular, if the straight line does not show a change in the downward direction (if the gradient of the trajectory of the reciprocal values remains constant) then there was no boosting in the economic growth. We obviously cannot claim a change of direction on an undisturbed straight line.

If the reciprocal values of data follow a decreasing straight line, the growth is not stagnant but hyperbolic. However, the concept of stagnation is not supported even if the reciprocal values of data do not decrease linearly. Any monotonically-decreasing trajectory will show that the postulate of stagnation followed by a takeoff at a certain time is not supported by data.

To prove the existence of the epoch of stagnation it is necessary to prove the presence of random fluctuations often described as Malthusian oscillations. Such random fluctuations should be clearly seen not only in the direct display of data but also in the display of their reciprocal values. It they are absent then there is no support in data for claiming the existence of the epoch of stagnation. Furthermore, if data do not show a clear takeoff from stagnation to growth at the postulated time, then there is no support for Galor's repeatedly-claimed takeoffs. However, if the reciprocal values of data follow a decreasing straight line, then they show, or at least strongly suggest, that the growth was hyperbolic.

If the straight line representing the reciprocal values of data remains unchanged, then obviously there is no change in the mechanism of growth. It is impossible to divide a straight line into different sections and claim different mechanism of growth for each of such arbitrarily selected sections. It is impossible to claim, for instance, a transition from stagnation to growth as repeatedly claimed by Galor in his Unified Growth Theory if the reciprocal values of data follow an undisturbed straight line. It is impossible to claim the existence differential takeoffs if there were no takeoffs. It is also impossible to claim that the Industrial Revolution changed the economic growth trajectory if the reciprocal values of data demonstrate that there was no change, i.e. that their linear trend remained undisturbed.

No-one has yet demonstrated the existence of Malthusian stagnation in the economic growth or in the growth of human population. For instance, Lee pointed out that "these models of Malthusian oscillations" are speculative when applied to the growth of human population (Lee, 1997, p. 1097). However, from the descriptions of Malthusian stagnation, its signature and the supposed escape from the Malthusian trap should be easy to identify. This signature is schematically presented in Figures 2 and 3.

For the direct display of GDP data (Figure 2), the signature of the regime of Malthusian stagnation can be identified by random fluctuations or oscillations around an approximately horizontal line. Over much longer sections of time, perhaps extending over thousands of years, fluctuations around the horizontal line might be replaced by fluctuations around a certain irregular trajectory (increasing, decreasing or randomly oscillating), which would be probably difficult to describe mathematically because the general concept of Malthusian stagnation is that it was controlled by random forces. Such random forces are hardly expected to generate monotonically-increasing distributions (Artzrouni & Komlos, 1985; Lagerlöf, 2006; McKeown, 2009; Komlos, 1989; van de Kaa, 2008). For the monotonically-increasing distributions, random forces are either too weak or they average out (Kapitza, 2006) and the growth is controlled by a certain dominant force, which could be constant (for the exponential growth), increasing with time or with the size of the growing entity (as for the hyperbolic growth) or even decreasing (as for the logistic growth).

The signature of the "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) to the sustained economic growth should be easily identified by a clear takeoff from the earlier stagnant distribution to a fast increasing growth. The supposed escape should occur around AD 1750 for developed regions and around AD 1900 for less-developed regions (Galor, 2008a, 2012a).

For the reciprocal values of data (Figure 3), the epoch of Malthusian stagnation can be again identified by random fluctuations around an approximately horizontal line or around an irregular trajectory but the escape from the Malthusian trap will be identified by a clear *downward* trend. It should be noted that in the display of the reciprocal values of GDP data, small fluctuations are magnified, which means that in this display, epoch of Malthusian stagnation should be easy to identify because it should be characterised by strong fluctuations.

Maddison's data are indispensable in studying the historical economic growth but they have a strongly-limited range because they contain a large gap between AD 1 and 1000, and between AD

1000 and 1500. The most useful sets of data are from AD 1500. However, this shortcoming is immaterial because all the action described by Galor's three regimes of growth takes place after AD 1500. Within the range of the good sets of data, i.e. commencing from AD 1500, we should see clearly all the hallmarks of Galor's postulate of the three regimes of growth. We should see the signature of the regime of Malthusian stagnation, the effects of the Industrial Revolution, which was supposed to have been "the prime engine of economic growth" (Galor, 2005a, p. 212), the signature of the escape from the supposed Malthusian trap and a clear evidence of the uninterrupted era of the fast-increasing and sustained economic growth after stagnation. All these features should be clearly displayed. If they are not, then there is no support in the data for Galor's interpretations of the historical economic growth based on such distorted presentations of data as shown in Figure 1. Such presentations have no place in the scientific research.

The discussion presented here is the extension of the mathematical analysis of the historical economic growth (Nielsen, 2016a). We have already demonstrated that the historical economic growth was hyperbolic and thus that implicitly it gives no support for the doctrine of the three regimes of growth. Now, we shall show it explicitly.

It is essential to understand the fundamental features of hyperbolic distributions (Nielsen, 2014). Hyperbolic growth *is* slow over a long time and fast over a short time, but it is still the same, monotonically-increasing distribution, which is *impossible* to divide into two or three different, mathematically-justified components. The easiest way to see it is by using the reciprocal values [see the eqn (2)] because the confusing hyperbolic growth is then represented by a decreasing straight line. It is then clear that it is impossible to divide such a straight line into distinctly different, mathematically-justified components and claim distinctly different mechanisms of growth for each of these arbitrarily selected components.

Even though hyperbolic growth is slow over a long time it is *not* stagnant. *Slow hyperbolic growth should never be interpreted as stagnant* because if we want to interpret the slow perceived part of hyperbolic growth as stagnant, and governed by the usually assumed multitude of random forces, we should use precisely the same mechanism to explain the perceived fast component. The perceived slow and fast components belong to the same, monotonically-increasing distribution. It is impossible to divide a monotonically-increasing hyperbolic distribution into the mathematically-justifiable slow and fast sections because it is obviously impossible to divide a straight line describing the reciprocal values and representing the hyperbolic distribution into distinctly-different and mathematically-justifiable sections (Nielsen, 2014). It is scientifically unjustified to use different mechanisms of growth for such arbitrarily selected sections. Hyperbolic distributions have to be interpreted as a whole and the same mechanism has to be applied to the apparent slow growth and to the apparent fast growth. There is no clearly defined transition between the apparent slow and the apparent fast growth.

These comments apply also to the income per capita distributions represented by the Gross Domestic Product per capita (GDP/cap). Such distributions are even more confusing than hyperbolic distributions. They are linearly-modulated hyperbolic distributions, i.e. the monotonically-increasing hyperbolic distributions representing the growth of the GDP modulated by the monotonically-decreasing linear distributions representing the reciprocal values of the size of the population (Nielsen, 2015b). A product or a ratio of monotonic distributions cannot generate a non-monotonic distribution.

Even though the GDP/cap distributions appear to be made of two or maybe even three different components, as claimed incorrectly by Galor, they are increasing *monotonically* and it is impossible to divide them into distinctly different, mathematically-justifiable components. We can demonstrate it by calculating gradients or the growth rates of the GDP/cap distributions and by showing that they increase monotonically (Nielsen, 2015b). Any attempt to divide the GDP/cap distributions into distinctly-different components is strongly subjective and mathematically unjustified.

Analysis of data for Western Europe

We shall analyse two sets of data for Western Europe: (1) the data for 12 selected countries and the data for the total of 30 countries. The 12 selected countries are made of Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Sweden, Switzerland and the United Kingdom. According to Maddison (2010), in 2008, these 12 countries accounted for 85% of the total GDP of the 30 countries of Western Europe. The total of the 30 countries includes also Ireland, Greece, Portugal, Spain and 14 other small west European countries.

The reason for analysing these two groups separately is that the listed 12 countries represent the most advanced economies, where the effects of the Industrial Revolution and the escape from the Malthusian trap should be most clearly visible. Consequently, for these 12 countries we should expect the best agreement between the Unified Growth Theory and the data.

Economic growth between AD 1 and 2008 in the 12 countries of Western Europe is shown in Figures 4 and 5. The growth in the total of 30 countries is shown in Figures 6 and 7.

Hyperbolic parameters describing economic growth in the 12 countries of Western Europe are: $a = 1.147 \times 10^{-1}$ and $k = 5.961 \times 10^{-5}$. The corresponding singularity is in 1923 but the economic growth was diverted to a slower trajectory around 1900, bypassing the singularity by about 23 years.

Hyperbolic fit to the data is remarkably good between AD 1500 and 1900 and acceptable below AD 1500. The point at AD 1 is only 27% higher than the fitted distribution and the point at AD 1000 is 54% lower. The critical range of time for testing the Unified Growth Theory is from AD 1500. It is in this range of time that we should be able to see transition from stagnation to growth and later a transition to the supposed sustained growth regime.

The data presented in Figures 4 and 5 clearly demonstrate that there is no support for the existence of the alleged regime of Malthusian stagnation. However, there is a convincing support for the hyperbolic growth at least between AD 1500 and 1900, the range of time where the signature of Malthusian stagnation should be still clearly displayed for about 300 years. The data show that during that time economic growth was following a steadily-increasing hyperbolic trajectory. There is no sign of the existence of Malthusian stagnation.

Absolutely nothing had happened at the end of the supposed Malthusian regime. There was no transition from stagnation to growth at any time. On the contrary, around the beginning of the postulated regime of sustained-growth, when the economic growth was supposed to have been launched from stagnation to a fast-increasing trajectory, the growth started to be diverted to a slower trajectory.

It is remarkable also that the Industrial Revolution had absolutely no impact on shaping the economic growth trajectory in these 12 countries. They should experience the greatest benefits of this revolution and they probably did but these benefits did not boost the economic growth. Technological innovations were used to sustain and propel economic growth but they did not change in the slightest the economic growth trajectory. In countries, where effects of the Industrial Revolution, "the prime engine of economic growth" (Galor, 2005a, p. 212), should have been most clearly reflected in the relevant data, we see no impacts of this engine.

This is an interesting issue, which should be studied and explained but it is futile to look for its explanation in the Unified Growth Theory.This interesting feature has not been even noticed by Galor, which is hardly surprising because it is hard or even impossible to carry out scientific research and draw reliable and scientifically-justified conclusions by repeatedly distorting data in such a way as shown in Figure 1.

Galor's Unified Growth Theory has no relevance to the description, let alone to the explanation of the mechanism of the economic growth, even in countries where his theory should be best fitted. Here, in the leading countries of Western Europe, where the effects of the Industrial Revolution should be most prominently displayed in the data describing economic growth, where the "remarkable" and "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) should be remarkably obvious, there are no signs of the impacts of the Industrial Revolution on the economic growth and no signs of any escape from the Malthusian trap, remarkable or less-remarkable, because there was no trap. Economic growth was increasing undisturbed and unconstrained along a hyperbolic trajectory until around 1900 when it started to be diverted to a slower but still fast-increasing trajectory.

Galor's three regimes of growth are totally dissociated from reality. They describe events that never happened.

Stories and explanations presented by Galor in his theory have no relevance to the explanation of the mechanism of the economic growth even in these 12 leading countries of Western Europe. His stories might be explaining or describing something else, e.g. social conditions or the style of living, but even then one wonders about the degree of reliability of such descriptions. His narrative does not explain the mechanism of the economic growth.

Results of the analysis of the economic growth in the total of 30 countries of Western Europe are presented in Figures 6 and 7. Hyperbolic parameters are: $a = 9.859 \times 10^{-2}$ and $k = 5.112 \times 10^{-5}$. The corresponding singularity is in 1929 but the economic growth was diverted to a slower trajectory around 1900, bypassing the singularity by about 29 years. The point at AD 1 is 42% higher than the calculated hyperbolic distribution and at AD 1000 it is 48% lover.

The analysis of the economic growth in the total of 30 countries of Western Europe leads to the same conclusions as for the 12 leading countries: Unified Growth Theory is contradicted by the economic growth data in Western Europe where the effects discussed by Galor should have been most convincingly confirmed. In contrast, they are convincingly contradicted.

Analysis of data for Eastern Europe

Results of the analysis of economic growth in Eastern Europe, based on using Maddison's data (Maddison, 2010), are presented in Figures 8 and 9. Hyperbolic parameters fitting the data are: $a = 7.749 \times 10^{-1}$ and $k = 4.048 \times 10^{-4}$. The point at AD 1 is 51% higher than the calculated curve. The singularity is in 1915 but the economic growth was diverted to a slower trajectory around 1890, bypassing the singularity by 25 years.

Unified Growth Theory is clearly contradicted by the economic growth data for Eastern Europe. The epoch of Malthusian stagnation did not exist within the range of the mathematically-analysable data. Outside of this range, any claim about the existence of the regime of Malthusian stagnation and about its effects on the economic growth has to be based on questionable conjectures. Such a claim would be also in conflict with the analysable data.

The data show no transition from stagnation to growth at any time because the growth was hyperbolic. There was no "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) because there was no trap. Industrial Revolution did not boost the economic growth in Eastern Europe.

There was also no boosting of the economic growth at the time of the transition from the supposed post-Malthusian regime to the supposed sustained growth regime. Soon after the commencement of this phantom sustained-growth regime, economic growth in Eastern Europe started to be diverted to a slower trajectory. Galor's regimes of growth are clearly dissociated from data. They do not describe the real world but the world of fancy created by preconceived ideas and supported by the habitually-distorted presentation of data (Ashraf, 2009; Galor, 2005a; 2005b; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

Analysis of data for Asia

Asia (excluding Japan) is made primarily, if not exclusively, of less-developed countries (BBC, 2014; Pereira, 2011). According to Galor, thisregion should have experienced the epoch of stagnation until around 1900 followed by the post-Malthusian regime commencing around that year. If Galor's claims are correct, we should see clear signs of stagnation in the data until around 1900 and a clear transition (a dramatic takeoff) from stagnation to growth around that year.

Economic growth in Asia between AD 1 and 2008 is presented in Figure 10. There is absolutely no correlation between the data and the three key events indicated in this figure: the Industrial Revolution, the supposed Malthusian regime and the supposed post-Malthusian regime, which were supposed to have been shaping the economic growth.

During the supposed Malthusian regime of stagnation, economic growth in Asia was increasing hyperbolically at least from AD 1000 but the point at AD 1 is also not far from the calculated hyperbolic distribution. Parameters fitting the data are $a = 2.303 \times 10^{-2}$ and $k = 1.129 \times 10^{-5}$.

The data show no signs of stagnation within their mathematically-analysable range, no signs of the Malthusian steady-state equilibrium and no signs of Malthusian oscillations. Assuming the existence of all such features is not only unnecessary but also scientifically unjustified because in science complicated interpretations are rejected in favour of simpler explanations. The data follow a steadily-increasing hyperbolic distribution, suggesting a simple mechanism of growth because hyperbolic distributions are described by a simple mathematical formula [see the eqn (1)].

The concept of stagnation is dramatically contradicted by data and so is the transition to the supposed post-Malthusian regime, which was supposed to have been a transition from stagnation to growth. We see no such transition but a continuation of the hyperbolic growth. The claimed by Galor takeoff did not happen. There was a minor and hard-to-notice disturbance in the economic growth around 1950 but the growth soon returned to its historical hyperbolic trajectory. The overall evidence in the data is that the propping-up structures (the supposed different regimes of growth) used by Galor are not only totally redundant but also strongly misleading. They can, and even should, be removed because the data reveal a totally different pattern of growth.

The data and their analysis show that nothing dramatic occurred during the supposed transition from the postulated Malthusian regime of stagnation to the supposed post-Malthusian regime, which is supposed to mark the escape from the postulated Malthusian trap and leading to a sustained growth regime. There was no escape from the trap because there was no trap. During the postulated Malthusian trap the economic growth was steadily increasing and it was obviously unconstrained. It is futile to claim random fluctuations and oscillations when there are none. Why should we even contemplate to make it all more complicated when the data show that the growth was much simpler?

If not for Maddison and his data, the established knowledge in the economic research would have remained established, but now it has to be revaluated and changed. However, new insights should be welcome, particularly if they suggest a simpler explanation of the historical economic growth.

Reciprocal values of the GDP data, 1/GDP, shown in Figure 11, also demonstrate that the Unified Growth Theory is contradicted by the same data, which were used during its development, the data published by Maddison in 2001 (Maddison, 2001) but later extended to include economic growth during the 21st century (Maddison, 2010).

During the supposed Malthusian regime of stagnation, reciprocal values of data were decreasing along a straight line indicating an undisturbed, hyperbolic economic growth. The data show also that nothing dramatic had happened at the end of this supposed epoch of stagnation. There was no transition to a new regime of growth. In particular, there was no transition from stagnation to growth, as claimed by Galor, but a continuation of the hyperbolic growth. The concept of the two regimes of growth is convincingly contradicted by data.

Analysis of data for the former USSR

Economic growth in the countries of the former USSR between AD 1 and 2008 is presented in Figure 12. Reciprocal values of the GDP data, 1/GDP, are shown in Figures 13 and 14. The growth was hyperbolic between AD 1 and around 1870. Parameters describing hyperbolic growth are $a = 6.547 \times 10^{-1}$ and $k = 3.452 \times 10^{-4}$.

During the entire range of the mathematically-analysable data the epoch of Malthusian stagnation did not exist. Galor's regimes of growth are hanging there without having any connection with data. The "remarkable" or "stunning" escape from the Malthusian trap did not happen because there was no trap. Galor's Malthusian regime ends in the middle of nowhere. Absolutely nothing (remarkable or less-remarkable, stunning or less stunning) happened on the border between the supposed Malthusian regime and the post-Malthusian regime. There was also no stunning or remarkable escape at the onset of the supposed sustained-growth regime. There was no dramatic increase in the economic growth. On the contrary, economic growth started to be diverted to a slower trajectory.

What is remarkable about the confrontation of Galor's theory with the empirical evidence is that there is such a consistently repeated and stunning disagreement between his theory and the data. The data also demonstrate that the Industrial Revolution had absolutely no impact on changing the economic growth trajectory in the countries of the former USSR. Here again we see that "the prime engine of economic growth" (Galor, 2005a, p. 212) did nothing to change to growth trajectory. Whatever this engine might have been doing, it certainly did not boost the economic growth. The data and their analysis give no support for the concept of Malthusian stagnation and for the assumption of the existence of the steady-state Malthusian equilibrium. Economic growth was increasing along a remarkably-stable hyperbolic trajectory. There was no escape from the Malthusian trap, let alone a "remarkable" or "stunning" escape as claimed by Galor (2005a, pp. 177, 220), because there was no trap. The growth was always unconstrained because the hyperbolic trajectory remained unimpeded.

The concept of stagnation is dramatically contradicted by data and so is the supposed transition from stagnation to growth. Such a transition never happened. On the contrary, from around 1870, economic growth in the countries of former USSR started to be diverted to a slower trajectory, away from its faster, historical hyperbolic trajectory.

Analysis of data for Africa

Africa is a perfect example of a cluster of countries, which belong to the group of less-developed and least-developed countries. Out of the total of 48 least-developed countries in the world, 34 are in Africa (Bangla News, 2015; UNCTAD, 2013). With just one minor exception, Africa is made entirely of less-developed and least-developed countries (BBC, 2014; Pereira, 2011). The exception is Western Sahara, a small country in transition made of around 586,000 people (UNDATA, 2015).

Maddison's data for Africa serve, therefore, as an excellent source of information to test Galor's hypothesis of the existence of the distinctly different regimes of economic growth in less-developed regions. We shall demonstrate that this hypothesis is dramatically and clearly contradicted by data.

Reciprocal values of data describing economic-growth in Africa are presented in Figure 15. Economic growth was clearly hyperbolic between AD 1 and around 1820 because the reciprocal values follow a straight line. There was definitely no stagnation. The concept of the regime of Malthusian stagnation is clearly contradicted by data. To prove its existence one would have to demonstrate a stagnant state of growth characterised by random Malthusian oscillation around an approximately horizontal line as shown in Figure 3. The data contain no such signature. On the contrary they show a steadily-increasing and remarkably-stable hyperbolic growth. There are no signs of any possible fluctuations, which in this representation of data should be strongly magnified.

Furthermore, Galor's concept of Malthusian stagnation extending to 1900 ignores not only the data between AD 1 and 1820 but also the clear and dramatic transition, which occurred around 1820. It was *not* a transition from stagnation to growth but *from growth to growth*, the transition from a slower but steadily-increasing hyperbolic growth to a faster and steadily-increasing hyperbolic growth. This pattern is in clear contradiction of the Unified Growth Theory (Galor, 2005a, 2008a, 2011, 2012a).

The concept of the regime of stagnation ignores the steadily-increasing economic growth before 1820, the dramatic change in the pattern of growth around that year and the new hyperbolic growth after 1820. The claim of Malthusian stagnation ending in 1900 for less-developed countries ignores also that absolutely nothing unusual had happened around that year. The economic growth continued undisturbed. The postulated Malthusian regime ends in the middle of nowhere. There is no justification for claiming the regime of Malthusian stagnation and no justification for terminating it in AD 1900 or at any other time because there was no stagnation.

In addition, the data demonstrate the existence of a feature, which is ignored by Galor: the diversion to a slower trajectory around 1950 indicated by the upward bending of the trajectory of the reciprocal values. According to Galor, the economic growth was supposed to have been boosted from stagnation to growth (at the end of his supposed Malthusian regime) and launched into a fast-increasing growth, but data present an entirely different interpretation: economic growth was increasing fast along a hyperbolic trajectory during the supposed regime of Malthusian stagnation but shortly after the time of the postulated transition to a faster growth the data started to follow a *slower* trajectory. Data tell one story, Galor tells another, and in science data have the priority.

The disagreement between Galor's theory and the data is also clearly demonstrated in Figures 17 and 18. Over the range of the mathematically-analysable data the Malthusian regime did not exist. The data show no evidence of the features characterising the epoch of Malthusian stagnation. In contrast, the data show steadily-increasing hyperbolic distributions.

In his description of economic growth, Galor did not even notice that there was a strong transition around AD 1820, let alone that it was a transition from one hyperbolic distribution to another. He also did not notice that that the postulated epoch of Malthusian stagnation ends in the middle of nowhere (see Figure 18).

Many important details are easily lost in the habitually distorted presentations of data (Ashraf, 2009; Galor, 2005a; 2005b; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008) as illustrated in Figure 1. It is hard or even impossible to draw reliable conclusions by using such distorted diagrams and by making no attempt to analyse data. Conclusions based on impressions are likely to be incorrect. It is hard or even impossible to do science without following the principles of scientific investigation.

Analysis of data for Latin America

Results of analysis of the economic growth in Latin America based on Maddison's data (Maddison, 2010) are shown in Figures 19 and 20.

The data suggest the existence of two hyperbolic growth trajectories: a slow trajectory between AD 1 and 1500 and a fast trajectory between AD 1600 and 1870. The slow trajectory is characterised by parameters $a = 4.421 \times 10^{-2}$ and $k = 2.093 \times 10^{-5}$. The singularity for this trajectory was at $t_s = 2113$. The fast trajectory is characterised by parameters $a = 1.570 \times 10^{-1}$ and $k = 8.224 \times 10^{-5}$. The singularity for this new trajectory was at $t_s = 1910$. However, from around 1870, i.e. from around the time of the supposed takeoff from stagnation to growth (Galor, 2008a;2012a), economic growth in Latin America started to be diverted to a slower trajectory bypassing the singularity by a safe margin of 40 years. The illusion of a *takeoff* is replaced by a diversion to a *slower* growth.

The characteristic features of the economic growth in Latin America are similar to the features in Africa. In both cases, a slow hyperbolic growth was followed by a much faster hyperbolic trajectory and this transition can be correlated with the intensified colonisation of Latin America (Bethell, 1984).

The data for Latin America are in clear disagreement with the Unified Growth Theory. The economic growth was slow before AD 1500 but there is no basis for claiming that it was stagnant. Hyperbolic trajectory between AD 1 and 1500 could be questioned but it is consistent with the similar, but much clearer, pattern in Africa and is in perfect agreement with the repeated evidence of hyperbolic growth in other regions. There is definitely no convincing support for the existence of the epoch of stagnation.

The data show a brief economic decline between AD 1500 and 1600, which appears to be coinciding with the commencement of the intensified Spanish conquest (Bethell, 1984). However, from around AD 1600, economic growth in Latin America was following a fast-increasing hyperbolic trajectory. The change from a slow to fast economic growth occurred about *300 years before the supposed takeoff around 1900*. Furthermore, as in Africa, *it was not a transition from stagnation to growth but from hyperbolic growth to hyperbolic growth*. This feature is ignored in the Unified Growth Theory. Remarkably also, at the time of the supposed "remarkable" escape from the Malthusian trap (Galor, 2005a, p. 177) around AD 1900, economic growth in Latin America was already diverted to a *slower* trajectory.

Unified Growth Theory presents a story, which is contradicted by data. There is no correlation between the data and the narrative of this theory. In his habitually crude display of data, Galor could not have seen all these important features. He appears to have been guided by the inherited ideas, which unfortunately he did not check by the rigorous analysis of the new and excellent data (Maddison, 2001) available to him at the time of the formulation of his theory. The updated compilation of the data describing the historical economic growth (Maddison, 2010) was also available to him even before the publication of his book (Galor, 2011) and certainly during his continuing dissemination of the same ideas after its publication. As mentioned earlier, any of these compilations can be used to show that Galor's theory is fundamentally incorrect because during the time when there were supposed to have been transitions between alleged regimes of growth the two compilation contain the same data and they show a clear disagreement withGalor's theory.

Summary and conclusions

We have analysed economic growth in Western Europe, Eastern Europe, Asia, former USSR, Africa and Latin America (Maddison, 2010). We have found that the fundamental concepts of the Unified Growth Theory (Galor, 2005a; 2011) are contradicted by the same data, which were used but never analysed during the formulation of this theory.

Whatever was wished-for did not happen. The real world refused to comply with the preconceived ideas and with the imagined interpretations, which were creating such an attractive story.

It seems to be obvious that the Industrial Revolution should have a strong and decisive effect on the economic growth but it did not. It seems to be obvious that a slow growth is stagnant but it is not. What seems to be obvious is not necessarily true. It is obvious that the Sun moves around the Earth but it does not. “It is clear that the earth does not move, and that it does not lie elsewhere than at the centre” (Aristotle).

Empirical evidence has to be methodically and carefully analysed; otherwise we shall be creating our own stories, which might be interesting, exciting and convincing but they will be stories of fiction. They will have nothing to do with science. In science we learn from nature. Any attempt to mould nature into the image fashioned by our creative imagination is bound to fail and the perfect example is the Unified Growth Theory.

Within the range of the mathematically-analysable data, the three regimes of growth, the Malthusian regime, the post-Malthusian regime and the sustained-growth regime did not exist. There is no correlation between the data and these three postulated regimes of growth. In particular, there was no escape from the Malthusian trap because there was no trap.

During the time described by the mathematically-analysable data, economic growth was hyperbolic and generally undisturbed. Only most recently, around the time when according to the Unified Growth Theory it should have been boosted from stagnation to growth, economic growth started to be diverted from the fast-increasing hyperbolic trajectories to slower trajectories. Unified Growth Theory does not explain, let alone describe the historical economic growth because it is based on the fundamentally incorrect premises.

The concept of the three regimes of growth was supported by the distorted presentation of data (Ashraf, 2009; Galor, 2005a; 2005b; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). When properly displayed and analysed, the same data show that the Unified Growth Theory is fundamentally incorrect.

The reliable and correct interpretation of the historical economic growth might appear to have no practical application because what was in the past is in the past. Why should the distant past have any influence on our present economic growth? However, the correct understanding of the past economic growth may well decide about our future.

Galor’s interpretations of the historical economic growth are not only scientifically unacceptable but also dangerously incorrect because they create the false sense of security. They present a picture of the unsustained economic growth in the past and of a transition to a new era of sustained economy after the "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220). At last, after the endless suffering, straggle, and deprivation, humans escaped the tyranny of the Malthusian regime and now they can enjoy the sustained economic growth with its prosperous future. This is a pleasing story but the opposite is true.

Rigorous analysis of data shows convincingly that the past economic growth was sustained and secure because it followed the remarkably stable hyperbolic trajectories (Nielsen, 2016a). This conclusion is in harmony with the study of ecological footprints, which shows that until the late 1900s global ecological footprint was lower than the ecological capacity (WWF, 2010). It was in the past that the economic growth was not only sustained but also sustainable. Now it is not, because it is supported by the increasing ecological deficit. Indeed, mathematical analysis of the economic growth shows that its future is insecure (Nielsen, 2015a).

Economic growth was not in a trap in the past but now it is in a trap of our continuing drive to increase not only the GDP but also the GDP/cap. We seem to see no limit to prosperity but the limit is imposed by the ecological limits and by the fast-increasing trajectories of economic growth. While the Unified Growth Theory suggests a prosperous future of the "sustained growth regime" after the supposed "Malthusian regime," the data indicate that unless we take decisive steps to control the current economic growth our future is insecure (Nielsen, 2015a).

In its present form, Galor's Unified Growth Theory is unacceptable. It has to be either thoroughly revised or rejected and replaced by a new theory aimed at explaining why the economic growth was hyperbolic in the past, why it was increasing along such remarkably stable trajectories, why it started to be diverted to slower, but still fast-increasing, trajectories and, most importantly, how to create a sustainable economic future.

Propelled by the gained momentum of the historical economic growth, the current growth continues to increase too fast. It has to be slowed down. The sustainable and secure economic growth has yet to be created. It has not been created automatically at the end of the supposed but non-existent Malthusian regime as suggested incorrectly by the Unified Growth Theory.

Appendix

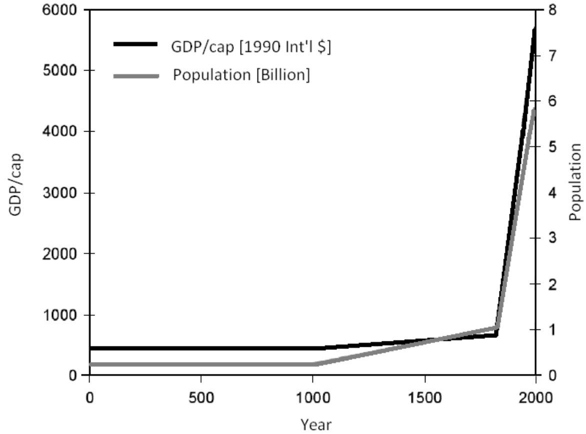

Figure 1. *Example of the ubiquitous, grossly-distorted and self-misleading diagrams used to create the Unified Growth Theory (Galor, 2005a, 2011). Madison's data (Maddison, 2001) were used during the formulation of this theory but they were never analysed. Such a state-of-the-art presentation of data was used to construct a system of scientifically-unsupported concepts, interpretations and explanations.*

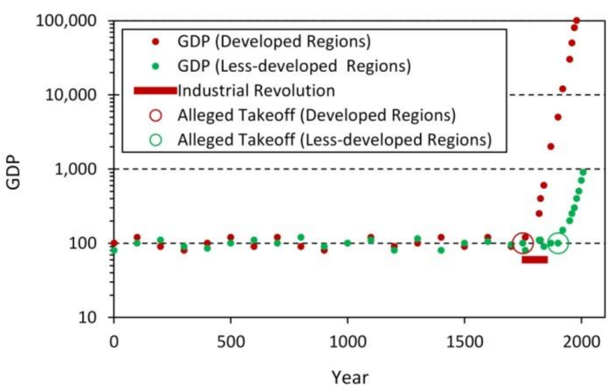

Figure 2. *Direct display of the hypothetical GDP data serving as the schematic representation of the signature of Malthusian stagnation (fluctuations around an approximately horizontal line) followed by the escape from the Malthusian trap into the sustained economic-growth regime around AD 1750 for developed regions and around AD 1900 for less-developed regions as claimed by Galor (2005a, 2008a, 2011, 2012a). If these signatures are missing, Unified Growth Theory is contradicted by data.*

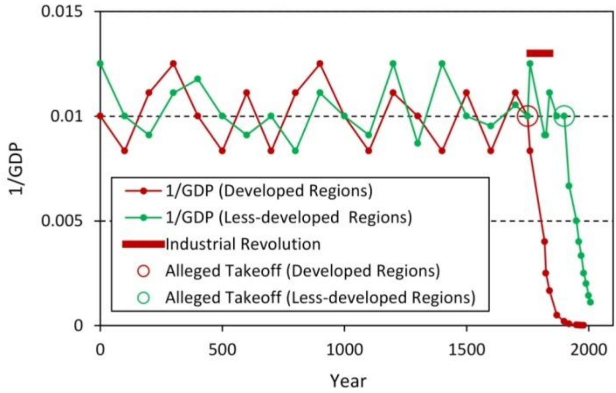

Figure 3. *Display of the reciprocal values of the same hypothetical data as shown in Figure 2, serving as the schematic representation of the signature of Malthusian stagnation (fluctuations around an approximately horizontal line) followed by the escape from the Malthusian trap into the sustained economic-growth regime around AD 1750 for developed regions and around AD 1900 for less-developed regions as claimed by Galor (2005a, 2008a, 2011, 2012a. If these signatures are absent, Unified Growth Theory is contradicted by data.*

Western Europe

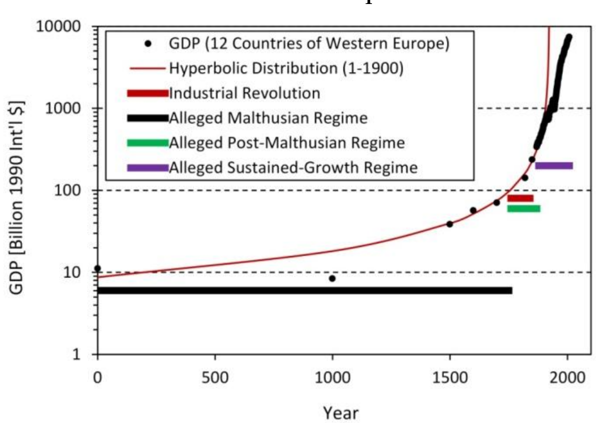

Figure 4. *Economic growth in the 12 selected countries of Western Europe representing the most-advanced economies where the Unified Growth Theory should have the strongest confirmation. There was no transition from stagnation to growth at any time. The growth was hyperbolic before and after the supposed transition around AD 1750. Industrial Revolution did not boost the economic growth. The "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) did not happen because there was no trap. Galor's three regimes of growth have no relevance to the description, let alone to the explanation, of the mechanism of the economic growth. During the supposed sustained growth regime, when the economic growth was supposed to follow a fast-increasing trajectory after the epoch of stagnation, economic growth was diverted to a slower trajectory.*

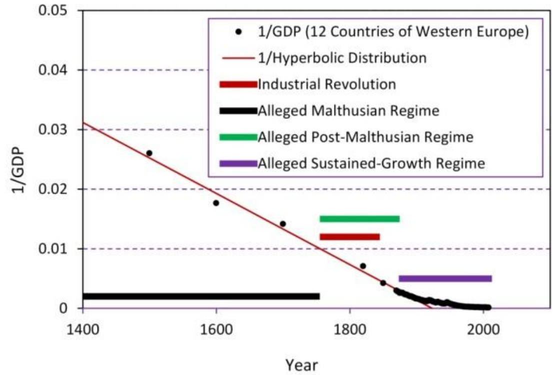

Figure 5. *Reciprocal values of the GDP data, 1/GDP, for the economic growth in the 12 selected countries of Western Europe. Unified Growth Theory (Galor, 2005a, 2011) is contradicted by Maddison's data (Maddison, 2010). Galor's three regimes of growth have no relevance to the description, let alone to the explanation, of the mechanism of the economic growth. There was no transition from stagnation to growth at any time because there was no stagnation. There was no "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) because there was no trap. Industrial Revolution did not boost the economic growth even in the countries where its effects should be most pronounced. During the supposed sustained growth regime, when the economic growth was supposed to follow a fast-increasing trajectory after the epoch of stagnation, economic growth was diverted to a slower trajectory, as indicated by the upward bending of the trajectory of the reciprocal values.*

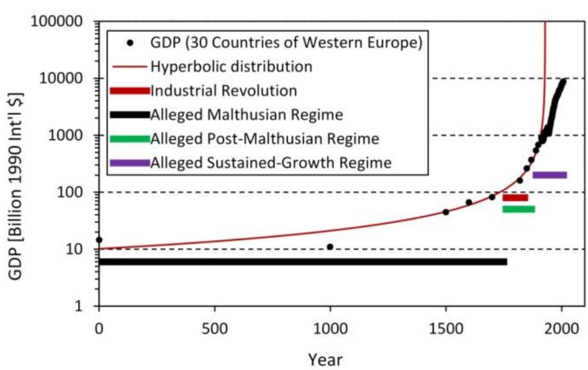

Figure 6. *Economic growth in the total of 30 countries of Western Europe. The data give no clear support for the existence of the supposed Malthusian regime of stagnation. Industrial Revolution did not boost the economic growth in Western Europe. The "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) did not happen because there was no trap. Galor's three regimes of growth have no relevance to the description or to the explanation of the mechanism of the economic growth in Western Europe. During the supposed sustained growth regime, when the economic growth was supposed to follow a fast-increasing trajectory after the epoch of stagnation, economic growth was diverted to a slower trajectory.*

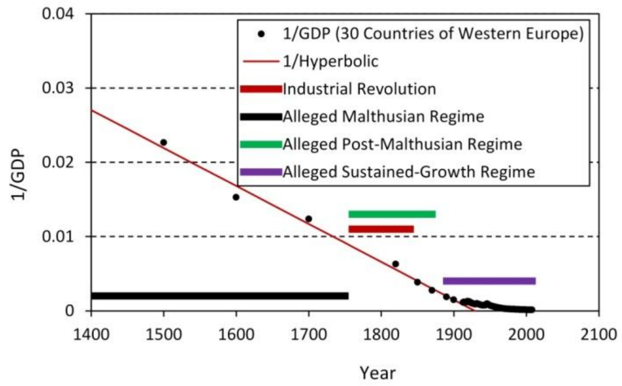

Figure 7. *Reciprocal values of the GDP data, 1/GDP, for the economic growth in the total of 30 countries of Western Europe. Unified Growth Theory (Galor, 2005a, 2011) is contradicted by Maddison's data (Maddison, 2010). Galor's three regimes of growth have no expected correlation with data. There was no transition from stagnation to growth at any time because there was no stagnation. There was no "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) because there was no trap. Industrial Revolution did not boost the economic growth in Western Europe. During the supposed sustained growth regime, when the economic growth was supposed to follow a fast-increasing trajectory after the epoch of stagnation, economic growth was diverted to a slower trajectory*

Eastern Europe

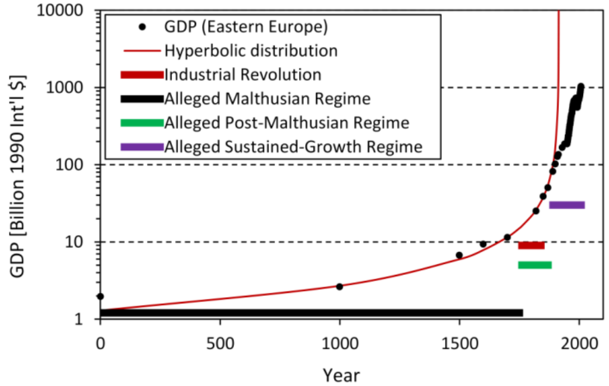

Figure 8. *Economic growth in Eastern Europe. Galor's three regimes of growth have no relevance to the description, let alone to the explanation, of the mechanism of the economic growth. Unified Growth Theory is contradicted by data. The supposed Malthusian regime of stagnation did not exist. Industrial Revolution did not boost the economic growth in Eastern Europe. The "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) did not happen because there was no trap. During the supposed sustained growth regime, when the economic growth was supposed to follow a fast-increasing trajectory after the epoch of stagnation, economic growth was diverted to a slower trajectory*

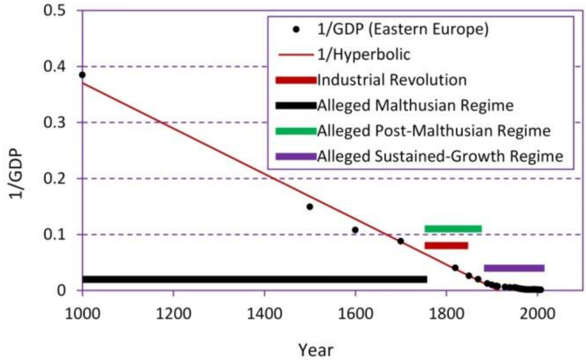

Figure 9. *The reciprocal values of the GDP data, 1/GDP, for the economic growth in Eastern Europe. Unified Growth Theory (Galor, 2005a, 2011) is contradicted by Maddison's data (Maddison, 2010). Galor's three regimes of growth have no expected connection with data. There was no transition from stagnation to growth at any time because there was no stagnation. There was no "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) because there was no trap. Industrial Revolution did not boost the economic growth in Eastern Europe. Galor's theory has no relevance to the description, let alone to the explanation, of the mechanism of the economic growth. During the supposed sustained growth regime, when the economic growth was supposed to follow a fast-increasing trajectory after the epoch of stagnation, economic growth was diverted to a slower trajectory, as indicated by the upward bending of the trajectory of the reciprocal values.*

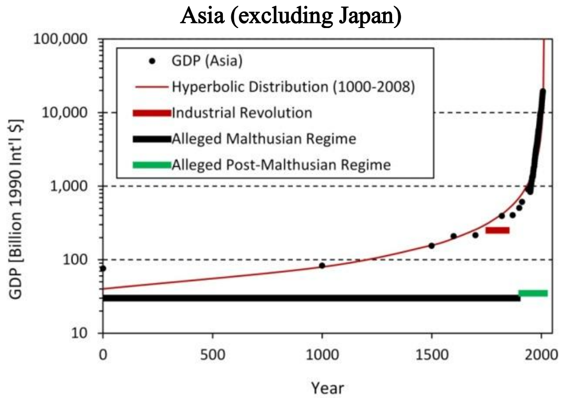

Figure 10. *Economic growth in Asia (excluding Japan) between AD 1 and 2008. Maddison's data (Maddison, 2010) are compared with the hyperbolic distribution and with their unsubstantiated interpretations promoted by Galor (Galor, 2005a, 2011). Economic growth was hyperbolic from at least AD 1000 until 2008. The minor delay after the Industrial Revolution was followed by the compensating recovery. The concept of the supposed Malthusian regime of stagnation is contradicted by data. The escape from the Malthusian trap never happened because there was no trap. There was no dramatic transition from stagnation to growth because there was no stagnation.*

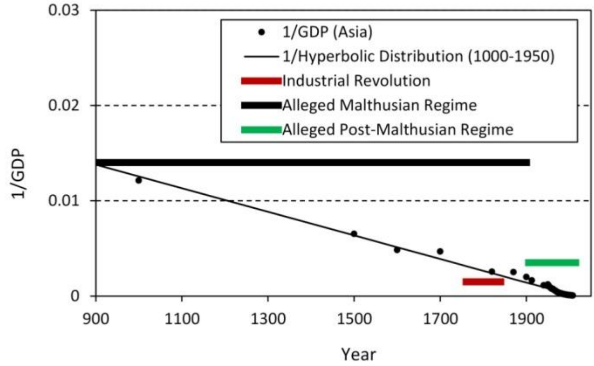

Figure 11. *Reciprocal values of the GDP data, 1/GDP, for Asia demonstrate that there is no correlation between the claimed events (Industrial Revolution, the supposed Malthusian regime of stagnation and the supposed post-Malthusian regime) and the data (Maddison, 2010). The postulated dramatic and remarkable takeoff around 1900 never happened. The Malthusian regime of stagnation and the post-Malthusian regime did not exist.*

Former USSR

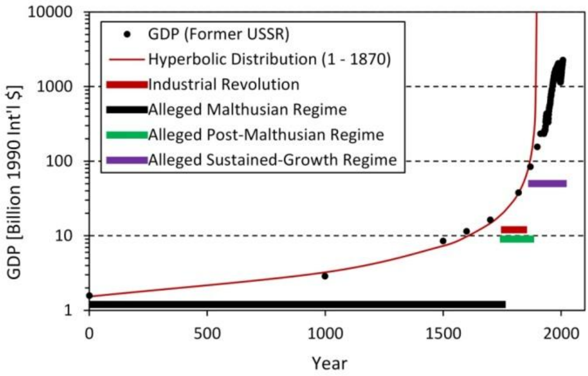

Figure 12. *Economic growth in the countries of the former USSR between AD 1 and 2008, as represented by Maddison's data (Maddison, 2010), is compared with the hyperbolic distribution and with the unsubstantiated interpretations of the mechanism of growth proposed by Galor (Galor, 2005a, 2008a, 2011, 2012a). The supposed Malthusian regime of stagnation did not exist and neither did the alleged post-Malthusian and sustained-growth regimes. The Industrial Revolution had absolutely no impact on changing the economic growth trajectory. There was also no dramatic transition to a new and faster economic growth after the supposed epoch of stagnation, no transition from stagnation to growth at any time because there was no stagnation. There was no escape from the Malthusian trap because there was no trap. In place of all these imaginary and wished-for features there was the undisturbed and well-sustained hyperbolic growth. During the supposed sustained growth regime, when the economic growth was supposed to follow a fast-increasing trajectory after the epoch of stagnation, economic growth was diverted to a slower trajectory.*

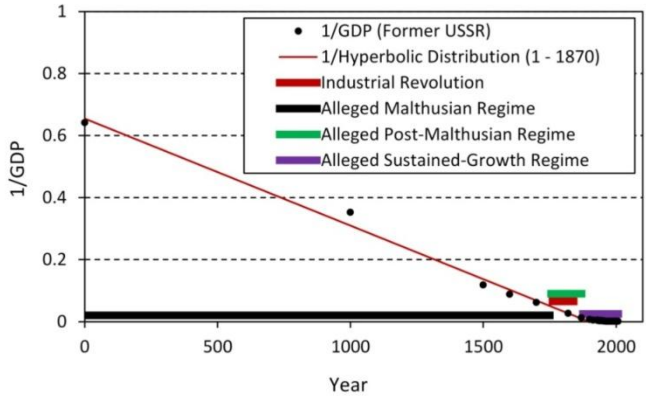

Figure 13. *Reciprocal values of the GDP data, 1/GDP, for the former USSR are compared with the hyperbolic distribution represented by the decreasing straight line. There was no stagnation. Throughout the entire range of the supposed Malthusian regime during the AD era, economic growth was hyperbolic.*

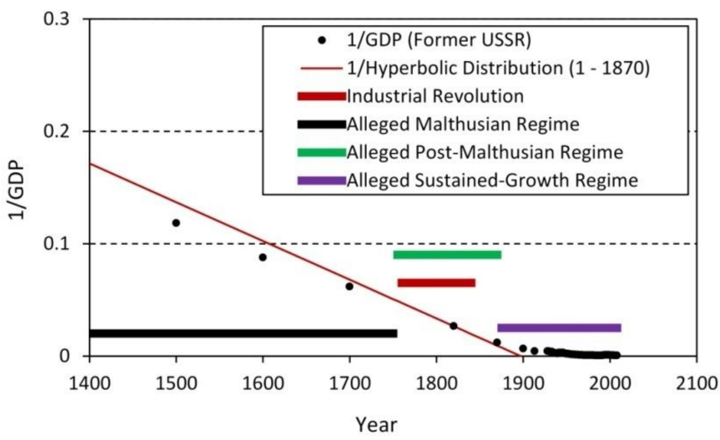

Figure 14. *The end part of the plot of the reciprocal values of the GDP data, 1/GDP, for the former USSR. Economic growth was hyperbolic until around AD 1870 when it started to be diverted to a slower trajectory indicated by an upward bending of the reciprocal values. Industrial Revolution did not boost the economic growth. The supposed Malthusian regime of stagnation did not exist and there was no transition from stagnation to growth at any time because there was no stagnation. The "stunning" or "remarkable" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) did not happen because there was no trap. During the supposed sustained growth regime, when the economic growth was supposed to follow a fast-increasing trajectory after the epoch of stagnation, economic growth was diverted to a slower trajectory.*

Africa

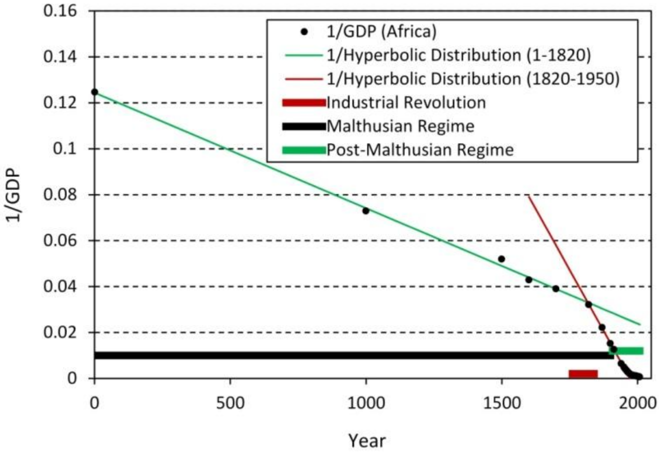

Figure 15. *Reciprocal values of the GDP data (Maddison, 2010) for Africa compared with the hyperbolic distributions represented by the decreasing straight lines. The two distinctly different regimes of growth postulated by Galor (2005a, 2008a, 2011, 2012a) did not exist. There was no transition from stagnation to growth at any time because there was no stagnation.*

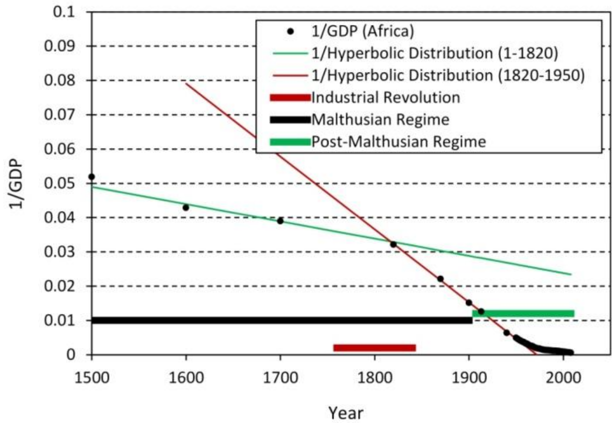

Figure 16. *Reciprocal values of the GDP data (Maddison, 2010) for Africa between AD 1500 and 2008 compared with the hyperbolic distributions represented by the decreasing straight lines. The two distinctly different regimes of growth postulated by Galor (2005a, 2008a, 2011, 2012a) did not exist. His postulate ignores the data. There was no transition from stagnation to growth because there was no stagnation. During the supposed post-Malthusian regime, when the economic growth was supposed to start to follow a fast-increasing trajectory after the supposed epoch of stagnation, economic growth was diverted to a slower trajectory.*

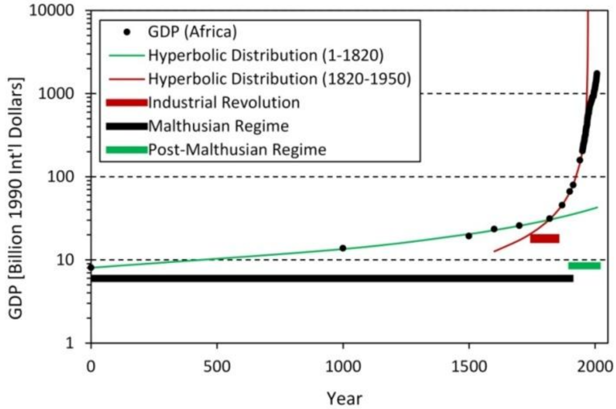

Figure 17. *GDP data (Maddison, 2010) for Africa between AD 1 and 2008 compared with hyperbolic distributions. The two distinctly different regimes of growth postulated by Galor (2005a, 2008a, 2011, 2012a) did not exist. His postulate ignores the data. There was no stagnation and no transition to a faster growth at the end of the supposed regime of Malthusian stagnation. There was no escape from the Malthusian trap because there was no trap. During the supposed post-Malthusian regime, when the economic growth was supposed to start to follow a fast-increasing trajectory after the supposed epoch of stagnation, economic growth was diverted to a slower trajectory*

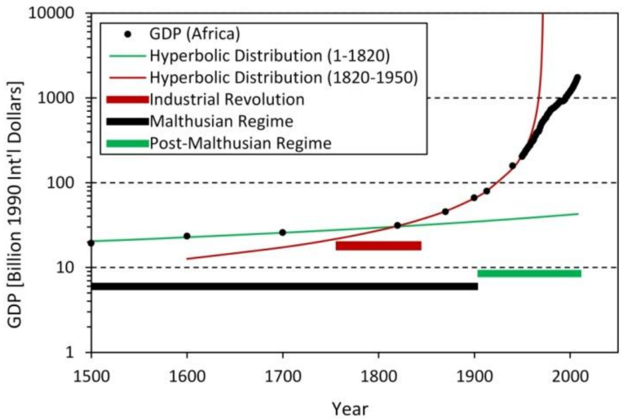

Figure 18. *GDP data (Maddison, 2010) for Africa between AD 1500 and 2008 compared with hyperbolic distributions. The two distinctly different regimes of growth postulated by Galor (2005a, 2008a, 2011, 2012a) did not exist. His postulate ignores the data. The data are in clear contradiction of Galor'stheory. There was no transition from stagnation to growth because there was no stagnation. During the supposed post-Malthusian regime, when the economic growth was supposed to start to follow a fast-increasing trajectory after the supposed epoch of stagnation, economic growth was diverted to a slower trajectory*

Latin America

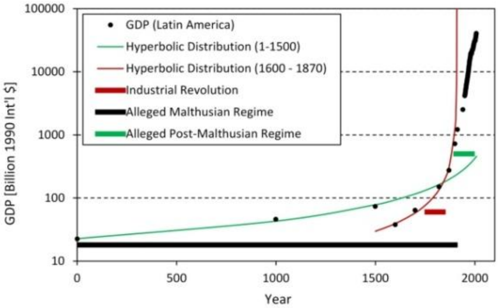

Figure 19. *Economic growth in Latin America between AD 1 and 2008. Maddison's data (Maddison, 2010) are compared with hyperbolic distributions and with their unsubstantiated interpretations proposed by Galor (2005a, 2008a, 2011, 2012a). The data suggest two hyperbolic distributions, the pattern similar to the economic growth in Africa. The supposed transition from stagnation to growth never happened because the economic growth was not stagnant but hyperbolic. Around the time of the postulated by Galor "remarkable" escape from the supposed Malthusian trap (Galor, 2005a, p. 177) at the end of the supposed regime of stagnation, the economic growth started to be diverted to a slower trajectory. There was no escape from the Malthusian trap because there was no trap.*

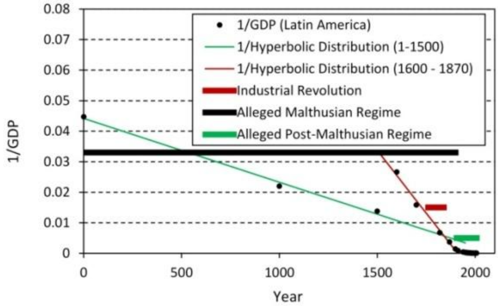

Figure 20. *Reciprocal values of the GDP data, 1/GDP, for Latin America between AD 1 and 2008. Maddison's data (Maddison, 2010) are compared with hyperbolic distributions represented by the decreasing straight lines and with their unsubstantiated interpretations proposed by Galor (2005a, 2008a, 2011, 2012a). During the supposed regime of stagnation, the growth was hyperbolic. The data suggest two hyperbolic distributions, the pattern similar to the economic growth in Africa. The supposed transition from stagnation to growth around AD 1900 did not happened because there was no stagnation. Around the time of the supposed takeoff from stagnation to growth, the economic growth started to be diverted from the fast-increasing hyperbolic trajectory to a slower trajectory as indicated by the upward bending of the trajectory of the reciprocal values. There was no escape from the Malthusian trap because there was no trap. The transition from the slow to fast growth occurred around 300 years before the expected takeoff in AD 1900 and it was not a transition from stagnation to growth but from growth to growth. This feature, as well as the diversion to a slower trajectory at the time of the claimed takeoff around AD 1900, is not even noticed in the Unified Growth Theory.*

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7. Unified Growth Theory contradicted by the analysis of income per capita distributions

Introduction

The aim of this publication is to present the direct proof that contrary to the fundamental postulate of the Unified Growth Theory (Galor, 2005a; 2011) distributions describing historical growth of income per capita cannot be divided into three, distinctly-different regimes of growth governed by distinctly different mechanisms. The indirect proof was presented earlier (Nielsen, 2016a; 2016c; 2016d) by showing that the historical growth of the Gross Domestic Product (GDP) and of human population were hyperbolic and that postulated by Galor three regimes of growth did not exist. Mathematical analysis of the latest data (Maddison, 2001; 2010) brings a new insight into the interpretation of the historical economic growth. Within the range of analysable data there was no Malthusian stagnation, no supposed takeoffs from stagnation to growth and no escapes from the hypothetical Malthusian trap because there was no trap (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f).

Unified Growth Theory serves as a good example of traditional interpretations of economic growth, interpretations revolving around the concept of Malthusian stagnation. It is also a theory, which appears to be based on Maddison's data (Maddison, 2001) but it is not. Ironically, even though these excellent data were used during the formulation of this theory, they were never mathematically analysed. Unified Growth Theory is *not* based on the scientific analysis of data but on impressions supported by the habitually distorted presentation of data (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Data were used unprofessionally and they were presented in such a way, intentionally or unintentionally, that they appear to support preconceived ideas. However, when closely analysed they are found to be in the direct contradiction of the promoted concepts, explanation and interpretations.

Historical economic growth and historical growth of human population were hyperbolic (Kapitza, 2006; Kremer, 1993; Nielsen, 2016a; 2016b; 2016d; Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975). Hyperbolic distributions are confusing and they are often misinterpreted in studies of economic growth and of the growth of human population. They present an image of a slow growth over a long time followed by a fast growth over a short time. These distributions are, therefore usually divided into two distinctly-different segments, slow and fast. The selected slow segment is then claimed to represent the epoch of Malthusian stagnation while the selected fast segment is assumed to represent an entirely new type of growth. The supposed transition between these two arbitrarily-selected segments is then described as explosion, takeoff, sudden spurt, sprint or the dramatic escape from the Malthusian trap. Distinctly-different mechanisms are also assigned for the two perceived segments of growth.

Often, however, interpretations of historical growth are not even based on any attempt to examine relevant data. Isolated examples are used to support the concept of stagnation followed by explosion. Even worse, more often than not, interpretations and explanations are just based on impressions and suppositions. Claims of the existence of Malthusian stagnation and transitions to different stages of growth are supported by a good dose of creative imagination.

There is no mathematically justifiable reason for dividing hyperbolic distributions into two or three distinctly-different components (Nielsen, 2014). It is mathematically *impossible* to divide hyperbolic distributions into slow and fast components. Hyperbolic distributions are slow over a long time and fast over a short time but they increase monotonically. Growth rate also increases monotonically without any unusual acceleration at any time. It increases hyperbolically with time or linearly with the size growing entity (Nielsen, 2016f). Concepts of stagnation and takeoffs from stagnation to growth are scientifically unjustified.

They are contradicted by the analysis of data describing economic growth and the growth of population (Kapitza, 2006; Kremer, 1993; Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975).

Hyperbolic distributions have to be interpreted as a whole and the same mechanism has to be used for the apparent slow and for the apparent fast segments. These segments do not exist even though they appear to exist. The best way to demonstrate that these segments do not exist is by using reciprocal values of hyperbolic distributions (Nielsen, 2014).

Historical economic growth is even more confusing than the historical growth of human population because economic growth is often described using income per capita represented by the Gross Domestic Product per capita (GDP/cap). It is a ratio of hyperbolic distributions and it creates an even stronger illusion of different stages of growth than the illusion created by hyperbolic distributions. It has been demonstrated (Nielsen, 2015a) that the characteristic features of the GDP/cap distributions, which are interpreted as the epoch of stagnation followed by a sudden takeoff, are nothing more than mathematical properties of dividing two hyperbolic distributions. It is incorrect to claim that these features characterise uniquely economic growth.

The ratio of two hyperbolic distributions, which includes the GDP/cap ratio, increases monotonically and there is no mathematically-justifiable reason for dividing them into distinctly different regimes of growth. There is no mathematical justification for assigning different mechanisms of growth to the two perceived but non-existing segments of income per capita distributions.

Growth of income per capita was slow over a long time and fast over a short time but it was increasing monotonically. The ratio of monotonically-increasing hyperbolic distributions can only produce a monotonically-changing distribution, increasing or decreasing, depending on the location of singularities of hyperbolic distributions (Nielsen, 2015a). Such a ratio cannot produce a distribution with a sudden discontinuity, which could be described as a takeoff.

The growth of income per capita has to be explained by using the same mechanism for the whole distribution, slow and fast. We shall now demonstrate that the empirical distributions describing income per capita were indeed increasing monotonically and that there were no sudden takeoffs from stagnation to growth as claimed incorrectly in the Unified Growth Theory (Galor, 2005a; 2011).

Unified Growth Theory

Maddison's data (Maddison, 2001, 2010) offer an unprecedented opportunity to test the past and present explanations of economic growth and of the growth of human population, explanations based on strongly-limited sources of empirical information and on creative imagination. Now, this rich body of data brings new and refreshing insights into the interpretation of the historical economic growth and of the growth of population, interpretations confirmed by other sources of relevant data (Biraben, 1980; Clark,1968; Cook, 1960; Durand, 1967; 1974; 1977; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994) and by the earlier research (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975)

It is both unfortunate and ironic that Galor had access to all this information but failed to use it to make useful contribution to science. He repeatedly distorted empirical distributions to support his preconceived ideas. An example of such distorted and self-misleading presentations of data is shown in Figure 1 but remarkably, precisely the same data, when properly analysed, demonstrate that the Unified Growth Theory is fundamentally incorrect.

Hyperbolic distributions do not have to be distorted to be confusing. They are already sufficiently confusing and it is easy to make mistakes with their interpretations. Distorted presentations, such as repeatedly used by Galor, make the interpretation of these distributions even more difficult. The example presented in Figure 1 is based on a figure presented by Galor (2005a, p. 181). Such self-misleading presentations of data can be expected to lead inevitably to incorrect conclusions. It is hard to understand why such distorted diagrams were repeatedly used by Galor because the analysis of hyperbolic distributions is trivially simple (Nielsen, 2014).

The fundamental postulates of the Unified Growth Theory are based on the assumption of the existence of three, distinctly-different regimes of economic growth: Malthusian regime of stagnation, post-Malthusian regime and the sustained-growth regime. According to Galor (2005a; 2008a; 2011; 2012a), Malthusian regime of stagnation was between 100,000 BC and AD 1750 for developed regions and between 100,000 BC and AD 1900 for less-developed regions. The post-Malthusian regime was supposedly between AD 1750 and 1850 for developed regions and from 1900 for less-developed regions. The sustained-growth regime was supposed to have commenced around 1850 for developed regions.

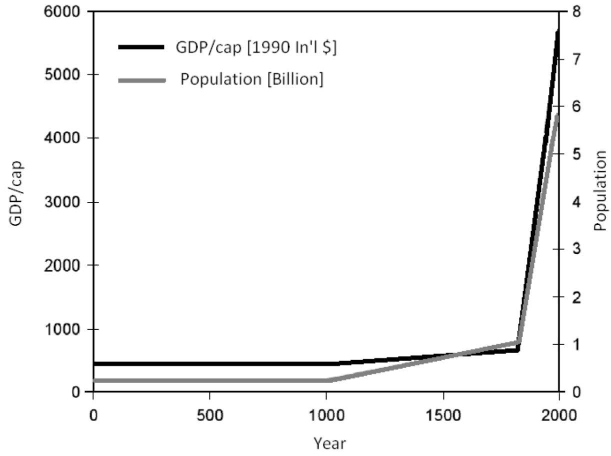

Figure 1. *Example of the ubiquitous, grossly-distorted and self-misleading diagrams used to create the Unified Growth Theory (Galor, 2005a; 2011). Maddison's data (Maddison, 2001) were used during the formulation of this theory but they were never analysed. Such state-of-the-art was used to construct a system of scientifically-unsupported interpretations, explanations and "mysteries of the growth process" (Galor, 2005a, p. 220).*

The end of the regime of Malthusian stagnation was supposed to have been characterised by dramatic takeoffs from stagnation to growth, described as a "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220). It is a signature, which cannot be missed. This change in the pattern of growth is described as "the sudden take-off from stagnation to growth" (Galor, 2005a, pp. 177, 220, 277) or as a "sudden spurt" (Galor, 2005a, 177, 220). According to Galor, the end of the Malthusian regime of stagnation for developed regions coincides with the Industrial Revolution. "The take-off of developed regions from the Malthusian Regime was associated with the Industrial Revolution" (Galor, 2005a, p. 185). Indeed, the Industrial Revolution is considered to have been "the prime engine of economic growth" (Galor, 2005a, p. 212).

For developed regions, the postulated sudden takeoffs from stagnation to growth is supposed to have occurred around AD 1750, or around the time of the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994). For less-developed regions, the takeoff was supposedly around 1900. A transition from growth to growth is not a signature of the postulated sudden takeoff from stagnation to growth. Thus, for instance, a transition from hyperbolic growth to another hyperbolic growth or to some other steadily-increasing trajectory is not a signature of the sudden takeoff from stagnation to growth. Likewise, a transition at a distinctly different time is not a confirmation of the theoretical expectations.

In the diagrams presented below, income per capita (GDP/cap) is in 1990 International Geary-Khamis dollars.

Hyperbolic growth

It has been shown earlier that over the range of analysable data historical growth of population and historical economic growth were hyperbolic (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f). For the economic growth, the range of analysable data extends down to AD 1 but for the growth of the world population it extends to 10,000 BC. These results are consistent with the analysis carried out over 50 years ago for the growth of the world population during the AD era (von Foerster, Mora, & Amiot, 1960) and with other similar studies (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975)

Demographic and economic research has to be based on the acceptance of hyperbolic descriptions of the historical growth of population and of the historical economic growth. Hyperbolic growth, confirmed repeatedly and consistently by data (Biraben, 1980; Clark,1968; Cook, 1960; Durand, 1967; 1974; 1977; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994) leaves no room for the outdated interpretations revolving around the concept of Malthusian stagnation followed by sudden takeoffs to a distinctly faster growth. Mathematical analysis of data contradicts consistently and repeatedly these hypothetical but unsupported concepts, including the concept that the Industrial Revolution changed the relevant growth trajectories. It did not.

Hyperbolic distribution describing growth is represented by a reciprocal of a linear function:

$$S(t) = \frac{1}{a - kt}, \qquad (1)$$

where $S(t)$ is the size of the hyperbolically growing entity (e.g. the GDP or the size of the population), while a and k are positive constants.

Distribution describing the time-dependence of income per capita (GDP/cap) is the ratio of two hyperbolic distributions: the hyperbolic distribution describing the growth of the GDP (Nielsen, 2016a) and the hyperbolic distribution describing the growth of population (Nielsen, 2016d). A GDP/cap distribution can be also interpreted as a ratio of two linearly decreasing distributions describing the respective reciprocal values or as a product of a hyperbolic distribution representing the GDP and a linear function representing the reciprocal values of the size of the population. Consequently, the GDP/cap ratio can be simply described as *the linearly-modulated hyperbolic distribution*, where linear modulation of the GDP distribution is done by the reciprocal values of the size of population (Nielsen, 2015a).

Growth of the world GDP/cap

Results of mathematical analysis of the world GDP/cap are presented in Figure 2. The fitted distribution represents the linearly-modulated GDP distribution (Nielsen, 2015a). Parameters describing the GDP data are $a_1 = 1.684 \times 10^{-2}$ and $k_1 = 8.539 \times 10^{-6}$ while the parameters describing the world population data are $a_2 = 7.739 \times 10^{0}$ and $k_2 = 3.765 \times 10^{-3}$.

For the growth of the world GDP/cap we should see the signature of two takeoffs: around AD 1750 for developed regions and around AD 1900 for less-developed regions, yet we see none of them. There was no stagnation before the Industrial Revolution and no transition from stagnation to growth around AD 1750 for developed regions or around AD 1900 for less-developed regions, as claimed by Galor (2008a; 2012a).

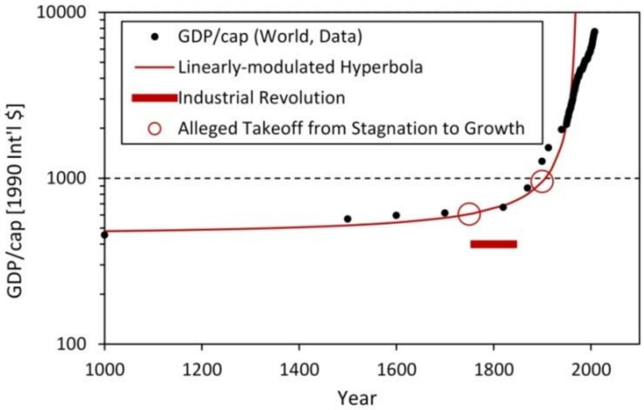

Figure 2. *Maddison's data (Maddison, 2010) describing growth of the world income per capita (GDP/cap) are compared with the linearly-modulated hyperbolic distribution (Nielsen, 2015a) obtained by fitting the GDP and population data (Nielsen, 2016a; 2016d). The supposed takeoffs from stagnation to growth around AD 1750 for developed regions and around 1900 for less-developed regions, as claimed by Galor (2008a; 2012a), did not happen. Industrial Revolution had no impact on changing the trajectory describing the growth of income per capita (GDP/cap).*

The data show a minor disturbance around AD 1900 which looks like a minor boosting. However, it is definitely not a "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) because (1) the growth deviated only slightly from the historical trajectory, (2) this minor deviation was not preceded by stagnation and (3) because it was only temporary disturbance and the growth soon returned to the original trajectory. Furthermore, rather than being permanently and spectacularly propelled along a distinctly new trajectory, as implied by Galor's claims of "remarkable" and "stunning" takeoffs (Galor, 2005a, pp. 177, 220), economic growth as described by data, started to be diverted to a *slower* trajectory. There was definitely no transition from stagnation to growth. There was no dramatic escape from the Malthusian trap because there was no trap in the growth of income per capita.

Western Europe

Growth of the GDP/cap in Western Europe is shown in Figure 3. Maddison's data (Maddison, 2010) are compared with the linearly-modulated hyperbolic distribution obtained by dividing two hyperbolic distributions: the distribution describing the growth of the GDP (Nielsen, 2016a) and the distribution describing the growth of population (Nielsen, 2016d). Parameters describing the displayed curve are: $a_1 = 9.859\times10^{-2}$ and $k_1 = 5.112\times10^{-5}$ for the GDP and $a_2 = 7.542\times10^{1}$ and $k_2 = 3.749\times10^{-2}$ for the growth of the population.

Results presented in Figure 3 are particularly important because they show that contrary to the generally accepted interpretations, Industrial Revolution had absolutely no impact on changing the growth trajectory of income per capita in the region where its impact should have been most pronounced. Galor's claim that the Industrial Revolution was "the prime engine of economic growth" (Galor, 2005a, p. 212) is remarkably contradicted by the same data, which he used during the formulation of his theory. This and other examples show how important Maddison's data are in correcting the outdated interpretations of the historical economic growth.

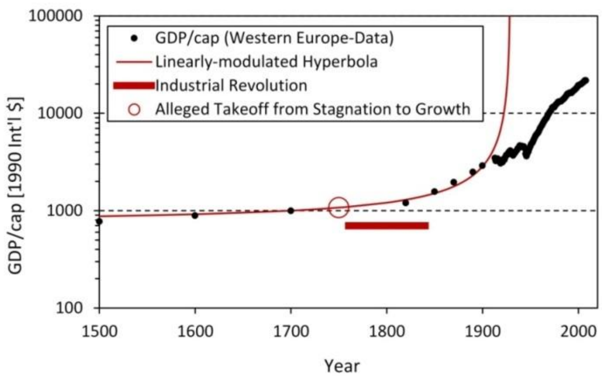

Figure 3. *Maddison's data (Maddison, 2010) describing the growth of income per capita (GDP/cap) in Western Europe are compared with the linearly-modulated hyperbolic distribution (Nielsen, 2015a) obtained by fitting the GDP and population data (Nielsen, 2016a; 2016d). The supposed takeoff from stagnation to growth around AD 1750 (Galor, 2008a; 2012a) did not happen. Industrial Revolution had no impact on the trajectory describing the growth of income per capita (GDP/cap). The analysis of data used by Galor shows that "the prime engine of economic growth" (Galor, 2005a, p. 212) had absolutely no impact on changing the economic growth trajectory in the region where this "engine" should have been most effective and where its impacts should have been most pronounced. From around AD 1900, the growth of the GDP/cap started to be diverted to a slower trajectory.*

It has been shown earlier (Nielsen, 2016a) that economic growth was hyperbolic not only for the total of 30 countries of Western Europe but also for the four countries, Denmark, France, the Netherlands and Sweden, described by the most complete sets of data and representing the most advanced economies. For these countries, hyperbolic growth was between AD 1 and 1875 when it started to be diverted to a *slower* trajectory. There was no Malthusian stagnation, no takeoff and no escape from the Malthusian trap, because there was no trap. Industrial Revolution had absolutely no impact on changing economic growth trajectories in these four progressive countries where the impact of this revolution should be clearly demonstrated in the economic growth data.

Analysis of Maddison's data (Maddison, 2010) demonstrates that the "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) never happened because there was no trap in the growth of income per capita. Whether expressed in terms of the GDP or GDP/cap, economic growth was remarkably undisturbed during the time of the Industrial Revolution and continued undisturbed until around 1900, when it started to be diverted to a slower trajectory.

Eastern Europe

Results of analysis of the growth of income per capita in Eastern Europe are summarized in Figure 4. Maddison's data (Maddison, 2010) are compared with the linearly-modulated hyperbolic distribution obtained by dividing two hyperbolic distributions: the distribution describing the growth of the GDP (Nielsen, 2016a) and the distribution describing the growth of population (Nielsen, 2016d). Parameters describing the fitted GDP/cap distribution are: $a_1 = 7.749 \times 10^{-1}$ and $k_1 = 4.048 \times 10^{-4}$ for the GDP and $a_2 = 3.055 \times 10^{2}$ and $k_2 = 1.525 \times 10^{-1}$ for the growth of the population.

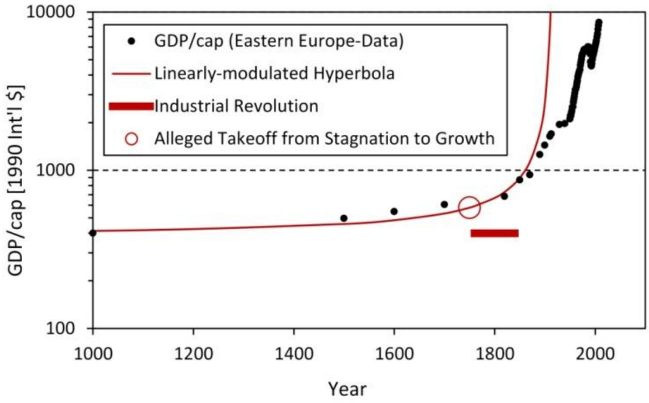

Figure 4. *Maddison's data (Maddison, 2010) describing the growth of income per capita (GDP/cap) in Eastern Europe are compared with the linearly-modulated hyperbolic distribution (Nielsen, 2015a) obtained by fitting the GDP and population data (Nielsen, 2016a; 2016d). The supposed takeoff from stagnation to growth around AD 1750 (Galor, 2008a; 2012a) did not happen. Industrial Revolution had no impact on the trajectory describing the growth of income per capita (GDP/cap). From around AD 1850, rather than being boosted by the Industrial Revolution, the growth of the GDP/cap started to be diverted to a slower trajectory.*

Growth of income per capita was slow but it was not stagnant. It was following the linearly-modulated hyperbolic distribution. Industrial Revolution had absolutely no impact on shaping the growth trajectory. The "stunning" takeoff postulated by Galor did not happen. His theory is repeatedly and consistently contradicted by the data he used during the formulation of his theory. Rather than being boosted by the Industrial Revolution, the growth of the GDP/cap started to be diverted to a *slower* trajectory from as early as around AD 1850.

Former USSR

Results of analysis of the growth of income per capita in the former USSR are summarized in Figure 5. Maddison's data (Maddison, 2010) are compared with the linearly-modulated hyperbolic distribution obtained by dividing two hyperbolic distributions: the distribution describing the growth of the GDP (Nielsen, 2016a) and the distribution describing the growth of population (Nielsen, 2016d). Parameters describing the fitted GDP/cap distribution are: $a_1 = 6.547 \times 10^{-1}$ and $k_1 = 3.452 \times 10^{-4}$

for the GDP and $a_2 = 2.618 \times 10^2$ and $k_2 = 1.333 \times 10^{-1}$ for the growth of the population.

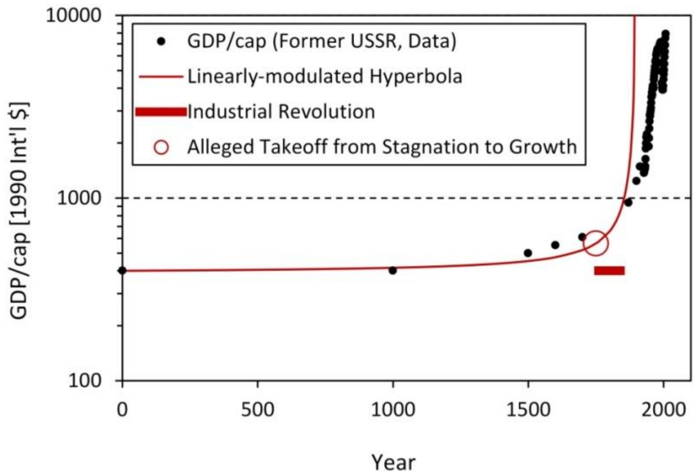

Figure 5. *Maddison's data (Maddison, 2010) describing the growth of income per capita (GDP/cap) in the former USSR are compared with the linearly-modulated hyperbolic distribution (Nielsen, 2015a) obtained by fitting the GDP and population data (Nielsen, 2016a; 2016d). The supposed takeoff from stagnation to growth around AD 1750 (Galor, 2008a; 2012a) did not happen. Industrial Revolution had no impact on the trajectory describing the growth of income per capita (GDP/cap). From around AD 1870, rather than being boosted by the Industrial Revolution, the growth of the GDP/cap started to be diverted to a slower trajectory.*

Growth of income per capita in the countries of the former USSR was following closely the linearly-modulated hyperbolic distribution from AD 1. The growth was slow but not stagnant. Growth of the GDP and population were monotonic (Nielsen, 2016a, 2016d) and consequently the growth of income per capita (GDP/cap) was also monotonic. The "remarkable" or "stunning" takeoff (Galor, 2005a, pp. 177, 220) claimed by Galor never happened. This wished-for feature is repeatedly contradicted by the analysis of economic and population data (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f) and by the analysis of the GDP/cap distributions. Soon after the alleged, but non-existent sudden takeoff from the non-existent stagnation to growth, the growth of income per capita in countries of the former USSR started to be diverted to a new and *slower* trajectory.

Asia

Analysis of the growth of income per capita (GDP/cap) in Asia (including Japan) is summarised in Figure 6. Maddison's data (Maddison, 2010) are compared with the linearly-modulated hyperbolic distribution obtained by dividing two hyperbolic distributions: the distribution describing the growth of the GDP (Nielsen, 2016a) and the distribution describing the growth of the population (Nielsen, 2016d). Parameters describing the fitted GDP/cap distribution are: $a_1 = 2.303 \times 10^{-2}$ and $k_1 = 1.129 \times 10^{-5}$ for the GDP and $a_2 = 1.068 \times 10^{1}$ and $k_2 = 4.999 \times 10^{-3}$ for the growth of population.

Asia is made primarily of less-developed countries (BBC, 2014; Pereira, 2011) so the supposed "stunning" takeoff from the supposed stagnation to growth should have occurred around AD 1900 (Galor, 2008a; 2012a). The data show a certain degree of boosting shortly after the time of the claimed "stunning" takeoff from stagnation to growth. However, this boosting is not a transition from stagnation to growth because the preceding trajectory was not stagnant and because the boosted trajectory follows closely the historical trend. It was obviously only a temporary boosting because the boosted trajectory is progressively coming closer to the historical trajectory and judging by its decreasing gradient it is likely to move to the other side.

The growth of income per capita in the past was slow but it is the mathematically-expected characteristic of dividing two hyperbolic trajectories (Nielsen, 2015a). To claim Malthusian stagnation, we would have to demonstrate Malthusian oscillation during that time. Obviously they are missing in this display and in the display of the GDP and population distributions (Nielsen, 2016a; 2016d).

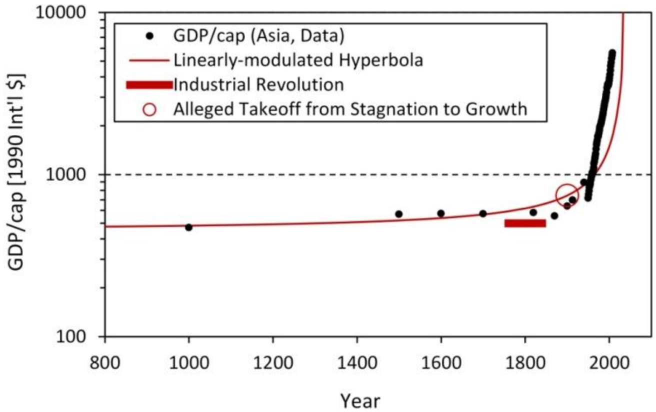

Figure 6. *Maddison's data (Maddison, 2010) describing the growth of income per capita (GDP/cap) in Asia are compared with the linearly-modulated hyperbolic distribution (Nielsen, 2015a) obtained by fitting the GDP and population data (Nielsen, 2016a; 2016d). The boosting, which commenced around AD 1950 follows closely the original historical trajectory and is likely to cross it and move to the other side. This temporary disturbance is a part of the commonly-observed recent transitions to slower trajectories. The only difference in this case is that the transition to a slower trajectory was preceded by a relatively minor boosting.*

The observed boosting could be probably explained by Japan's contribution to the total GDP/cap. Until 1900, Japan's contribution was less than 5% but by 1950 it gradually increased to 12% and by 2000 it climbed to 20%. Japan belongs to the more-developed countries so according to Galor (2008a; 2012a) it should have experienced "remarkable" and "stunning" takeoff (Galor, 2005a, pp. 177, 220) in its GDP/cap around 1750 but it did not. On the other hand, Asia should have experienced a sudden explosion in the GDP/cap growth around 1900 but it did not. It did experience a minor boosting close to that time but as already pointed out this minor boosting is probably associated with the increased economic activity in Japan. There was no dramatic transition from stagnation to growth as claimed by Galor but only a transition from the non-stagnant, linearly-modulated hyperbolic trajectory to a temporarily faster growth, which appears to have been caused primarily, if not entirely, by the increasing contribution of Japan's economy, the contribution, which should have commenced explosively around 1750 but it did not. Impressions prompted by wished-for features and reinforced by distorted presentations of data such as shown in

Figure 1 could be persuasive but they can be also strongly misleading. Data have to be rigorously analysed.

Africa

Results of analysis of the growth of income per capita in Africa are presented in Figure 7. As demonstrated earlier (Nielsen, 2016a, 2016d), the GDP and population data for Africa can be fitted using two hyperbolic distributions, a slow distribution followed by a fast distribution. The transition from the slow to fast distribution occurred around 1820 for the growth of the GDP and around 1840 for the growth of population.

Parameters describing the fitted GDP/cap distribution between AD 1 and 1820 are: $a_1 = 1.244 \times 10^{-1}$ and $k_1 = 5.030 \times 10^{-5}$ for the GDP and $a_2 = 5.794 \times 10^{1}$ and $k_2 = 2.473 \times 10^{-2}$ for the growth of the population. For the GDP/cap distribution from AD 1840, parameters are: $a_1 = 4.192 \times 10^{-1}$ and $k_1 = 2.126 \times 10^{-4}$ for the GDP and $a_2 = 1.571 \times 10^{2}$ and $k_2 = 7.834 \times 10^{-2}$. The fit to the transient region between AD 1820 and 1840 was obtained by polynomial interpolation.

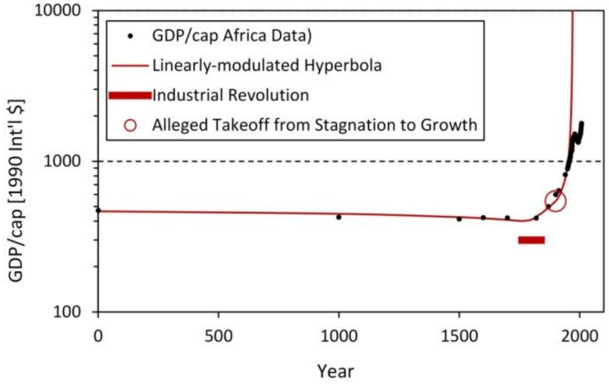

Figure 7. *Maddison's data (Maddison, 2010) describing the growth of income per capita (GDP/cap) in Africa are compared with the linearly-modulated hyperbolic distributions (Nielsen, 2015a) obtained by fitting the GDP and population data (Nielsen, 2016a; 2016d). The supposed takeoff from stagnation to growth around AD 1900 for less-developed regions (Galor, 2008a; 2012a) did not happen because the GDP/cap trajectory was not stagnant before that year and because it continued undisturbed after this year until around 1950 when it started to be diverted to a slower trajectory.*

Africa presents an interesting and unique case when the singularity for the hyperbolic trajectory describing the growth of population between AD 1 and 1840 is earlier than the singularity for the hyperbolic trajectory describing the growth of the GDP between AD 1 and 1820. For the growth of the population, the point of singularity is at $t = 2343$ while for the growth of the GDP it is at $t = 2473$ (Nielsen, 2016a, 2016d).

For the linearly-modulated hyperbolic distribution the escape to infinity at a fixed time occurs when the singularity for the growth of the GDP is earlier than the singularity for the growth of population (Nielsen, 2015a). If the singularity for the growth of population occurs earlier, as in Africa, then the GDP/cap ratio *decreases* slowly with time and then escapes rapidly to zero at the time of the singularity for the growth of population. The decreasing GDP/cap distribution between AD 1 and the early 1800s in Africa does not represent an unusual and distinctly new mechanism of economic growth but simply the mathematical property of dividing two hyperbolic distributions describing the growth of the GDP and population. In particular, it does not represent Malthusian stagnation because both the GDP and the population were increasing hyperbolically (Nielsen, 2016a, 2016d). It simply shows that the growth of population was just slightly faster than the economic growth. In all other regions, and globally, the past economic growth was faster than the growth of population, the unwelcome situation because if both trajectories are hyperbolic, such a growth leads eventually to excessively fast-increasing trajectories of income per capita, which, if unchecked, increase to infinity at a fixed time.

The best but still not the ideal solution would be to keep the economic growth and the growth of population increasing at approximately the same rate. In such a case, we could avoid approaching a critical growth in the income per capita. However, the problem of the growth of population and of the economic growth would still remain unsolved. Both cannot growth indefinitely and it does not matter whether they follow hyperbolic trajectories or some other increasing trajectories.

The past economic growth in Africa was close to a perfect balance between the economic growth and the growth of population. However, between AD 1820 and 1840, this nearly perfect balance changed dramatically to resemble the generally experienced trend of reckless and irresponsible economic growth. It does not mean that a secure economic growth can be only achieved by adopting primitive living conditions. Progress can be made but there is a limit to how much such a progress can be supported by the availability of natural resources. The current global economic growth is insecure (Nielsen, 2015b)

Africa is made of less-developed countries (BBC, 2014; Pereira, 2011) so according to Galor (2008a; 2012a) it should have experienced stagnation until around AD 1900 followed by a clear takeoff around that year. These expectations are contradicted by data.

In contradiction of Galor's interpretations of economic growth (Galor, 2005a, 2008a; 2011; 2012a), the Malthusian regime of stagnation did not exist. The GDP and population were increasing hyperbolically during the entire time of the supposed but non-existent regime of Malthusian stagnation, from AD 1 to 1900 and even after that year. Unrecognised by Galor (because he did not analyse data but preferred to use distorted diagrams) there was a transition between two hyperbolic trajectories during his assumed but non-existent regime of Malthusian stagnation, transition from a slowly decreasing income per capita described by the linearly-modulated hyperbolic trajectory to a fast increasing trajectory. When studied separately, it was a transition from slow hyperbolic growth of the GDP or population to a fast increasing hyperbolic growth. It was not a transition from stagnation to growth, claimed erroneously by Galor, but transition from growth to growth.

Africa is the only region where the economic growth was boosted at the time of the Industrial Revolution but it is also the poorest region, where the claimed Malthusian stagnation should have been most clearly demonstrated. According to Galor, Malthusian stagnation should have prevailed in Africa until around 1900 (Galor, 2008a; 2012a). This hypothesis, which appears to have been confirmed by his manipulation of data, is clearly and convincingly contradicted by their mathematical analysis.

Analysis of data describing the GDP and population in Africa shows that there was no stagnation over the entire range of the AD era (Nielsen, 2016a, 2016d). Economic growth (as described by the GDP) and the growth of population were following the steadily-increasing and undisturbed hyperbolic trajectories but around the time of the Industrial Revolution they were diverted to faster hyperbolic trajectories. There are no signs of Malthusian stagnation before and after the Industrial Revolution and before AD 1900, which was supposed to mark the end of the epoch of Malthusian stagnation. Hyperbolic growth, even if slow, does not represent Malthusian stagnation. Convincing signature of Malthusian stagnation is in the form of random fluctuations often described as Malthusian oscillations. This signature is missing in the data for

Africa and for other regions. Data for Africa and for other regions, show steadily-increasing hyperbolic distributions describing economic growth and the growth of population (Nielsen, 2016a, 2016d).

Analysis of the GDP/cap data for Africa shows that after a transition from a slowly-decreasing trajectory before around 1840 (which as we have pointed out does not represent Malthusian stagnation but the mathematical property of dividing monotonically-increasing hyperbolic trajectories) the growth of the GDP/cap in Africa was following a vigorously-increasing trajectory during the supposed Malthusian stagnation. So while the suitable manipulation of data (Galor, 2008a; 2012a) appears to be confirming preconceived ideas, mathematical analysis of precisely the same data shows that the preconceived ideas are clearly incorrect.

Here it might be a good place to point out that harsh and primitive living conditions in the past should not be immediately interpreted as a proof of the existence of Malthusians stagnation. Africa is at present the poorest region. It is also the only region where the growth of income per capita was negative over hundreds of years, between AD 1 and 1840 but there was no Malthusian stagnation in the growth of population and in the economic growth in Africa (Nielsen, 2016a; 2016d). There was also no Malthusian stagnation in other regions. Living conditions in the past in all regions were primitive and often harsh by modern standards and yet in all of them there was no Malthusian stagnation in the economic growth and in the growth of population. Worldwide, living conditions in the distant past were harsh and primitive and yet for as long as we can trace it by using analysable data, for a time extending down to 10,000 BC, there was no Malthusian stagnation in the growth of human population. With the exception of just two past and relatively brief transitions, the world growth of population was hyperbolic for nearly 12,000 years (Nielsen, 2016b).

Mathematical analysis of data finds no confirmation of the existence of the hypothetical epoch of Malthusian stagnation. It is a vague concept, which has no application in the explanation of the dynamics of economic and demographic growth. Its continuing presence in academic discussions as a tool to explain the dynamics of growth is not only irrelevant but also counterproductive because it diverts attention form finding correct explanations of the mechanism of the growth process.

It is also useful to compare results of the analysis for Africa with the results for Western Europe. Industrial Revolution, “the prime engine of economic growth" (Galor, 2005a, p. 212) should have worked most efficiently in Western Europe and its effects should have been most convincingly confirmed by data in this region, but these effects are convincingly contradicted by data: the supposed engine did not change the economic growth or the growth of population trajectories in Western Europe. Likewise, Malthusian stagnation should have been most prominently confirmed in Africa but it is not. There was never any form of stagnation in the economic growth in Africa.

Furthermore, while in Western Europe, Industrial Revolution had absolutely no impact on changing the economic growth trajectory, in Africa there was a spectacular acceleration of growth during the time of the Industrial Revolution but it was not the acceleration from stagnation to growth but from growth to growth. The wished-for features are contradicted by data showing that even plausible stories and explanations should not be accepted in science unless they can be confirmed by relevant data; otherwise they are at best just interesting stories with no scientific merit.

The supposed sudden acceleration (takeoff) in income per capita is supposed to have been associated with the benefits of progress such as better health care, better housing, better education, higher standard of living and generally better living conditions. However, data show that in Europe there was no takeoff in the income per capita at the time of the Industrial Revolution, while in Africa there was a dramatic acceleration without a dramatic improvement in the style of living. On the contrary, this dramatic boosting in income per capita in Africa at the time of the Industrial Revolution appears to coincide with the dramatic *deterioration* of living conditions of native populations. It occurred around the time of the intensified colonisation of Africa (Duignan & Gunn, 1973; McKay, et al., 2012; Pakenham, 1992).

If a sudden takeoff is supposed to mark the "remarkable" or "stunning" escape from the Malthusian trap (Galor, 2005a, pp. 177, 220), then the only such takeoff occurred in Africa. However, this dramatic takeoff did not mark the transition from stagnation to growth, because there was no stagnation. It also did not mark the dramatic escape from the Malthusian trap because there was no Malthusian trap in the economic growth or in the growth of population in Africa: both were increasing hyperbolically before the claimed but non-existent takeoff around AD 1900. The sudden and steep change in the economic growth and in the growth of population in Africa between AD 1820 and 1840did not mark the dramatic escape from the hypothetical Malthusian trap but the transition from freedom and independence to the trap of misery, deprivation and suffering of the native population. Poetic explanations could be interesting but they are not necessarily scientifically sound. Creative imagination can play an important role in science as long as it is tested by the scientific process of investigation.

Latin America

Results of analysis of economic growth in Latin America are presented in Figure 8. Data for Latin America contain a discontinuity in the growth of the GDP and population between AD 1500 and 1600 (Nielsen, 2016a, 2016d). This discontinuity is reflected in the discontinuity of the growth of income per capita (GDP/cap).

Parameters describing the slowly-increasing, linearly-modulated hyperbolic trajectory are $a_1 = 4.421 \times 10^{-1}$ and $k_1 = 2.093 \times 10^{-4}$ for the GDP and $a_2 = 1.765 \times 10^{2}$ and $k_2 = 8.242 \times 10^{-2}$ for the population. The fast-increasing trajectory from AD 1600 is described by the following parameters: $a_1 = 1.570 \times 10^{0}$ and $k_1 = 8.224 \times 10^{-4}$ for the GDP and $a_2 = 6.561 \times 10^{2}$ and $k_2 = 3.371 \times 10^{-1}$ for the population. The discontinuity in the economic growth and in the growth of population coincides with the onset of Spanish conquest (Bethell, 1984). However, after this relatively brief delay, economic growth and the growth of human population were following fast-increasing hyperbolic trajectories.

Latin America is also made of less-developed countries (BBC, 2014; Pereira, 2011) so again, according to Galor (2008a; 2012a), the growth of income per capita (GDP/cap) in this region should have been stagnant until around AD 1900 and fast from around that year. This pattern of growth is contradicted by data. Data show a diametrically different pattern: (1) there is no convincing evidence of the existence of stagnation over the entire range of time between AD 1 and 1900 (convincing evidence of Malthusian stagnation requires the presence of random fluctuations) but there is a sufficiently convincing evidence of the linearly-modulated hyperbolic growth particularly between AD 1600 and 1900; (2) there was no takeoff from stagnation to growth at any time; and (3) at the time of the postulated takeoff in 1900 the growth of income per capita started to be diverted to a slower trajectory. The wished-for takeoff is replaced by a *slower* growth. However, even if we had a takeoff around that time it would have been a takeoff of a different kind, not a takeoff from stagnation to growth as required by the Unified Growth Theory (Galor, 2005a; 2011) but a takeoff from growth to growth.

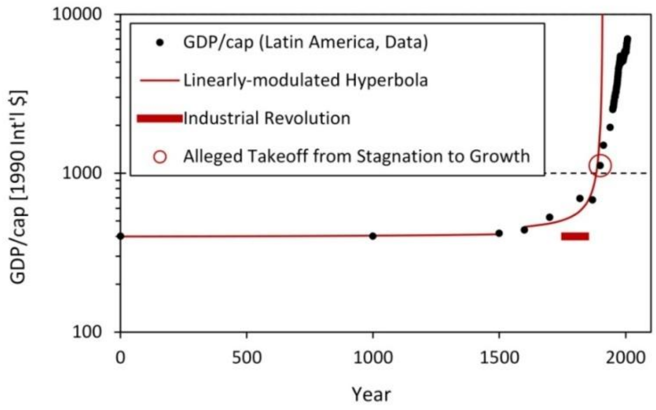

Figure 8. *Maddison's data (Maddison, 2010) describing growth of income per capita (GDP/cap) in Latin America are compared with the linearly-modulated hyperbolic distributions (Nielsen, 2015a) obtained by fitting the GDP and population data (Nielsen, 2016a; 2016d). There was a discontinuity in the economic growth and in the growth of population between AD 1500 and 1600 reflected in the discontinuity in the GDP/cap distribution. The supposed takeoff from stagnation to growth around AD 1900 (Galor, 2008a; 2012a) did not happen because the GDP/cap trajectory was not stagnant before that year and because there was no sudden acceleration in growth around that year. On the contrary, around the supposed takeoff there was a transition to a* slower *trajectory.*

Summary and conclusions

Results of mathematical analysis of the historical income per capita (GDP/cap) distributions are summarised in Table 1. Listed parameters (a_1 , k_1 , a_2 , k_2)describe the fitted, linearly-modulated hyperbolic trajectories (Nielsen, 2015a) represented by the ratios of hyperbolic distributions describing the growth of the GDP and population. Parameters a_1 and k_1 describe hyperbolic distributions fitting the GDP data (Nielsen, 2016a), while parameters a_2 and k_2 describe hyperbolic distributions fitting population data (Nielsen, 2016d).

Results of this analysis demonstrate explicitly that the postulated by Galor (2005a; 2008a; 2011; 2012a) takeoffs in the income per capita (GDP/cap) did not happen. There were no transitions from stagnation to growth because within the mathematically-analysable data Galor's regime of Malthusian stagnation did not exist in the growth of income per capita. Growth of income per capita was following the linearly-modulated hyperbolic distributions until recently when it started to be diverted to slower trajectories.

Galor's Unified Growth Theory (Galor, 2005a; 2011) is contradicted yet again by data. The "remarkable" and "stunning' escape from the Malthusian trap (Galor, 2005a, pp. 177, 220) never happened because there was no trap in the economic growth and in the growth of population. His claim of the existence of the differential timing of takeoffs is also contradicted by data because we cannot have differential timing of takeoffs without takeoffs. Galor describes phenomena that did not exist. His explanations of economic growth are based on phantom features created by hyperbolic illusions and magnified by his habitually distorted presentations of data such as illustrated in Figure 1. His theory is irrelevant and misleading.

Galor had access to the excellent data of Maddison (2001). He even used them during the formulation of his theory but he did not attempt to analyse them, which is surprising because their analysis is trivially simple (Nielsen, 2014). Now, precisely the same data can be used to demonstrate that his Unified Growth Theory (Galor, 2005a; 2011) is repeatedly contradicted by data (Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f).

Table 1. *Summary of the mathematical analysis of the historical income per capita (GDP/cap) distributions*

Region	a_1	k_1	a_2	k_2	Stagnation	Takeoff
World	1.684×10^{-2}	8.539×10^{-6}	7.739×10^{0}	3.765×10^{-3}	X	X
Western Europe	9.859×10^{-2}	5.112×10^{-5}	7.542×10^{1}	3.749×10^{-2}	X	X
Eastern Europe	7.749×10^{-1}	4.048×10^{-4}	3.055×10^{2}	1.525×10^{-1}	X	X
Former USSR	6.547×10^{-1}	3.452×10^{-4}	2.618×10^{2}	1.333×10^{-1}	X	X
Asia	2.303×10^{-2}	1.129×10^{-5}	1.068×10^{1}	4.999×10^{-3}	X	X
Africa	1.244×10^{-1}	5.030×10^{-5}	5.794×10^{1}	2.473×10^{-2}	X	X
	4.192×10^{-1}	2.126×10^{-4}	1.571×10^{2}	7.834×10^{-2}		
Latin America	4.421×10^{-1}	2.093×10^{-4}	1.765×10^{2}	8.242×10^{-2}	X	X
	1.570×10^{0}	8.224×10^{-4}	6.561×10^{2}	3.371×10^{-1}		

Notes: a_1, k_1, a_2, k_2 – Parameters describing linearly-modulated hyperbolic distributions (ratios of hyperbolic distributions). Parameters a_1, k_1 describe hyperbolic growth of the GDP, while a_2, k_2 describe hyperbolic growth of population [see eqn (1)]. X – No stagnation/takeoff. Within the range of the mathematically-analysable data the claimed by Galor (2005a; 2008a; 2011; 2012a) Malthusian regime of stagnation did not exist. The claimed takeoffs from stagnation to growth never happened.

Unified Growth Theory is fundamentally incorrect and scientifically unacceptable. It is a theory based on scientifically unsupported concepts created by impressions and reinforced by the manipulation of data. Excellent data of Maddison (2001) were not analysed but presented repeatedly using distorted and misleading diagrams such as shown in Figure 1. Such distorted presentation of data appears not only ubiquitously in the Unified Growth Theory but also in other related publications (Ashraf, 2009; Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). Selected but meaningless and misleading values of data were also repeatedly quoted to support the concept of stagnation followed by takeoffs from stagnation to growth. This is an unscientific approach to research.

Galor's handling of data is puzzling. Maybe he does not know how to analyse data, but this conclusion is hard to accept because he appears to be familiar with mathematics and the analysis of hyperbolic distributions is trivially simple (Nielsen, 2014). It looks as if, for whatever reason, he purposefully manipulated data to support his preconceived ideas.

Assisted by the excellent data of Maddison (2001) available to him at the time of the formulation of his theory, Galor was on the verge of making an important contribution to science by showing that economic growth was hyperbolic, in agreement with what has been already known about the growth of population (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962; 2002; von Hoerner, 1975). He would have discovered that there was no Malthusian stagnation and no takeoffs from stagnation to growth the economic growth, the features which were not confirmed in the published results describing the growth of population. However, he missed this excellent opportunity because he failed to follow the fundamental principles of scientific investigation, which require that theories should be tested by data and that research should be guided and moderated by data.

The most plausible explanation why Galor appears to have been reluctant to be guided by data and why he apparently manipulated data to support his preconceived ideas is that he was blinded by prejudice. It is a kind of fear or reluctance to accept evidence contradicting the established knowledge. Psychologists describe it as cascade behaviour, information cascade, informational avalanche, illusion of truth, illusory truth, illusion of familiarity, running with the pack, following the crowd, herding behaviour, bandwagons and path depending choice (Anderson & Holt, 1997; Begg, Anas & Farinacci, 1992; Bikhchandani, Hirshleifer & Welch, 1992; De Vany & Lee, 2008; De Vany & Walls, 1999; Easley & Kleinberg, 2010; Grebe, Schmid & Stiehler, 2008; Ondrias, 1999; Parks & Tooth, 2006; Ramsey, Raafat, Chater & Frith, 2009; Walden & Browne, 2003).

In the demographic and economic research this phenomenon is demonstrated by the reluctance to accept the compelling contradicting evidence in data simply because many demographers or economists would not agree with the contradicting evidence. It is safer to follow the crowd and run with the pack. Tradition is stronger than science and only an outsider who has not been blinded by prejudice and who is not afraid of being rejected by the crowd might dare to show that the accepted doctrines are incorrect. He or she is then likely to be ridiculed and rejected but science is a self-correcting discipline so sooner or later such resistance to accept the overwhelming empirical evidence will have to be broken, but it would be better for science and scientists if the required change in the paradigm is accepted sooner rather than later.

The evidence contradicting currently-accepted interpretations is overwhelming: historical economic growth and historical growth of population were hyperbolic (Kapitza, 2006; Kremer, 1993; Nielsen, 2014; 2015a; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; Podlazov, 2002; Shklovskii, 1962; 2002; von Foerster, Mora & Amiot, 1960; von Hoerner, 1975). Hyperbolic growth should be the basis for explaining the mechanism of the historical growth of population and of the historical economic growth.

Interpretations revolving around the concept of Malthusian stagnation and around assumed transitions from stagnation to growth are repeatedly and consistently contradicted by data and by their mathematical analyses. Historical economic growth and historical growth of population cannot be divided into distinctly different regimes governed by distinctly different mechanisms of growth. Hyperbolic growth has to be explained as a whole. The same mechanism has to be applied to the slow and fast hyperbolic growth because it is mathematically impossible to divide hyperbolic distributions into distinctly different sections (Nielsen, 2014). Once we can explain properly the mechanism of the past growth we might be able to understand better the current growth and how it should be controlled.

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