Ron W. Nielsen

Evidence-based

UNIFIED GROWTH THEORY

Vol.3

Mechanism of the growth of population and of economic growth in the past 2,000,000 years explained

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Ron W. Nielsen
Australian National University, Australia

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Preface

The true laboratory is the mind, where behind illusions we uncover the laws of truth.
— Sir Jagadish Chandra Bose

Duration is not a test of true or false.
— Anne Morrow Lindbergh

If they don't depend on true evidence, scientists are no better than gossips.
— Penelope Fitzgerald

In science it often happens that scientists say, 'You know that's a really good argument; my position is mistaken,' and then they would actually change their minds and you never hear that old view from them again. They really do it. It doesn't happen as often as it should, because scientists are human and change is sometimes painful. But it happens every day.
— Carl Sagan

Unified Growth Theory[1] published by Oded Galor is called unified because it puts together earlier attempts to explain the historical economic growth and the historical growth of population. These attempts were made over many years and by now they form the established knowledge in economics and in demography.

Unfortunately, the past research was difficult because (1) access to data was strongly limited and (2) growth turns out to be represented by strongly deceptive distributions. They create an illusion of stagnation followed by a sudden explosion, while in fact they increase monotonically all the time and there is no sudden transition from a slow to fast growth. Data represented by these distributions have to be carefully and methodically analysed; otherwise conclusions are based on illusions.

Galor was in a far better position than many of the past researchers because he had access to superb and extensive sets of data made available by the world renown economist, Angus Maddison. These data describe economic growth and the growth of population, global, regional and even in individual countries. They

[1] Galor, O. (2011). *Unified Growth Theory.* Princeton, New Jersey: Princeton University Press.

are a rich source of information, which Galor failed to use. He made no attempt to analyse them.

There is no explanation for his neglect to analyse data mathematically because (1) he uses mathematics in his theory and thus he is familiar with mathematical procedures and (2) because trajectories describing growth of population and economic growth, while being deceptive, are trivially easy to analyse. No great skill is needed to analyse these distributions. Indeed, there is even no need to analyse them mathematically. Reliable conclusions can be reached just by using different plots of data. However, mathematical analysis, which is simple and easy, helps in a better understanding of the mechanism of growth.

Galor ignored also the earlier evidence published in 1960 that the growth of population during the AD section of time was hyperbolic. Using this information, the obvious next step would be to check whether the same type of growth is applicable to the economic growth.

Rather than using the previously published evidence, he systematically presented data in a suitably distorted way to support preconceived ideas. He could have made an important discovery but he did not. His theory presents nothing new. It is just a repetition of old interpretations of the growth of population and of economic growth, incorrect interpretations because they are contradicted by data. Unified Growth Theory is repeatedly contradicted even by the same data, which were used during its formulation.

The presented here *Evidence-based Unified Growth Theory* is firmly supported by a rigorous, mathematical analysis of data describing economic growth and the growth of population. It is also called *unified* because it presents a unified explanation of the growth of population and of economic growth in the past 2,000,000 years.

The terms *Malthusian stagnation, Malthusian regime* and *Malthusian trap* will be used in the presented here discussion but it should be remembered that they are incorrect, because Malthus never claimed that his positive checks were causing stagnation or creating a certain regime of growth or a trap. On the contrary, he observed that they stimulated growth and he even suggested that this curious phenomenon should be further investigated. Unfortunately, his observation was ignored, dubious concepts were later introduced and the name of Malthus was questionably attached to them, which Malthus would probably not approve. These phrases are used only because in this form, they are repeatedly used in the published literature.

This book is a compilation of my articles describing the investigation of the growth of population and of economic growth. I start by showing why the established knowledge is scientifically unacceptable. I follow this chapter by the introduction of a simple method of reciprocal values, which makes the analysis of hyperbolic distributions trivially simple. These two introductory chapters are followed by the explanation how the Unified Growth Theory is contradicted by data. These chapters are in turn followed by a detailed study of the growth of human population and of economic growth in the past 2,000,000 years; by the discussion of earlier attempts to explain the mechanism of hyperbolic growth; by the examination of the impacts of Malthusian positive checks; by the examination of impacts of demographic catastrophes; by the examination of the relation between the growth rate and growth trajectories, the essential step leading to the explanation of the mechanism of growth; by the formulation of the general law of growth; and by the explanation of the mechanism of the hyperbolic growth of human population and of the economic growth.

Ron W. Nielsen
Gold Coast, Australia
July, 2018

Contents

Introduction
(1)

Chapter 1
Economic Growth and the growth of human population in the past 2,000,000 years
(5)

Chapter 2
Earlier interpretations of hyperbolic growth
(33)

Chapter 3
Examination of Malthusian positive checks
(78)

Chapter 4
Demographic catastrophes did not shape the growth of human population
(110)

Chapter 5
Mathematical analysis of income per capita in the United Kingdom
(141)

Chapter 6
The law of growth
(156)

Chapter 7
Mechanism of hyperbolic growth explained
(168)

Chapter 8
New direction for the economic and demographic research
(193)

Introduction

This introduction is designed as a guide to the topics discussed in this book.

The spontaneous (unconstrained and undisturbed) growth of human population is not exponential, as was expected by Malthus, but hyperbolic. The same applies to the economic growth. This conclusion is in harmony with the earlier investigation carried out by von Foerster, Mora and Amiot[2] who studied the growth of the world population during the AD section of time. However, the study presented here extends the analysis to the BC time and to the economic growth. It also includes the analysis of regional growth of population and regional economic growth.

Results presented here are also in harmony with the earlier study of Deevey[3], who observed that growth of human population in the past 1,000,000 years was in three stages. However, he postulated that each stage was reaching an equilibrium. Results presented here confirmed the three stages of growth but demonstrated that each stage was hyperbolic. Rather than reaching an equilibrium, each stage had a potential to increase to infinity and was at a certain time terminated.

Two well-known theories, the Unified Growth Theory and the Demographic Transition Theory[4], are contradicted by the same data, which were used in their support.

[2] von Foerster, H., Mora, P., & Amiot, L. (1960). Doomsday: Friday, 13 November, A.D. 2026. *Science, 132*, 1291-1295.

[3] Deevey, E. S. Jr (1960). The human population. *Scientific American,203*(9), 195-204.

[4] For references see Vol.2, Chapter 5.

In the case of the Demographic Transition Theory, data, which appeared to be in support of this theory, were never analysed. Conclusions are based on impressions. However, in addition, contradicting data are systematically ignored.

In the case of the Unified Growth Theory, data were also never analysed but they were suitably distorted to support preconceived ideas. This deliberately distorted and misleading presentation of data is used in many other related publications.

There is no convincing explanation why the Author of the Unified Growth theory failed to analyse data mathematically and why he was systematically presenting them in a distorted way, because (1) he used mathematics in his theory and thus he is familiar with mathematical procedures, (2) hyperbolic growth was demonstrated as early as in 1960, (3) it is hard to imagine that he is not familiar with the fundamental properties of hyperbolic distributions, that they increase slowly over a long time and fast over a short time but that they increase monotonically, and (4) mathematical analysis of hyperbolic distributions is trivially simple.

Precisely the same data, which in their deliberately distorted way were used to support the Unified Growth Theory, are in fact in its direct contradiction. It is hard to understand why so much work was devoted to support the earlier erroneous interpretations of the mechanism of growth and why data were not properly analysed to check whether these interpretations, which were earlier based on limited data and on illusions, could be still supported.

Income per capita distributions show puzzling characteristics. They show that over a long time, income per capita was approximately constant but then, most recently, it was increasing extremely rapidly. The analysis of data presented here explains these puzzling characteristic features. They reflect nothing more than mathematical properties of dividing two hyperbolic distributions. They do not represent some peculiar mechanism applicable only to the economic growth but the feature, which applies to any two hyperbolic distributions, with only one condition that the singularity of the numerator is earlier than the singularity of the denominator.

Galor describes certain mysteries of growth in his Unified Growth Theory and indicates that they should be studied and explained. They have now been explained. They have nothing to do with the growth of population or with the economic growth. They were created by his distorted representations of data.

Galor describes a puzzling phenomenon of great divergence. This claimed phenomenon is also nothing more than a feature created by his distorted representations of data. There was no great divergence and there is nothing to explain except to explain how the great divergence was created by Galor.

Industrial Revolution had no impact on changing growth trajectories describing growth of population and economic growth, even in Western Europe and even in the United Kingdom. Forces associated with the Industrial Revolution are reflected in changing socio-economic conditions but they did not shape growth trajectories of the growth of population and of economic growth.

With the exception of just one event, demographic catastrophes had no impact on shaping the growth of population. The one and only exceptional event in the past 2,000,000 years, as presented by data, was an unusual convergence of five strong demographic catastrophes between AD 1195 and 1470. However, even this unusual event caused only a minor disturbance in the growth trajectory. When this exceptionally strong crisis was over, growth of population was even faster than before.

Survey of demographic catastrophes indicated that they were, in general, too weak to cause a major disruption in the growth of the world population even if they had strong local impacts. Analysis of Malthusian positive checks also added to the explanation why demographic catastrophes did not shape the growth of the world population.

It is interesting that Malthus noticed the dichotomous property of his positive checks, i.e. their destructive and regenerating effects. He even suggested that the regenerating effects should be further investigated. Unfortunately, the original observation of Malthus was ignored and the destructive aspect of his positive checks was blown out of proportion and used to explain the claimed prolonged stagnation, that never happened, while no effort was made to understand their regenerating property, which is in fact common in nature.

Mathematical analysis of the effects of Malthusian positive checks has now been carried out and it demonstrated that Malthus was right. His positive checks increase mortality rates but they also increase fertility rates, with the combined effect of increasing the growth rate. The regeneration process, or the growth stimulating property, is so efficient that the growth is even faster. This is a well-known phenomenon but it is an inconvenient property for those who created the concept of the prolonged epoch of stagnation used in the Demographic Growth Theory and in the Unified Growth Theory.

As a part of the presented here investigations, general law of growth was formulated and used to explain the mechanism of hyperbolic growth of population and of economic growth. It turns out that the mechanism is exceptionally simple, which is hardly surprising because hyperbolic growth is described by an exceptionally simple mathematical formula.

With the exception of two major transitions (46,000 - 27,000 BC and 425 BC – AD 510) and one minor disturbance (AD 1195 – 1470), growth of the world population in the past 2,000,000 years was consistently hyperbolic. It was steadily increasing without any signs of a random behaviour or of a sudden rapid increase towards the end of this long time. There was no stagnation and no sudden explosion. The same applies to the economic growth, which for the most part of the past 2,000,000 years was directly proportional to the size of human population. Explanation of the dynamics of growth is much simpler than presented in the Unified Growth Theory or in the Demographic Growth Theory or in many other published discussions, which ignore the earlier evidence of hyperbolic growth and which are not supported by a rigorous analysis of data but by impressions and conjectures.

1. Economic Growth and the growth of human population in the past 2,000,000 years

Introduction

The aim of this publication is to analyse the growth of human population and the associated economic growth in the past 2,000,000 years. This work is an extension of our previous analysis of the growth of human population in the past 12,000 years (Nielsen, 2016a) and of the analysis of economic growth during the AD era (Nielsen, 2016b). These earlier studies demonstrated that the natural tendency for the growth of human population and for the economic growth is to follow hyperbolic distributions. Hyperbolic growth can be faster or slower but it is always prompted by the fundamentally the same mechanism.

We have shown that the mechanism of hyperbolic growth of human population can be easily explained (Nielsen, 2016c). It is a growth prompted by just one indispensable force, the biologically-controlled force of procreation expressed as a difference between the biologically-controlled force of sex drive and the biologically-controlled process of aging and dying. No other forces are needed. A change in the growth trajectory occurs only if other forces interfere substantially with this fundamental force of growth. In the past 12,000 years, there was *only one strong* interference, around AD 1, and *one minor* interference, around AD 1300. Each time, the fundamental character of the growth trajectory was not changed. There was only a transition from one hyperbolic trajectory to another. The first time, it was a transition from a fast to a slow hyperbolic growth, while the second time it was a transition to only a slightly faster growth. With the exception of these two, relatively brief transitions, the growth was always hyperbolic. In addition to these past transitions we now experience a new strong interference reflected in the gradual slowing down growth. The growth of population is no longer hyperbolic but it is still close to the historical hyperbolic growth. Now, we are going to demonstrate that hyperbolic growth prevailed not only in the past 12,000 years but also during the past 2,000,000 years.

Data for the early growth of population

If data for the BC era down to 10,000 BC are scarce, data beyond that time are even more difficult to find. However, we now have a few estimates from reputable sources and we can use them to extend the analysis of the growth of human population down to 2,000,000 million years ago.

The earliest estimates were made by Deevey (1960). He estimated that during the Lower Palaeolithic (around 1,000,000 years ago) the size of population was 0.125 million, during the Middle Palaeolithic (around 300,000 years ago) it was 1 million and 3.34 million during the Upper Palaeolithic (around 25,000 years ago). Birdsell (1972) estimated 0.4, 1 and 2.2 million for the same years, respectively, while Hassan (1981) estimated 0.6, 1.2 and 6. In 2002, he estimated 0.4, 0.8, 1.2 and 3.3 million at 1,500,000, 1,000,000, 100,000 and 14,000 years ago, respectively (Hassan, 2002). In our calculations, we shall use his updated estimates (Hassan, 2002). Incidentally, it should be noted that his two values listed in his Table 17.2 (Hassan, 2002, p. 684) are clearly misplaced. The values of 0.4 and 0.8 million should have been aligned with 1,500,000 and 1,000,000 respectively. However, his diagram presented as Figure 17.2 is correct.

All these estimates are listed in Table 1. The corresponding years are expressed as BC. The expression *years ago* or *before present* are interpreted as before 2000. The years 1,500,000, 1,000,000 and 300,000 years ago or before present are interpreted as 1,500,000 BC, 1,000,000 BC and 300,000 BC. The values for the years after100,000 BC were reduced by 2000.

Table 1. *Estimates of the size of population before 10,000 BC (in million)*

Year (BC)	Deevey (1960)	Birdsell (1972)	Hassan (2002)	Average Values
1,500,000			0.4	0.4
1,000,000	0.125	0.4	0.8	0.44
300,000	1	1		1
100,000			1.2	1.2
23,000	3.34	2.2		2.77
12,000			3.3	3.3

Reciprocal values of these data are shown in Figure 1. This figure shows also data used in the earlier analysis (Nielsen, 2016a). As pointed out earlier (Nielsen, 2014), linearly decreasing reciprocal values identify hyperbolic growth, because hyperbolic growth is described by the reciprocal of a decreasing straight line:

$$S(t) = \frac{1}{a - kt} \tag{1}$$

where $S(t)$ is the size of the growing entity, in our case the size of population or the size of economic growth, t is the time and a and k are positive constants.

In Figure 1 we see two straight lines. They cross at 34,350 BC. Around that time there was a transition from a slow hyperbolic trajectory to a significantly faster growth, also described by hyperbolic trajectory. This transition was one of only two *major* transitions in the past 2,000,000 years. The later major transition was around AD 1 (Nielsen, 2016a). A closer view of this first earlier transition is shown in Figure 2.

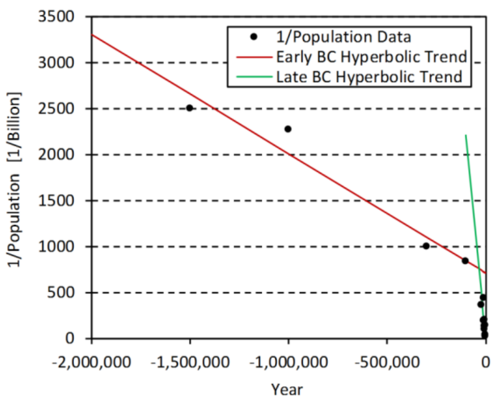

Figure 1. *Reciprocal values of the size of population between 1,500,000 BC and 1000 BC. Decreasing straight lines for the reciprocal values identify hyperbolic trajectories (Nielsen, 2014). The two trajectories cross at 34,350 BC marking a transition from a slow to a fast hyperbolic trajectory. The late BC trajectory was discussed earlier (Nielsen, 2016a). The BC years are represented by negative numbers.*

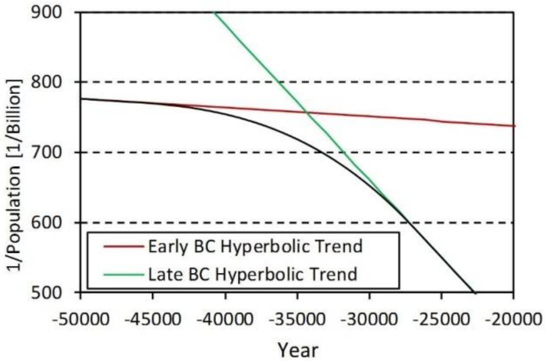

Figure 2. *The first major demographic transition. It occurred between 46,000 BC and 27,000 BC. It was a transition from a slow hyperbolic growth to a significantly faster hyperbolic growth, which prevailed until 425 BC to be replaced in AD 510 by a significantly slower hyperbolic growth during the AD era (Nielsen, 2016a). The BC years are represented by negative numbers.*

This early transition commenced around 46,000 BC and continued until around 27,000 BC. From around that year, the growth of the world population started to follow a significantly faster hyperbolic trajectory. These new data confirm that the natural tendency for the growth of population is to follow hyperbolic distributions.

Growth of human population in the past 2,000,000 years

Overview

In Figure 3 we show the average values of data describing the growth of the world population in the past 2,000,000 years (Biraben, 1980; Birdsell, 1972; Clark, 1968; Cook,1960; Deevey, 1960; Durand, 1974; Gallant, 1990; Hassan, 2002; Haub, 1995; Livi-Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973; 1999; 2013; US Census Bureau, 2017). The time scale is in years before 2100. We also display the best fit to the data, which most of the time is hyperbolic. We can see these hyperbolic distributions more clearly in Figure 4. The fit presented in Figure 3, combined with the exceptionally slow growth during the first stage, allows for the extension of the growth of population to 2,000,000 years before 2100 or to approximately to 2,000,000 BC.

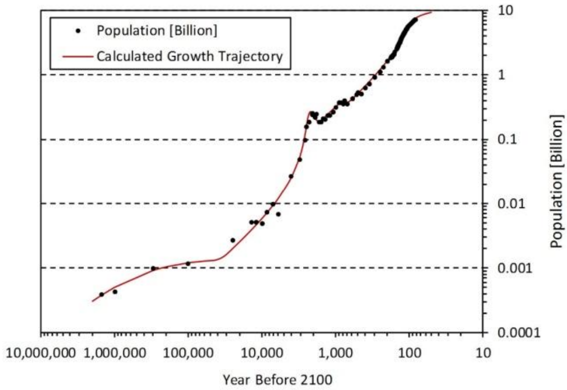

Figure 3. *Growth of human population in the past 2,000,000 years.*

Growth of human population in the past 2,000,000 was in three major stages but it was not in the stages imagined by Deevey (1960). It is remarkable that based on a strongly limited information he did realise that the growth of population was in three major stages. However, while being close to the correct interpretation of the growth of population, Deevey imagined the three stages incorrectly. He imagined that each stage was leading to an equilibrium, i.e. to a plateau in the growth of population as shown in Figure 5. This figure is based on his conceptual diagram (Deevey, 1960, p. 198). If we compare his interpretation of growth shown in Figure 5 with the growth presented in Figure 3, we can see that the growth was indeed in three stages as suggested by Deevey but the details of the growth trajectories are clearly different. Only the first stage looks similar to the stage suggested by Deevey. However, as we shall explain later, this stage also did not lead to an equilibrium.

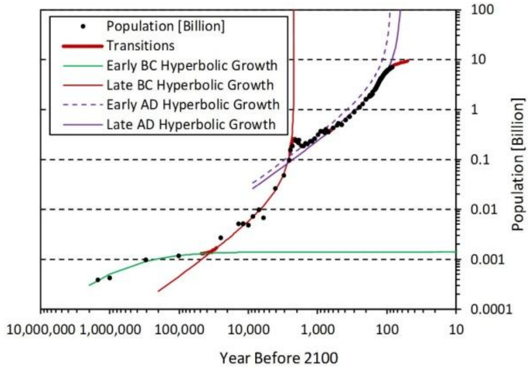

Figure 4. *The three major stages of growth of the world population in the past 2,000,000 years: (1) between 2,000,000 BC and 27,000 BC, (2) between 27,000 BC and AD 510 and (3) between AD 510 and present. The last stage experienced a minor distortion between around AD 1195 and 1470. This distortion caused a small shift in the hyperbolic growth.*

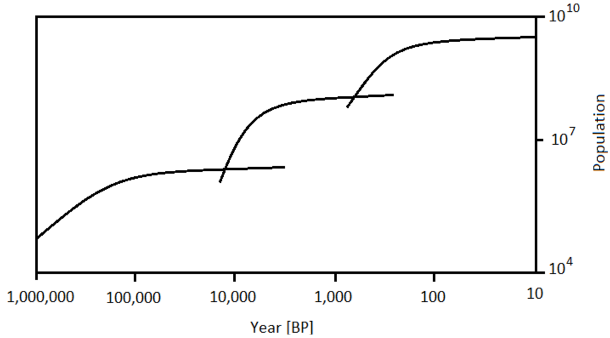

Figure 5. *The "three population surges" as imagined by Deevey (1960, p. 198).*

It is remarkable, that the currently established knowledge (Nielsen, 2016g), which is based on the doctrine of Malthusian stagnation, ignores not only results of von Foerster, Mora & Amiot (1960) but also the observation of Deevey (1960). These two early studies clearly demonstrated that there was no stagnation in the growth of population. They indicated that there was a regular and well-defined pattern of growth, which contradicts the doctrine of Malthusian stagnation.

Results of von Foerster, Mora & Amiot (1960) demonstrated that the growth of human population during the AD era was hyperbolic, and consequently not stagnant. It followed a monotonically increasing trajectory. Hyperbolic growth is in the direct contradiction of the concept of Malthusian stagnation.

Hyperbolic growth is slow over a long time (but not stagnant) and fast over a short time, so fast that it escapes to infinity at a fixed time. It is the growth, which is governed by the same mechanism when it is slow and when it is fast. If we want to interpret the slow growth as stagnant, we should apply the same interpretation to the fast growth. It would be obviously ludicrous to describe the growth escaping to infinity at a fixed time as stagnant, but because it is always the same growth, then it is also ludicrous to describe it as stagnant when it is slow. The concept of Malthusian stagnation is based on the incorrect interpretation of hyperbolic growth and has no place in science.

Results of Deevey indicated that over a longer time, extending as far back as to 1,000,000 years ago, the growth was in three distinctly different stages. It is again a clearly different pattern than the pattern suggested by the concept of Malthusian stagnation. The doctrine of Malthusian stagnation claims an endless stagnant state of growth, characterised by unpredictable, random fluctuations often described as Malthusian oscillations. Hyperbolic growth is definitely predictable and consequently it suggests an entirely different interpretation of the mechanism of growth.

Studies of von Foerster, Mora and Amiot (1960) and of Deevey (1960) indicated that there was nothing chaotic about the growth of population. They indicated that there was a certain regular pattern. Such a regular pattern can hardly be expected to be produced by random forces of growth.

In order to understand the growth of population in the past 2,000,000 years, it is useful to discuss separately its three stages of growth as presented in Figure 4: (1) between 2,000,000 BC and 27,000 BC, (2) between 27,000 BC and AD 510, and (3) between AD 510 and present. Each of these stages is described by hyperbolic growth followed by a transition to the next stage. However, the last stage contains a fine structure expressed as a slight shift in the hyperbolic distribution.

Mathematics of growth

Parameters describing the growth of population in the past 2,000,000 years are listed in Table 2. They are: a and k, for the hyperbolic growth and a_i ($i = 0$ to n) and b_i ($i = 0$ to $n+1$) for transitions. They can be used to calculate the size of population $S(t)$ and the growth rate $R(t)$ at any given time. For these parameters, the size of population is in billions. The time is in years and it is positive for the AD era and negative for the BC era.

Table 2 presents also the range of $S(t)$ and $R(t)$ values for hyperbolic distributions, which are also the range of values for transitions, because the end of a given hyperbolic growth is the beginning of a transition while the beginning of a hyperbolic growth is the end of a preceding transition.

Mathematics of growth of population is exceptionally simple. As discussed earlier (Nielsen, 2016a, 2016c) and as shown in Figures 3 and 4, the growth was hyperbolic, except when there was a relatively brief transition. Hyperbolic growth is described by a very simple mathematical expression, presented as eqn (1), which is a solution of a very simple differential equation:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = kS(t)\,. \tag{2}$$

Transitions are described by a similar differential equation:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = k(t)S(t)\,. \tag{3}$$

Table 2. *Parameters describing growth of human population in the past 2,000,000 years.*

Hyperbolic Growth		Transitions	
$S(t) = (a - kt)^{-1}$; $R(t) = kS(t)$		$S(t) = \left[\sum_{i=0}^{n+1} b_i t^i\right]^{-1}$; $k(t) = \sum_{i=0}^{n} a_i t^i$; $R(t) = k(t)S(t)$	
Years	**Parameters**	**Years**	**Parameters**
2.000,000 – 46,000 BC	$a = 7.120\times10^{2}$	46,000 – 27,000 BC	$b_0 = -9.247\times10^{2}$
	$k = 1.296\times10^{-3}$		$b_1 = -9.990\times10^{-2}$
2,000,000 BC	$S(t) = 3.027\times10^{5}$		$b_2 = -1.966\times10^{-6}$
	$R(t) = 3.923\times10^{-5}\%$		$b_3 = -1.295\times10^{-11}$
46,000 BC	$S(t) = 1.296\times10^{6}$		$a_0 = 9.990\times10^{-2}$
	$R(t) = 1.680\times10^{-4}\%$		$a_1 = -1.808\times10^{-1}$
			$a_2 = 3.885\times10^{-11}$
27,000 – 425 BC	$a = 2.282\times10^{0}$	425 BC – AD 510	$b_0 = 3.834\times10^{0}$
	$k = 2.210\times10^{-2}$		$b_1 = 2.347\times10^{-3}$
27,000 BC	$S(t) = 1.682\times10^{6}$		

	$R(t)=3.718\times10^{-3}\%$		$b_2=1.330\times10^{-5}$
425 BC	$S(t)=1.406\times10^{8}$		$b_3=-2.493\times10^{-8}$
	$R(t)=3.108\times10^{-1}\%$		$a_0=-2.347\times10^{-3}$
			$a_1=-2.659\times10^{-5}$
			$a_2=7.479\times10^{-8}$
AD 510 – 1195	$a=6.940\times10^{0}$	AD 1195 – 1470	$b_0=-2.903\times10^{2}$
	$k=3.448\times10^{-3}$		$b_1=1.022\times10^{0}$
AD 510	$S(t)=1.930\times10^{8}$		$b_2=-1.309\times10^{-3}$
	$R(t)=6.654\times10^{-2}\%$		$b_3=7.326\times10^{-7}$
AD 1195	$S(t)=3.546\times10^{8}$		$b_4=-1.517\times10^{-10}$
	$R(t)=1.223\times10^{-1}\%$		$a_0=-1.022\times10^{0}$
			$a_1=2.618\times10^{-3}$
			$a_2=-2.198\times10^{-6}$
			$a_3=6.068\times10^{-10}$
AD 1470 – 1950	$a=9.123\times10^{0}$	AD 1950 – 2016	$b_0=2.001\times10^{3}$
	$k=4.478\times10^{-3}$		$b_1=-2.928\times10^{0}$
AD 1470	$S(t)=3.935\times10^{8}$		$b_2=1.428\times10^{-3}$
	$R(t)=1.762\times10^{-1}\%$		$b_3=-2.323\times10^{-7}$
AD 1950	$S(t)=2.550\times10^{9}$		$a_0=2.928\times10^{0}$
	$R(t)=1.142\times10^{0}\%$		$a_1=-2.856\times10^{-3}$
			$a_2=6.968\times10^{-7}$

$S(t)$ - the size of population. $R(t)$ - the growth rate. In mathematical formulae, time is in years and it has positive values for the AD era and negative for the BC era. Furthermore, for the listed parameters, the size of population is in billions. The growth rate given by the mathematical formulae is not expressed in per cent.

Parameter *k*, whether constant or dependent on time, is the driving force divided by the resistance to growth (Nielsen, 2016c). For the growth of population, the driving force is the force of procreation given by the difference between the *biologically controlled* force of sex drive and the *biologically controlled* aging and dying. It is a spontaneous, unrestrained and fundamental force of growth, which has to be considered in any attempt to explain the mechanism of growth of human population. Other forces may be added but only if necessary, i.e. if this fundamental force is unable to explain the mechanism of growth. The study presented here and in earlier publications demonstrates that this force alone explains why the growth of population was, most of the time, hyperbolic (Nielsen, 2016a; 2016c; 2016d; von Foerster, Mora & Amiot, 1960).

During transitions, the fundamental force of procreation does not change. There is no need to assume that it does. Only the resistance to growth is changing and this change is described by $k(t)$.

In the past, every change in the resistance to growth was leading to a new, constant resistance and consequently to a new hyperbolic growth. The current transition, which commenced around 1950, also describes a change in the resistance to growth but the future trajectory is unknown.

The solution of the eqn (3) is given by the following expression:

$$S(t) = -\frac{1}{\int k(t)dt}. \tag{4}$$

In the simplest case, when $k(t) = k = const$, the eqn (3) is the same as eqn (2) and the solution (4) is the same as eqn (1). It is the reciprocal of a linear function.

If we assume that $k(t)$ is represented by the *n*-order polynomial, if

$$k(t) = \sum_{i=0}^{n} a_i t^i \tag{5}$$

then

$$S(t) = \left[\sum_{i=0}^{n+1} b_i t^i \right]^{-1}. \tag{6}$$

We should also notice that eqns (2) and (3) describe the growth rate $R(t)$. Thus, if we know the size of the population and k or $k(t)$, we can also calculate the corresponding growth rate at a given time:

$$R(t) = k(t)S(t). \tag{7}$$

For the hyperbolic growth [i.e. for the first-order hyperbolic growth given by the eqn (1)] $k(t) = k = const$ and the growth rate is directly-proportional to the size of population.

Stage 1: 2,000,000-27,000 BC

This stage is made of a hyperbolic growth between 2,000,000 BC and 46,000 BC followed by a transition to the next stage. The transition was between 46,000 BC and 27,000 BC (see Figures 2 and 4).

In Figures 3 and 4, this stage looks different than the other two stages and it resembles the distribution outlined by Deevey (see Figure 5). However, it is just an illusion created by using logarithmic scales of reference and Deevey appears to have been misguided by this illusion. He imagined that it was a fast growth followed by an equilibrium, or a plateau. However, the calculated curve shown in Figures 3 and 4 is hyperbolic. It was not a fast growth followed by equilibrium but a *monotonically increasing* growth.

We know that the growth in the first stage was hyperbolic because we have shown earlier (see Figure 1) that the reciprocal values of the size of the population during that time were following closely a straight line, which identifies hyperbolic growth (Nielsen, 2014). Why then does this stage look so much different? How to explain the peculiar shape presented in Figures 3 and 4?

First, it is important to notice that hyperbolic growth during this first stage was exceptionally slow, so slow that if continued it would not escape to infinity until around AD 549,391. *Second*, we have to remember that logarithmic scales, while being useful in displaying a wide range of data, they also introduce unavoidable distortions. In Figure 3 and 4 we have *double distortion* because we are using *two* logarithmic scales. The further we go back in time, the stronger is the compression of the displayed data, but there is also an increasing compression of the displayed size of the population as we move up along the vertical scale.

Every marked section of the first (left-most) cycle of the horizontal scale represents a compression of 1,000,000 years. The vertical scale introduces similar distortion but in reverse order. Here the first cycle represents an exceptionally stretched scale. This compressing and stretching, combined with the exceptionally slow growth during the first stage creates an illusion of a fast growth followed by an equilibrium, illusion so strong that it caused Deevey not only to see an incorrect pattern but also to try to explain its mechanism.

A simple way to dispel this illusion is to use linear scales as shown in Figure 6. In this figure, we present precisely the same data (for the Stage 1) and the same hyperbolic distribution as shown in Figures 3 and 4 but now we use linear scales for the time and for the size of the population.

We can now see clearly that the growth of population was *increasing monotonically*. There is obviously no sign of any plateau or equilibrium and no hope of having such a plateau in the future because the growth was hyperbolic, escaping to infinity at a fixed time. Deevey's claim of plateaus and his attempts to explain their mechanism was based on illusion.

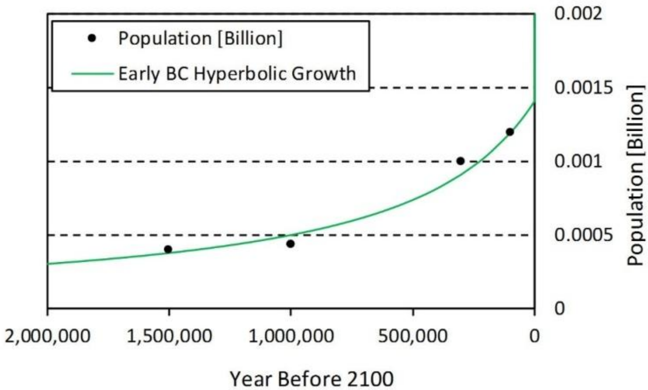

Figure 6. *The first stage of hyperbolic growth between 2,000,000 BC and 46,000 BC displayed using linear scales for the time and for the size of population. The illusion of a fast growth followed by an equilibrium created by the double-logarithmic scales used by Deevey (1960, p.198) and in Figures 3 and 4 has now disappeared. It is now clear that the growth is increasing monotonically and that it does not lead to an equilibrium.*

During this early stage of the BC growth, the size of population increased from the estimated 0.4 million in 1,500,000 BC to 1.3 million in 46,000 BC. The calculated value in 1,500,000 BC is 0.38 million. If we extend the fitted hyperbolic distribution to 2,000,000 BC, then the calculated size of population in that year is 0.3 million. If continued, the size of population would increase to one billion in AD 548,620.

Reciprocal values of data shown in Figure 1 demonstrates that this exceedingly slow hyperbolic growth was replaced by a much faster growth. The transition occurred between 46,000 BC and 27,000 BC (see Figure 2).

The size of the population during this transition increased from 1.3 million in 46,000 BC to 1.7 million in 27,000 BC. This transition converted the exceedingly slow hyperbolic growth during the first stage to a 17 times faster hyperbolic growth (as measured by the parameter *k*) during the second stage, the difference in the intensity of growth reflected in the distinctly different values of the gradient of the reciprocal values of the size of the population shown in Figure 1. During that time, the resistance to growth *decreased* by a huge factor of around 17 and starting from around 27,000 BC the growth of population was much faster than before 46,000 BC.

The timing of this transition agrees well with archaeological and anthropological data. Even though the emergence of modern humans is claimed to have been between 150,000 and 200,000 years ago (Mellars, *et al.,* 2007) the progress in their development was slow until around 50,000 BC, as demonstrated by archaeological evidence (Klein, 1989; 1995; Mellars, 1989; Stringer & Gamble, 1993). Human evolution appears to have experienced a great leap forward around that time.

For a long time since their emergence, modern humans were not much different than other hominins "and it was only around 50,000-40,000 years ago that a major behavioral difference developed" (Klein, 1995, p. 167). This first transition in the hyperbolic growth appears to coincide also with the extinction of Neanderthals, first in Europe and later in and later in other parts of the world, marking the beginning of the undisputed domination of *Homo sapiens* (Higham, *et al*., 2014).

Forces operating during the first transition between 46,000 BC and 27,000 BC from an earlier large resistance to growth before around 46,000 BC to significantly smaller resistance after around 27,000 BC were of a social and intellectual nature. The long race between different representatives of hominins was over. One by one they were left behind and became extinct. Finally, the last two remaining were *Homo floresiensis* and *Homo neanderthalensis* but they also were eliminated or virtually eliminated around the time of the beginning of the first transition, i.e. around 50,000 BC. Now, only modern humans, represented by *Homo sapiens*, remained. The first transition, between 46,000 BC and 27,000 BC was a transition to a new era of the exceptionally fast and long-lasting hyperbolic growth, the unique growth which was never to be repeated.

This complete freedom of growth was eventually restricted, not by forces of nature and not by the competition with other representatives of the genus *Homo* because they were extinct for a long time but by the strong competition between humans. However, in 27,000 BC, at the end of the first transition, this change was still long time into the future. The gained momentum of the free growth was to propel the growth of human population for many thousands of years.

Stage 2: 27,000 BC - AD 510

Stage 2 is made of a fast hyperbolic growth between 27,000 BC and 425 BC, followed by a transition to a slower hyperbolic trajectory during the AD era. The transition took place between 425 BC and AD 510. (In our earlier publications, we have labelled this transition as being roughly between 500 BC and AD 500.)

Hyperbolic growth between 27,000 BC and 425 BC was the fastest growth (as defined by the parameter k) in the past 2,000,000 years. During that time, the size of population increased from 1.7 in 27,000 BC million to 140 million in 425 BC, representing a nearly 82-fold increase. In contrast, there was only around 38-fold increase between AD 510 and present. If continued, this fast BC growth would escape to infinity at the end of 104 BC. We have come very close to experiencing the so-called population explosion at the end of the BC era.

In deciding which hyperbolic growth is fast we should not be confused by the growth during the AD era. It reached a higher size of population in a shorter time but we should remember that it also started with a significantly larger size of population, around 190 million, compared with only 1.7 million for the hyperbolic growth between 27,000 BC and 425 BC.

The transition between 425 BC and AD 510 can be described by the reciprocal of the third-order polynomial. During this transition, the resistance to growth *increased* by a factor of 6.4. As discussed earlier (Nielsen, 2016c), forces shaping this transition appear to have been of political nature. This transition coincides with the domination of Roman Empire over large areas surrounding the Mediterranean Sea. It also coincides with the accelerated process of the formation of countries in various parts of the world and with the rapidly changing political landscape (Teeple, 2002). From the complete freedom in around 27,000 BC, humans became slaves of their own design. They have invented many ways of self-destruction, bondage and oppression, which eventually led to a new hyperbolic growth characterised now by a larger resistance to growth. Humans appear to be their own best enemies and they might eventually cause their own extermination.

Stage 3: AD 510 - present

This stage is also made of a hyperbolic growth followed by a transition, which commenced around 1950. We have shown earlier (Nielsen, 2016a) that the growth of population between AD 510 and 1950 can be well described using a single hyperbolic distribution. However, we have also pointed out that there was a minor disturbance in this hyperbolic growth between around AD 1200 and 1400. This disturbance caused only a small shift in the hyperbolic growth (see Figure 4).

The best description of data between AD 510 and 1950 is given by two, approximately parallel hyperbolic trajectories separated by a small transition between around AD 1195 and 1470. The two hyperbolic trajectories, before AD 1195 and after AD 1470 are virtually identical. Measured by the parameter *k*, hyperbolic growth after AD 1470 was only 30% faster than the hyperbolic growth before AD 1195.

As discussed earlier (Nielsen, 2016a), this minor transition between AD 1195 and 1470 coincides with a unique event of a convergence of *five* demographic catastrophes. This is the only example showing a correlation between demographic catastrophes and the growth of population. However, the combined impact was small.

From around 1950, there was at first a small surge in the growth of population followed soon by a consistently slowing down growth. The data for the world population from that year are well documented by the US Bureau of Census (2017) but they can be also described using third-order polynomial with parameters listed in Table 2 (see Figure 7).

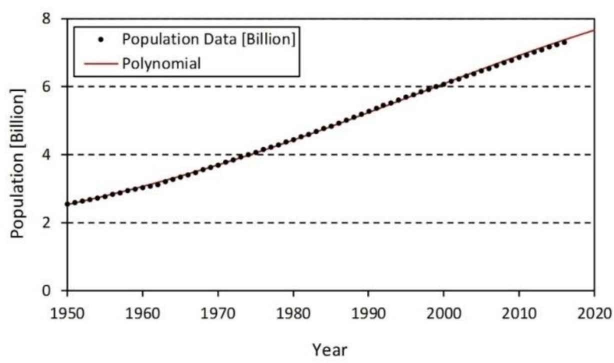

Figure 7. *Population data (US Census Bureau, 2017) are compared with the third-order polynomial distribution. Its parameters are listed in Table 2. This is just a mathematical description of data.*

This current transition appears to be associated with the increasing impact of Malthusian preventative checks (Malthus, 1798). The outcome of this transition is unknown. If the past pattern of growth is repeated, it could be a transition to a new hyperbolic trajectory. However, hyperbolic growth of population is possible only if the dominating force of growth is the biologically-controlled force of procreation. It is unlikely that this force alone will control the future growth of population. Under new conditions, with the increasing awareness of the need to control growth, the future growth of population could follow an entirely different trajectory. For the first time in human existence it will probably not be a hyperbolic growth.

The fitted distribution shown in Figure 3 with parameters listed in Table 2 can be now used to calculate the size of population at any time in the past 2,000,000 years. The calculated values are listed in Tables A1-A3 in the Appendix.

Economic growth in the past 2,000,000 years

De Long (1998) pointed out that income per capita (GDP/cap) can be used to estimate the past economic growth expressed in terms of the Gross Domestic Product (GDP). It is because the GDP/cap values quickly converge to an approximately constant value when we move back in time (see Figure 8). This property is nothing more than the mathematical property of dividing two hyperbolic distributions (Nielsen, 2017a) but it is useful for calculating the GDP values from the population data. What it simply means is that as we move back in time, the size of the GDP becomes approximately directly proportional to the size of the population. They follow virtually the same trajectories but displaced by an approximately constant factor.

Parameters describing the fitted GDP/cap distribution shown in Figure 8 are $a = 1.684\times10^{-2}$ and $k = 8.539\times10^{-6}$ for the GDP expressed in billions of the 1990 international Geary-Khamis dollars and $a = 7.739\times10^{0}$ and $k = 3.765\times10^{-3}$ for the Maddison's population data expressed in billions.

The fitted curve is a linearly modulated hyperbolic distribution (Nielsen, 2017a), which increases to infinity at a fixed time. For the distribution displayed in Figure 8, the point of singularity is in 1971. The growth of income per capita came close to this critical point but it bypassed it by a small margin of about 20 years. Income per capita continues now to increase along a new trajectory.

We can see that the calculated curve and the data representing the GDP/cap values quickly converge to a constant value when we move back in time. We can use this property to estimate the size of the GDP down to 2,000,000 BC. Results are presented in Tables A4-A6 in the Appendix.

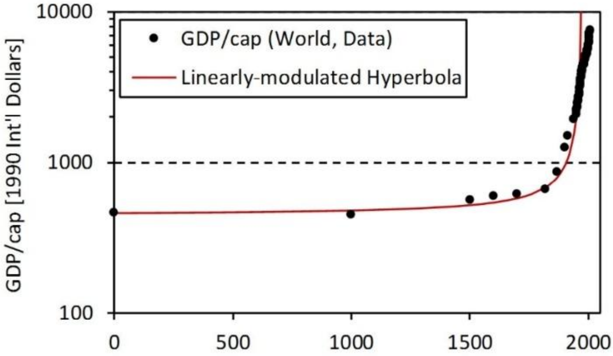

Figure 8. *Growth of the Gross Domestic Product per capita (GDP/cap) during the AD era. The full circles represent Maddison's data (Maddison, 2010) and the line is the best fit to the data (Nielsen, 2016e). The calculated curve is the linearly modulated hyperbolic distribution (Nielsen, 2017a).*

De Long (1998) carried out similar calculations. However, he used population data listed by Kremer (1993), which were taken from two sources: McEvedy & Jones (1978) and Deevey (1960). Our results are based on the analysis of all available data.

Furthermore, De Long assumed a constant GDP/cap value below AD 1500. This is good approximation below AD 1000 but not between AD 1000 and 1500. We shall use consistently the fitted trajectories but only below 1950 for two reasons: (1) good year-by-year data for the GDP starting from AD 1950 are already available (Maddison, 2010; GGDC, 2013) and (2) the calculated distribution of income per capita reproduces the data only up to 1950. From 1950, the GDP/cap values do not follow the linearly modulated hyperbolic distribution.

The GDP values presented in Tables A4-A6 are based on the best fits to the population data and to the GDP/cap data up to 1950. From 1950, the GDP values are as listed by Maddison (2010) for the years of up to 2008 and as calculated from the GDP/cap data listed by GGDC (2013) for 2009 and 2010 by using population data of the US Census Bureau (2017).

Summary and conclusions

We have carried out analysis of the growth of population in the past 2,000,000 years using data from a variety of sources (Biraben, 1980; Birdsell, 1972; Clark, 1968; Cook,1960; Deevey, 1960; Durand, 1974; Gallant, 1990; Hassan, 2002; Haub, 1995; Livi-

Bacci, 1997; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973; 1999; 2013; US Census Bureau, 2017). We have confirmed the earlier observation of Deevey (1960) that the growth of the world population was in three major stages. However, our analysis reveals that Deevey made a mistake by imagining that each stage was at first fast but then was reaching a certain equilibrium. Our analysis shows that each stage was *hyperbolic*. Each stage was *increasing monotonically* and was never levelling off to any form of equilibrium. On the contrary, if not terminated, hyperbolic distributions increase to infinity at a fixed time.

Nothing can increase to infinity. Consequently, any hyperbolic growth has to be, at a certain stage, terminated, which is not unusual because many other types of growth not only can but also are at a certain stage terminated. For instance, the better known exponential growth does not increase to infinity at a fixed time but if continued over a sufficiently long enough time, it leads to such large values that it becomes unsustainable.

The three stages of growth are: (1) 2,000,000 BC to 27,000 BC; (2) 27,000 BC to AD 510, and (3) AD 510 to present. Each of the listed stages includes a transition to a new growth. The transitions, as revealed by the analysis of data, are: (1) 46,000 BC to 27,000 BC, (2) 425 BC to AD 510, and (3) AD 1950 to present. During the third stage of growth, there was a minor transition between AD 1195 and 1470 but it only produced a slight shift in the hyperbolic trajectory.

Hyperbolic growth of population is generated by only one predominant force, the force of procreation, which is expressed as the difference between the ever-present, *biologically-controlled* force of sex drive and the *biologically-controlled* force of aging and dying (Nielsen, 2016c). This essential force has to be included in any explanation of the mechanism of growth of human population and it turns out that this force alone generates hyperbolic growth. As long as the growth remains hyperbolic, there is no need to include any other force. When a hyperbolic growth is being terminated or strongly disturbed, as between AD 1195 and 1470, other forces are strong enough to interfere with the usually dominant, biologically controlled, force of procreation.

Hyperbolic growth is characterised uniquely by parameter k [see eqn (1)]. This parameter is the ratio of the force of growth and of the resistance to growth. Working on the fundamental scientific principle of parsimony we can assume that during each hyperbolic growth the fundamental force of procreation per person remains unchanged and only resistance to growth is different. Transitions are associated with changing the resistance to growth. This change is described by the time-dependent parameter $k(t)$ [see eqn (3) and Table 2].

Each, of the first two stages of growth of human population in the past 2,000,000 years was terminated by a transition to a new hyperbolic growth. The third stage is now also being terminated. This transition commenced around 1950 but its outcome is unknown.

The first hyperbolic stage of growth was slow but during the first transition the resistance to growth decreased by a factor of around 17. The second stage was characterised by a fast hyperbolic growth, so fast that if continued it would have escaped to infinity around 104 BC. Fortunately, this fast hyperbolic growth was terminated. During the second transition, between 425 BC and AD 510, the resistance to growth increased by an approximate factor of 6.4. The new hyperbolic trajectory was significantly slower than the immediately preceding BC trajectory.

Each of the past two major transitions, as well as the current transition, appears to be associated with significant changes in the style of living. The first transition between 46,000 BC and 27,000 BC appears to have been associated with the surge in the evolution of *Homo Sapiens* (Klein, 1989; 1995; Mellars, 1989; Stringer & Gamble, 1993). Forces, which eventually reduced substantially the resistance to growth appear to have been of social and intellectual character. The second major transition between 425 BC and AD 510 appears to have been of political nature as reflected in the apparently intensified changes in the political landscape (Teeple, 2002). The current third major transition appears to be moulded predominantly, if not exclusively, by the Malthusian preventative checks (Malthus, 1798).

The minor transition between AD 1195 and 1470 appears to have been of an entirely different nature. It was not associated with the change in the style of living but rather with the one and only example of a strong impact of demographic catastrophes caused by an unusual convergence of five major catastrophic events (Nielsen, 2016a; 2017b). This transition caused a 30% *decrease* in the resistance to growth, reflecting the efficient action of the regeneration process triggered by the Malthusian positive checks (Malthus, 1798; Nielsen, 2016f).

Using the best fit to the data we have calculated the size of human population in the past 2,000,000 years. These values are listed in Tables A1-A3. Using results of our earlier analysis (Nielsen, 2016e) of the Gross Domestic Product per capita (GDP/cap) and the current analysis of population data, we have also listed the estimated values of the GDP in the past 2,000,000 years until 1950. The GDP values from 1950 to 2008 were taken directly from the publication of Maddison (2010). The last two values, for 2009 and 2010 were calculated using the GDP/cap values listed by GGDC (2013) and the population data of the US Census Bureau (2017). All these values, expressed in billions of 1990 international Geary-Khamis dollars are listed in Tables A4-A6.

Appendices

Table A1. *Growth of human population from 2,000,000 BC to 1BC*

Year [BC]	Population [Million]	Year [BC]	Population [Million]	Year [BC]	Population [Million]
2,000,000	0.30	7000	6.56	380	160.53
1,500,000	0.38	6000	7.67	370	165.35
1,000,000	0.50	5500	8.38	360	170.22
800,000	0.57	5000	9.24	350	175.15
600,000	0.67	4500	10.29	340	180.11
400,000	0.81	4000	11.61	330	185.09
200,000	1.03	3500	13.32	320	190.09
100,000	1.19	3000	15.62	310	195.07
80,000	1.23	2800	16.78	300	200.04
60,000	1.27	2600	18.12	290	204.96
50,000	1.29	2400	19.70	280	209.82
46,000	1.30	2200	21.58	270	214.61
42,000	1.31	2000	23.85	260	219.29
40,000	1.32	1900	25.18	250	223.85
38,000	1.35	1800	26.66	240	228.28
36,000	1.37	1700	28.33	230	232.54
34,000	1.41	1600	30.23	220	236.63
32,000	1.46	1500	32.39	210	240.51
30,000	1.53	1400	34.89	200	244.18
28,000	1.62	1300	37.80	190	247.62
27,000	1.68	1200	41.25	180	250.80
26,000	1.75	1100	45.39	170	253.73
25,000	1.82	1000	50.45	160	256.38
24,000	1.89	900	56.78	150	258.75
23,000	1.98	800	64.93	140	260.83
22,000	2.07	700	75.81	130	262.61
21,000	2.17	600	91.08	120	264.10
20,000	2.27	500	114.03	110	265.29
19,000	2.39	490	116.98	100	266.19
18,000	2.53	480	120.08	90	266.80
17,000	2.68	470	123.36	80	267.12
16,000	2.85	460	126.82	70	267.17
15,000	3.04	450	130.47	60	266.95
14,000	3.26	440	134.35	50	266.49
13,000	3.51	430	138.46	40	265.78
12,000	3.80	425	140.61	30	264.85
11,000	4.15	420	142.05	20	263.71
10,000	4.57	410	146.54	10	262.37
9000	5.09	400	151.12	1	261.01
8000	5.73	390	155.78		

Table A2. *Growth of human population from AD 1 to 1330*

Year [AD]	Population [Million]	Year [AD]	Population [Million]	Year [AD]	Population [Million]
1	260.69	450	188.30	900	260.63
10	259.18	460	188.67	910	262.99
20	257.36	470	189.19	920	265.40
30	255.41	480	189.87	930	267.85
40	253.35	490	190.72	940	270.34
50	251.19	500	191.75	950	272.89
60	248.95	510	192.99	960	275.48
70	246.64	520	194.28	970	278.12
80	244.28	530	195.59	980	280.82
90	241.88	540	196.92	990	283.56
100	239.45	550	198.27	1000	286.36
110	237.00	560	199.63	1010	289.22
120	234.54	570	201.02	1020	292.13
130	232.09	580	202.42	1030	295.10
140	229.66	590	203.84	1040	298.14
150	227.24	600	205.28	1050	301.23
160	224.85	610	206.75	1060	304.39
170	222.50	620	208.23	1070	307.62
180	220.19	630	209.74	1080	310.92
190	217.94	640	211.27	1090	314.29
200	215.73	650	212.82	1100	317.73
210	213.59	660	214.39	1110	321.25
220	211.51	670	215.99	1120	324.85
230	209.49	680	217.61	1130	328.53
240	207.55	690	219.25	1140	332.30
250	205.68	700	220.92	1150	336.15
260	203.90	710	222.62	1160	340.09
270	202.19	720	224.34	1170	344.12
280	200.56	730	226.09	1180	348.26
290	199.03	740	227.86	1190	352.49
300	197.58	750	229.67	1195	354.64
310	196.22	760	231.50	1200	355.85
320	194.96	770	233.36	1210	359.73
330	193.79	780	235.26	1220	363.23
340	192.72	790	237.18	1230	366.33
350	191.75	800	239.14	1240	369.01
360	190.89	810	241.13	1250	371.27
370	190.13	820	243.15	1260	373.11
380	189.48	830	245.20	1270	374.55
390	188.95	840	247.29	1280	375.59
400	188.52	850	249.42	1290	376.28
410	188.22	860	251.58	1300	376.64
420	188.05	870	253.79	1310	376.73
430	188.00	880	256.03	1320	376.58
440	188.08	890	258.31	1330	376.26

Table A3. *Growth of human population from AD 1340 to 2016*

Year [AD]	Population [Million]	Year [AD]	Population [Million]	Year [AD]	Population [Million]
1340	375.83	1770	834.64	1974	3,984.30
1350	375.35	1780	867.04	1975	4,057.11
1360	374.90	1790	902.06	1976	4,130.85
1370	374.54	1800	940.03	1977	4,205.51
1380	374.36	1810	981.33	1978	4,281.06
1390	374.45	1820	1,026.44	1979	4,357.47
1400	374.90	1830	1,075.88	1980	4,434.73
1410	375.80	1840	1,130.33	1981	4,512.79
1420	377.26	1850	1,190.59	1982	4,591.63
1430	379.41	1860	1,257.63	1983	4,671.22
1440	382.38	1870	1,332.68	1984	4,751.52
1450	386.34	1880	1,417.25	1985	4,832.48
1460	391.48	1890	1,513.28	1986	4,914.08
1470	393.49	1900	1,623.26	1987	4,996.26
1480	400.54	1910	1,750.49	1988	5,078.98
1490	407.86	1920	1,899.36	1989	5,162.20
1500	415.45	1930	2,075.91	1990	5,245.86
1510	423.32	1940	2,288.63	1991	5,329.92
1520	431.50	1945	2,412.23	1992	5,414.31
1530	440.00	1950	2,538.51	1993	5,498.99
1540	448.84	1951	2,587.24	1994	5,583.89
1550	458.05	1952	2,636.93	1995	5,668.96
1560	467.64	1953	2,687.57	1996	5,754.14
1570	477.64	1954	2,739.19	1997	5,839.36
1580	488.08	1955	2,791.79	1998	5,924.57
1590	498.98	1956	2,845.38	1999	6,009.70
1600	510.39	1957	2,899.97	2000	6,094.69
1610	522.32	1958	2,955.57	2001	6,179.48
1620	534.83	1959	3,012.18	2002	6,263.99
1630	547.95	1960	3,069.82	2003	6,348.16
1640	561.73	1961	3,128.47	2004	6,431.94
1650	576.23	1962	3,188.16	2005	6,515.25
1660	591.49	1963	3,248.87	2006	6,598.04
1670	607.58	1964	3,310.62	2007	6,680.24
1680	624.57	1965	3,373.41	2008	6,761.79
1690	642.54	1966	3,437.22	2009	6,842.64
1700	661.57	1967	3,502.07	2010	6,922.73
1710	681.77	1968	3,567.94	2011	7,002.00
1720	703.24	1969	3,634.83	2012	7,080.40
1730	726.10	1970	3,702.74	2013	7,157.89
1740	750.50	1971	3,771.65	2014	7,234.42
1750	776.60	1972	3,841.55	2015	7,309.94
1760	804.57	1973	3,912.44	2016	7,384.42

Table A4. *Economic growth from 2,000,000 BC to 1BC*

Year	GDP	Year	GDP	Year	GDP
2,000,000	0.13	7000	2.92	380	73.30
1,500,000	0.17	6000	3.42	370	75.51
1,000,000	0.22	5500	3.74	360	77.75
800,000	0.25	5000	4.12	350	80.01
600,000	0.30	4500	4.60	340	82.29
400,000	0.36	4000	5.19	330	84.58
200,000	0.45	3500	5.96	320	86.87
100,000	0.52	3000	7.00	310	89.17
80,000	0.54	2800	7.53	300	91.45
60,000	0.56	2600	8.14	290	93.71
50,000	0.57	2400	8.85	280	95.95
46,000	0.57	2200	9.70	270	98.16
42,000	0.58	2000	10.74	260	100.31
40,000	0.59	1900	11.34	250	102.42
38,000	0.59	1800	12.02	240	104.46
36,000	0.61	1700	12.78	230	106.43
34,000	0.62	1600	13.64	220	108.31
32,000	0.65	1500	14.63	210	110.11
30,000	0.68	1400	15.76	200	111.81
28,000	0.72	1300	17.09	190	113.40
27,000	0.74	1200	18.67	180	114.88
26,000	0.77	1100	20.56	170	116.24
25,000	0.80	1000	22.87	160	117.48
24,000	0.84	900	25.77	150	118.59
23,000	0.87	800	29.49	140	119.56
22,000	0.91	700	34.47	130	120.40
21,000	0.96	600	41.46	120	121.11
20,000	1.01	500	51.98	110	121.67
19,000	1.06	490	53.33	100	122.11
18,000	1.12	480	54.75	90	122.41
17,000	1.19	470	56.25	80	122.58
16,000	1.26	460	57.84	70	122.63
15,000	1.35	450	59.52	60	122.55
14,000	1.44	440	61.29	50	122.36
13,000	1.56	430	63.18	40	122.06
12,000	1.69	425	64.16	30	121.66
11,000	1.84	420	64.82	20	121.16
10,000	2.03	410	66.88	10	120.57
9000	2.26	400	68.98	1	119.97
8000	2.55	390	71.12		

Year: BC; GDP: Gross Domestic Product, billion 1990 international Geary-Khamis dollars.

Table A5. *Economic growth from AD 1 to 1330*

Year	GDP	Year	GDP	Year	GDP
1	119.83	450	87.59	900	123.89
10	119.15	460	87.79	910	125.10
20	118.34	470	88.07	920	126.33
30	117.47	480	88.42	930	127.59
40	116.55	490	88.84	940	128.87
50	115.58	500	89.36	950	130.18
60	114.57	510	89.96	960	131.52
70	113.53	520	90.60	970	132.88
80	112.47	530	91.25	980	134.27
90	111.39	540	91.90	990	135.69
100	110.30	550	92.57	1000	137.14
110	109.19	560	93.24	1010	138.62
120	108.09	570	93.92	1020	140.14
130	106.98	580	94.62	1030	141.68
140	105.89	590	95.32	1040	143.27
150	104.80	600	96.04	1050	144.88
160	103.72	610	96.76	1060	146.54
170	102.66	620	97.50	1070	148.23
180	101.62	630	98.25	1080	149.96
190	100.60	640	99.01	1090	151.73
200	99.61	650	99.78	1100	153.55
210	98.65	660	100.56	1110	155.41
220	97.71	670	101.36	1120	157.31
230	96.81	680	102.16	1130	159.26
240	95.94	690	102.99	1140	161.26
250	95.10	700	103.82	1150	163.31
260	94.30	710	104.67	1160	165.42
270	93.53	720	105.53	1170	167.58
280	92.81	730	106.41	1180	169.79
290	92.12	740	107.30	1190	172.06
300	91.48	750	108.20	1195	173.22
310	90.88	760	109.12	1200	173.92
320	90.32	770	110.06	1210	176.04
330	89.80	780	111.02	1220	177.99
340	89.33	790	111.99	1230	179.75
350	88.91	800	112.97	1240	181.32
360	88.54	810	113.98	1250	182.69
370	88.21	820	115.00	1260	183.87
380	87.94	830	116.04	1270	184.85
390	87.72	840	117.10	1280	185.65
400	87.55	850	118.18	1290	186.29
410	87.44	860	119.28	1300	186.77
420	87.38	870	120.40	1310	187.12
430	87.39	880	121.54	1320	187.37
440	87.46	890	122.71	1330	187.55

Year: AD; GDP: Gross Domestic Product, billion 1990 international Geary-Khamis dollars.

Table A6. *Economic growth from AD 1340 to 2010*

Year	GDP	Year	GDP	Year	GDP
1340	187.67	1750	471.71	1970	13,765.94
1350	187.79	1760	495.02	1971	14,336.49
1360	187.92	1770	520.73	1972	15,018.42
1370	188.12	1780	549.22	1973	16,015.15
1380	188.42	1790	580.95	1974	16,388.00
1390	188.87	1800	616.51	1975	16,637.92
1400	189.51	1810	656.64	1976	17,449.53
1410	190.39	1820	702.24	1977	18,157.09
1420	191.59	1830	754.52	1978	18,955.43
1430	193.14	1840	815.03	1979	19,633.16
1440	195.15	1850	885.85	1980	20,029.99
1450	197.68	1860	969.81	1981	20,422.61
1460	200.85	1870	1,070.89	1982	20,648.35
1470	202.45	1880	1,194.80	1983	21,235.64
1480	206.68	1890	1,350.09	1984	22,204.27
1490	211.09	1900	1,550.16	1985	22,969.60
1500	215.69	1910	1,817.13	1986	23,781.92
1510	220.50	1920	2,190.37	1987	24,693.77
1520	225.52	1930	2,747.20	1988	25,753.18
1530	230.78	1940	3,662.62	1989	26,576.36
1540	236.29	1945	4,381.03	1990	27,134.08
1550	242.06	1950	5,335.86	1991	27,494.23
1560	248.12	1951	5,649.96	1992	28,077.30
1570	254.50	1952	5,911.28	1993	28,693.57
1580	261.20	1953	6,208.99	1994	29,697.95
1590	268.27	1954	6,421.22	1995	30,942.24
1600	275.73	1955	6,830.52	1996	31,990.50
1610	283.61	1956	7,151.72	1997	33,241.79
1620	291.96	1957	7,423.90	1998	33,803.49
1630	300.80	1958	7,662.29	1999	34,997.33
1640	310.20	1959	8,013.45	2000	36,688.28
1650	320.19	1960	8,432.82	2001	37,739.37
1660	330.85	1961	8,725.32	2002	39,021.27
1670	342.23	1962	9,136.47	2003	40,809.56
1680	354.42	1963	9,533.55	2004	42,950.18
1690	367.51	1964	10,224.89	2005	44,982.59
1700	381.58	1965	10,760.25	2006	47,340.58
1710	396.77	1966	11,346.93	2007	49,411.11
1720	413.20	1967	11,769.15	2008	50,973.94
1730	431.04	1968	12,416.76	2009	50,762.92
1740	450.47	1969	13,101.91	2010	53,650.54

Year: AD; GDP: Gross Domestic Product, billion 1990 international Geary-Khamis dollars. From 1950, the data are as listed by Maddison (2010) up to 2008. The two values for 2009 and 2010 were calculated using the GDP/cap values listed by GGDC (2013) and the population data of the US Census Bureau (2017).

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2. Earlier interpretations of hyperbolic growth

Introduction

Historical economic growth and the growth of population were hyperbolic (Nielsen, 2014, 2016a, 2016b, 2016c). Hyperbolic growth is described by an exceptionally simple mathematical formula. It is just the reciprocal of a linear function. Many attempts were made to understand hyperbolic growth or to give an alternative mathematical description. These descriptions or interpretations tend to be complicated, maybe because hyperbolic distributions appear to be complicated. Furthermore, they do not explain the mechanism of growth. They also do not give better description of data than the description furnished by the simple mathematical equation. We shall present here a few examples of earlier attempts to explain or to describe hyperbolic distributions.

Technology and the growth of population

Using correlations between two processes might be tempting in order to explain the mechanism of growth but correlations could be spurious and misleading. Just because there is a correlation between two processes it does not mean that one process influences another. It does not mean that there is a cause-effect relation between two observed processes. One has to be on guard when using such correlations because they can lead easily to loops and to the incorrect interpretation of the mechanism of growth.

The correlation between technology and the growth of human population is deceptively misleading and it leads quickly to a dubious loop (Korotayev, Malkov, & Khaltourina, 2006a): technology increases the carrying capacity, the increased carrying capacity promotes population growth, population growth promotes the growth of technology, technology increases the carrying capacity, and so on. It is explaining one unknown mechanism by another unknown mechanism. It is going in circles and explaining nothing.

Technology might be helpful in supporting the existence of people but is it essential? When trying to explain the mechanism of growth it is necessary to consider first the most obvious and most essential force or forces. Other forces may be added if the essential force is insufficient to explain growth.

It is obvious that the essential force controlling the growth of population is the force of procreation. Technology does not produce children and it is even not essential to support growth, as demonstrated by the fast growth of population in poor countries.

Even if we briefly agree that technology supports or limits the growth of human population, such an "explanation" ignores the obvious and *indispensable* force of growth of human population, the force of procreation. It ignores the abundant evidence that even without advanced technology people can still produce children and support them.

This closed-loop explanation is supported by the assertion that "throughout most of human history the world population was limited by the technologically determined *ceiling* of the carrying capacity of land" (Korotayev, Malkov, & Khaltourina, 2006a, p. 18. Italics added.). It is a typical claim based of pure imagination, a statement that has to be accepted by faith. How can we possibly prove that over thousands of years and all over the world, the growth of human population was so *finely tuned* to the "the technologically determined ceiling of the carrying capacity of land"?

When this statement was published and when the associated closed-loop explanation was proposed it was already well known that the growth of human population was hyperbolic, at least during the AD era (Kapitza,1992, 1996, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962, 2002; von Foerster, Mora, & Amiot, 1960; von Hoerner, 1975). Evidence-based indication is that hyperbolic growth was in general unconstrained and surprisingly robust over a long time. This type of growth contradicts the concept of the limiting effects of the ceiling of the carrying capacity. This ceiling appears to have been always much higher than required for supporting growth, the conclusion being in agreement with the study of the ecological capacity and ecological footprints showing that only recently we have crossed the ecological limit of our planet (Ewing, *et al.* 2010).

To accept this closed-loop explanation we would have to accept, without a proof, that each component in this loop was not only finely tuned but also that they were all for some mysterious and unexplained reason increasing hyperbolically: the population was increasing hyperbolically, the technology was increasing hyperbolically, the carrying capacity was increasing hyperbolically and all of them were so finely tuned as to increase in unison, in such perfect harmony and so close to each other. The size of the population would have to be all the time close to the limiting ceiling of the carrying capacity, which would be so mysteriously increasing.

The proposed closed-loop explanation breaks also down already in the first step. What if the carrying capacity was already so large that the assumed contribution from technology was inconsequential? The size of the population in the past was small over a long time. It is hard to accept that our planet was incapable to support the increasing population.

With the exception of just two demographic transitions in the past 12,000 years (Nielsen, 2016a), the growth of human population was increasing without any major disturbance. With the small number of people and with the huge resources of our planet we can reasonably expect that the carrying capacity was much higher than the size of human population.

It would be unrealistic and unconvincing to assume that the growth of human population over such a long time was so precisely adjusted to the carrying capacity. It would be unrealistic to expect that this *fine tuning* was done so precisely by technological development. To make such a claim we would first have to prove that the growth of human population was always limited by the carrying capacity of our planet but we have no such proof and probably we shall never have. Any theory, which attempts to explain the mechanism of growth of human population by *fine tuning* of the size of population to the carrying capacity by technology, economic growth or by any other means is either unscientific (because it is based on untestable assumptions) or at least strongly questionable.

We would have to have some incredibly advanced technology to *measure* the carrying capacity and to *adjust* the growth of human population so precisely to its "ceiling." But even then, we could hardly expect such a regular hyperbolic growth. By using this advanced technology, we would also have to control precisely three interacting processes: technological development, the increase in the carrying capacity and the growth of human population. We would have to make sure that these three processes are perfectly synchronised and that they follow the closely coupled hyperbolic trajectories.

To justify the closed-loop process we would have to explain it without assuming that it was controlled by any advanced technology. Without such explanation, the mechanism of the proposed closed-loop remains unexplained and consequently, it does not explain the mechanism of the growth of human population.

We can also have other questions about this first step in the postulated closed loop. What is the carrying capacity of our planet? What was the carrying capacity of our planet over the past 12,000 years or longer? What was the contribution of technology to the carrying capacity? Even if we assume that technology increases the carrying capacity, is this assumed increase so essential to support the growth of human population? It is well known that people can survive on very little and that even then they can still procreate and support children. All they need is basic food, body cover and shelter.

How much damage is caused by technology? How is the technology *reducing* the carrying capacity? Can we ignore, for instance, that carbon footprint accounts for about 50% of our total ecological footprint? (Ewing, *et al.* 2010). Can we ignore the pollution of not only the atmosphere but also of the land and water? Can we ignore climate change, the ever-increasing weather-related economic losses, the decreasing carrying capacity of people living on islands, the increasing deforestation, the continuing human-induced extinction of species, the continuing loss of arable land, the overuse of pesticides, herbicides, artificial fertilisers and other agricultural chemicals? Can we ignore the ever-increasing urban population and their increasing dependence on food supply, which comes from the decreasing land resources? Can we ignore how the huge and the well-stocked arsenal of weapons is relentlessly used to destroy the carrying capacity? Can we ignore the never-decreasing stream of displaced population?

If we want to claim that technology increases the carrying capacity, we should also consider how this carrying capacity is decreased by technology. But the essential point is to show that technology was indeed playing the crucial role in shaping the growth of population, that this assumed force of growth has to be added to the essential and indispensable force of procreation, that without technology population would not have been increasing or that it would not have been increasing hyperbolically.

Another problem with linking technological development with the growth of human population is the misinterpretation of the fundamental mechanism of technological development. Technological growth is not prompted by the sheer number of people but by concepts, ideas and solutions. This is the driving force of technological development. People are just *carriers* of these concepts, ideas and solutions, or more precisely, carriers of the genetic ability to generate concepts, ideas and solutions.

Is the technological development dependent on the *number* of people? While it is true that with a larger number of people we can expect a greater number of ideas and solution, it is also true that the growth of human population is now slowing down. Does it mean that technological development is also slowing down because of the slowing down growth of human population? If the growth of human population is going to reach a maximum and stop growing, will the technology also reach a certain maximum and stop growing? Will people stop thinking and inventing?

The growth of technology is not determined by the number of people but by the number of creative ideas, inventions and solutions, which do not appear to be directly proportional to the number of people. Consequently, even if the size of population is going to be constant, people will not stop being intellectually active.

The correlation between technology and the growth of human population was investigated by Kremer (1963). He claims that there is a close correlation between the growth of population and technological development, which is hardly surprising. However, by observing a correlation between two processes we can only tell that there is a correlation. The correlation alone does not explain the mechanism of growth of any of the correlated processes. Correlations can be strongly misleading and they have to be handled with care.

Kremer claims that the growth rate of human population during the AD era was approximately proportional to the size of human population indicating that the growth was hyperbolic but he did not explain *why* it was hyperbolic. He suggests the correlation between the growth of human population and the growth of technology but this correlation does not explain the mechanism of growth of any of them. It does not explain why these two correlated processes are hyperbolic. It is like with the finely-tuned closed-loop mechanism proposed by Korotayev, Malkov & Khaltourina, (2006a): one process is explained by another without explaining any of them. The growth of human population is hyperbolic because the growth of technology is hyperbolic, and the growth of technology is hyperbolic because the growth of population is hyperbolic. It is also explaining one unknown mechanism by another unknown mechanism and going in circles.

The primary, if not the only, force driving the growth of population is the force of procreation, which in its simplest representation is the biologically controlled sex drive and biologically controlled mortality. Until recently, children were not produced by technology. Mortality was also not controlled by technology. Maybe technology could be claimed to give a better chance of survival but it is definitely not the primary force of growth. Likewise, the primary force driving technological development can be identified as concepts and ideas created by people, combined with the efficiency of sharing information.

The primary force of the growth of population is represented by the biological processes controlling birth and death. The primary force controlling the growth of technology is represented by concepts, ideas and generally by creative activities of human population. Biological process controlling birth and death apply not only to humans but also to other species. The force of creative thinking applies specifically only to humans. There might be some examples of creative thinking in other species, particularly in primates, but they are on such a low level that they do not initiate some new lines of technological development.

If the force responsible for the growth of technology were determined by the sheer number of people, i.e. by the number of members of this particular species, there would be no reason for excluding other species from this process. They should be also expected to develop technology but they do not. The growth of technology and the growth of population are controlled by different forces of growth. Their fundamental mechanisms of growth are distinctly different.

There is a close correlation between the growth of technology and the growth of human population only because creative concepts come from humans. Explaining the growth of population by technology and technology by the growth of population is going in circles and explaining nothing.

Convoluted construction

Hyperbolic growth is described by a simple formula:

$$S(t) = \frac{1}{C - kt}, \qquad (1)$$

where $S(t)$ is the size of the growing entity, such as population or the Gross Domestic Product (GDP), C is the constant of integration, k is a positive constant and t is time.

This expression is a solution of a very simple differential equation:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = kS(t)\,. \tag{2}$$

Normally, the next step would be to explain why the growth is hyperbolic. To this end, we would have to start with some simple and easily acceptable assumptions and *derive* the hyperbolic formula based on these assumptions. Maybe we could also start with acceptable assumptions and derive and alternative formula, which would give a better description of data. However, if we derived a more complicated formula, which would not give a better description of data we could then decide that we were on the wrong track and we would have to try another approach.

In contrast, in the demographic and economic research there appears to be a tendency to *construct* mathematical formulae and to try to make them as complicated as possible. Here is one such example (Johansen & Sornette, 2001).

Start with the logistic equation of growth

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = b[K - S(t)]. \tag{3}$$

This is already a questionable starting point because we know that population and the GDP do not grow logistically but hyperbolically. Even now, they do not yet level off (Nielsen, 2016d) to suggest a conversion to a logistic-type of growth.

Assume that the limit to growth K depends on time.

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = b[K(t) - S(t)]. \tag{4}$$

For no apparent reason, delete $S(t)$ from the right-hand side of the eqn (4).

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = b[K(t)]. \tag{5}$$

Again, for no apparently good reason, assume that

$$K(t) = [S(t)]^{\delta}, \tag{6}$$

where $\delta > 1$.

Under this assumption, eqn (6) is now changed to

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = b[S(t)]^{\delta}. \tag{7}$$

This equation can be presented as

$$\frac{dS(t)}{dt} = b[S(t)]^{\delta+1}. \tag{8}$$

We can solve it by substitution $S = Z^{-1}$. The solution is

$$\frac{1}{\delta}S^{-\delta} = c - bt \ , \tag{9}$$

where c is the constant of integration. So now we have

$$S(t) = (b\delta)^{z}(t_c - t)^{z}, \tag{10}$$

where $z = -1/\delta$ and $t_c = c/b$ is the time of singularity when $S(t)$ escapes to infinity.

Replace $(b\delta)^{z}$ by an arbitrary and adjustable parameter B and add another arbitrary and adjustable parameter A to construct

$$S(t) = A + B(t_c - t)^{z}. \tag{11}$$

Assume that the parameter z is a complex number

$$z = -(\beta + i\omega) \tag{12}$$

So now we have

$$S(t) = A + B(t_c - t)^{\beta + i\omega}. \tag{13}$$

Find the real component of $(t_c - t)^{\beta + i\omega}$.

This is an easy exercise that can be completed using two well-known formulae:

$$x^y = e^{y \ln x} \tag{14}$$

and

$$e^{i\varphi} = \cos\varphi + i \sin\varphi. \tag{15}$$

The answer is

$$\mathrm{Re}(t_c - t)^{\beta + i\omega} = (t_c - t)^{\beta} \cos[\omega \ln(t_c - t)]. \tag{16}$$

Assuming that both A and B are real, the formula for $S(t)$ can be now expressed as

$$\mathrm{Re}\, S(t) = A + B\, \mathrm{Re}(t_c - t)^{\beta + i\omega}, \tag{17}$$

which with the help of the eqn (16) gives

$$\mathrm{Re}\, S(t) = A + B(t_c - t)^{\beta} \cos[\omega \ln(t_c - t)]. \tag{18}$$

Use the eqn (13) again but now delete $i\omega$.

$$S(t) = A + B(t_c - t)^{\beta}. \tag{19}$$

Return to the eqn (16) and multiply the right-hand side of this equation by a constant *D.*

$$\mathrm{Re}(t_c - t)^{\beta + i\omega} = D(t_c - t)^{\beta} \cos[\omega \ln(t_c - t)]. \tag{20}$$

Add a phase shift in the eqn (20).

$$\mathrm{Re}(t_c - t)^{\beta + i\omega} = D(t_c - t)^{\beta} \cos[\omega \ln(t_c - t) + \phi]. \tag{21}$$

Return to the equation (19) and add to it the right-hand side of the eqn (21). We have now *constructed* the equation published by Johansen & Sornette (2001).

$$S(t) = A + B(t_c - t)^{\beta} + D(t_c - t)^{\beta} \cos[\omega \ln(t_c - t) + \phi] \quad (22)$$

This equation contains seven adjustable parameters but we do not know how they are supposed to be linked with the mechanism of growth. We know how we *constructed* (not derived) this complicated and impressive formula but we do not know why we have it and indeed why we should be interested in using it except perhaps to draw a line through data points, which we could do equally successfully using pen and paper and obtain equally meaningless result.

It is always good to look for mathematical description of data because it could help in understanding the nature of the observed phenomenon. However, if complicated description is not better than description given by a simple mathematical formula there is obviously no advantage in using the complicated description.

Hyperbolic growth described by the eqn (1) gives a satisfactory description of the growth of population and of the economic growth (Nielsen, 2014, 2016a, 2016b, 2016c). This is a simple formula, which could be expected to have a simple explanation. But now, we have a significantly more complicated formula. So, rather than making our task of explaining the mechanism of growth easier we have made it even more complicated.

In Figure 1, the distribution generated by the complicated eqn (22) is compared with the first-order hyperbolic distribution described by the eqn (1) and with data. As explained elsewhere (Nielsen, 2016a), fitting data around AD 1 by using hyperbolic distribution is pointless because around that time there was a transition from a fast to a slow hyperbolic trajectory. However, if we replace the complicated formula of Johansen and Sornette by a significantly simpler reciprocal of the second order polynomial

$$S(t) = (a_0 + a_1 t + a_2 t^2)^{-1} \quad (23)$$

we can generate a virtually identical distribution. There is no clear advantage in using the complicated formula of Johansen and Sornette. Simple description using the first-order hyperbolic distribution given by the eqn (1) gives acceptable representation of data but we can also replicate the complicated seven-parameter calculations but using just three parameters.

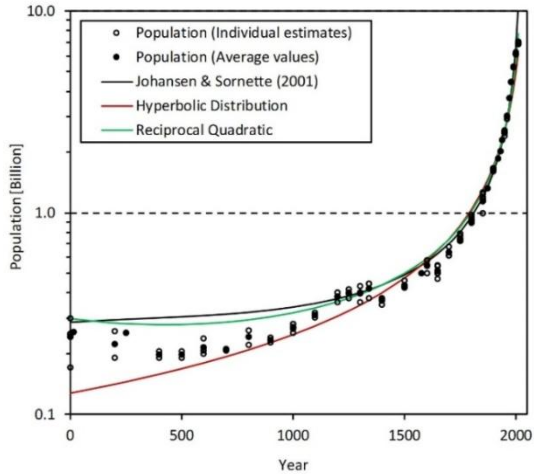

Figure 1. *Growth of the world population calculated using the Johansen & Sornette's (2001) constructed formula (22) is compared with the calculations based on significantly simpler formulae given by the eqns (1) and (24). Population data come from numerous sources compiled by Manning (2008) and by the US Census Bureau (2016). The parameters for the distribution of Johansen and Sornette given by the eqn (22) are: A≈0, B≈1624, D ≈−127, z≈−1:4, t_c≈2056; ω ≈6:3 and ϕ ≈5.1. Parameters for the hyperbolic distribution given by the eqn (1) are:* $C = 7.875\times10^{0}$ *and* $k = 3.834\times10^{-3}$. *Parameters for the reciprocal second-order polynomial distribution given by the eqn (24) are:* $a_0 = 3.367\times10^{0}$, $a_1 = 1.172\times10^{-3}$ *and* $a_2 = -1.382\times10^{-6}$.

The aim of constructing this complicated formula appears to be misplaced because even Johansen and Sornette used a significantly simpler formula in their analysis of a wide range of data presented in their Figs 9-32. The formula they used was

$$S(t) = a(t_c - t)^z . \qquad (24)$$

However, even in this simplified form it is already unnecessarily more complicated than the eqn (1) because $S(t)$ is no longer represented by the reciprocal of a linear function but by the time difference taken to the power of *z*. This expression is linear only if $z = 1$. For integer values of $z > 1$ it describes higher-order polynomials. For integers $z < -1$ it describes reciprocals of higher order polynomials. However, z can be also any other number greater or smaller than zero.

The homeostatic simulation model

In conformity with the generally accepted established knowledge in demography and in economic research (Nielsen, 2016d), Artzrouni & Komlos (1985) imagined that the growth of population can be divided into two distinctly different regimes: Malthusian stagnation and explosion. These regimes of growth are assumed by be controlled by two distinctly different mechanisms of growth. They assumed incorrectly that the growth before the Industrial Revolution was controlled by random forces such as wars, famines and diseases, the mechanism causing presumably stagnation in the growth of population. They also assume, incorrectly, that the growth after around the Industrial Revolution was exponential.

They should have known that their assumptions were unrealistic and incorrect because many years earlier it has been shown that the growth of population was hyperbolic (von Foerster, Mora, & Amiot, 1960; von Hoerner, 1975). Hyperbolic growth cannot be divided into two regimes of growth, slow and fast. For this type of growth, Malthusian regime does not exist and the apparent explosion is just the natural continuation of hyperbolic growth. There was no stagnation in the growth of human population and in the economic growth and there were no takeoffs leading to distinctly different explosive growth (Nielsen, 2014, 2015, 2016a, 2016b, 2016c, 2016e, 2016f, 2016g, 2016h).

Their work is important because, unknown to them, they have demonstrated that the established knowledge is contradicted by science. They did not realise that they made an important discovery because typically for the research carried out within the constraints of the established knowledge they did not compare results of their research with data.

To generate the growth of population before the Industrial Revolution, Artzrouni and Komlos carried out Monte Carlo simulations of the supposed Malthusian regime of stagnation. To describe the supposed population explosion, they simply assumed exponential growth after the Industrial Revolution. In their model, the growth of population is given by

$$\frac{\Delta S(t)}{\Delta t} = rS(t)\,. \tag{25}$$

For the constant r, this equation would describe exponential growth. However, in their calculations, the growth rate r is either constant (after the Industrial Revolution) or time-dependent (before the Industrial Revolution).

So, more explicitly, they consider two stages of growth. Before the Industrial Revolution the growth is given by:

$$\frac{\Delta S(t)}{\Delta t} = r(t)S(t), \tag{26}$$

whereas after the Industrial Revolution it is given by

$$\frac{\Delta S(t)}{\Delta t} = r_e S(t), \tag{27}$$

where r_e is a certain constant "escape rate" (Artzrouni & Komlos, 1985, p 27), escape from nowhere because there was no escape, or more accurately there was nothing to escape from, because the mythical Malthusian trap did not exist. The growth of population was monotonically hyperbolic, and the Industrial Revolution had no impact on changing the growth trajectory. However, according to the established but erroneous knowledge, there was an escape.

Fluctuations in the growth rate $r(t)$ before the Industrial Revolution are determined by $e(t)$ described as "a non negative random variable generated by a Monte Carlo type of simulation" (Artzrouni & Komlos, 1985, p. 27). For no apparent reason, this variable is defined by the following equation:

$$e(t) = 0.1v(t)U(t)\left\{1 + e^{0.15[y(t)-5]}\right\}, \tag{28}$$

where $v(t)$ is a random number drawn from a normal distribution with the mean 0 and variance 1, $y(t)$ is the number of decades the population was in the assumed Malthusian crisis and $U(t)$ is defined (again for no clear reason) as

$$U(t) = \frac{1}{1 + 4e^{-400[P_T \cdot P(t)]}}. \tag{29}$$

The population is divided into two sectors: the subsistence sector ("rural") and the capital producing sector ("urban"). In the eqn (29), $P(t)$ represents the per capita output (production) of the subsistence sector. If the per capita output is below a certain threshold defined by P_T, i.e. if $P(t) < P_T$, the population is assumed to be in the Malthusian crisis and cannot grow. If $P(t) \geq P_T$, the population is assumed to be out of crisis and can increase.

The per capita output in the subsistence sector is defined as

$$P(t) = C_2[K(t)]^{\beta_1} \frac{[S_R(t)]^{\beta_2}}{S(t)}, \tag{30}$$

where C_2, β_1 and β_2 are positive constants with $\beta_1 + \beta_2 = 1$, $K(t)$ is the capital stock and $S_R(t)$ is the population in the subsistence ("rural") sector.

The total output in the subsistence sector is given by

$$Q_R(t) = C_2[K(t)]^{\beta_1}[S_R(t)]^{\beta_2}. \tag{31}$$

Likewise, the total production in the capital producing sector ("urban") is given by

$$Q_U(t) = C_1[K(t)]^{\alpha_1}[S_U(t)]^{\alpha_2}, \tag{32}$$

where C_1, α_1 and α_2 are positive constants with $\alpha_1 + \alpha_2 = 1$.

The total population is then given by

$$S(t) = S_R(t) + S_U(t). \tag{33}$$

Returning to the eqn (28) we should notice that the function $U(t)$ defining the time-dependent parameter $e(t)$, which plays the essential role in the Monte Carlo simulations, depends on $P(t)$, which in turn depends on the capital stock $K(t)$. The growth of the capital stock is described as

$$\frac{\Delta K(t)}{\Delta t} = \lambda(t) Q_U(t) , \tag{34}$$

where $\lambda(t)$ is defined as

$$\lambda(t) = 0.01 + 1.778 \cdot 10^{-26} e^{0.05756t} . \tag{35}$$

The process of Monte Carlo simulations is well described by Artzrouni & Komlos (1985). These simulations produced most interesting results. Designed to demonstrate the existence of Malthusian regime of stagnation, *the model shows that the regime of Malthusian stagnation did not exist*. The assumed mechanism of stagnation does not produce stagnation but a steadily-increasing growth. Furthermore, the growth generated by the model during this imaginary regime of stagnation does not fit the data (see Figure 2).

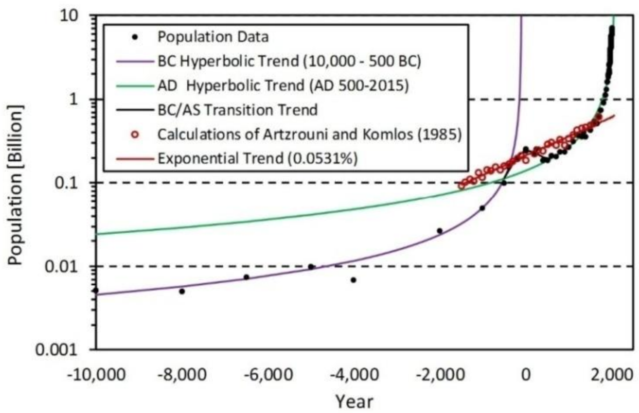

Figure 2. *The established knowledge in demography is contradicted by science. Simulations of the mechanism of Malthusian stagnation carried out by Artzrouni & Komlos (1985) do not produce stagnation but a steadily increasing exponential growth. They also do not fit date. These calculations are compared with hyperbolic distributions (Nielsen, 2016a). The data represent the average values of the size of population calculated using the compilations of Manning (2008) and of the US Census Bureau (2016).*

Parameters describing hyperbolic distributions shown in Figure 2 and defined in the eqn (1) are: $C = -2.282 \times 10^{0}$ and $k = 2.210 \times 10^{-2}$ for the BC era and $C = 7.061 \times 10^{0}$ and

$k = 3.398 \times 10^{-3}$. The data come from a variety of sources compiled by Manning (2008) and the US Census Bureau (2016).

The model of Artzrouni & Komlos's (1985), designed to reproduce the "well documented fluctuations experienced by the world's population throughout history" (Artzrouni & Komlos, 1985, p. 24), produced instead a steadily increasing growth along exponential trajectory. (In the semilogarithmic display, exponential growth is represented by an increasing straight line.) Furthermore, their model calculations do not fit the data. Their results show clearly that the model of Malthusian stagnation does not work. The mechanism of Malthusian stagnation does not describe the growth of population. The model based on the assumption of the mechanism of Malthusian stagnation did not generate the required fluctuations in the growth of population let alone fluctuations that "were, to a large extend, brought about by randomly determined demographic crises (wars, famines, epidemics, etc.)" (Artzrouni & Komlos, 1985, p. 24).

Thus, if Artzrouni and Komlos took the final step normally expected in scientific investigations, if they compared theory with data, even with the data used by von Foerster, Mora & Amiot (1960), they would have made an important discovery that the fundamental concepts of the established knowledge in demography are incorrect. They would then be able to suggest new lines of research.

It is essential to notice that even though Monte Carlo simulations based on the assumption of the mechanism of Malthusian stagnation produced exponential growth it would be incorrect to claim that the mechanism of Malthusian stagnation generates exponential growth. Equation (26) makes it clear that Artzrouni & Komlos (1985) *assumed* exponential growth. They *assumed* that Monte Carlo calculations were fluctuating around the growth rate describing exponential growth because eqn (26) describes modulated exponential growth. If we assume exponential growth it is hardly surprising that we get exponential growth. Fluctuations in the growth rate are not readily reflected as fluctuations of the growth of population or the GDP (Nielsen, 2016i, 2016k).

Lagerlöf's model of growth

Lagerlöf's model of growth (2003a, 2003b) belongs to the so-called OLG (overlapping generations) models (Aliprantis, Brown & Burkinshaw, 1990) used for instance by Becker, Murphy & Tamura (1990) and by Galor (2005a, 2011) to look at the growth of the population from the economic perspective. The central idea of this approach is to try to explain the growth of population by considering human capital defined as "embodied knowledge and skills" (Becker, Murphy & Tamura, 1990, p. S13). The growth is on the favourable rates of return.

> When human capital is abundant, rates of return on human capital investments are high relative to return on children, whereas when human capital is scarce, rates of return on human capital are low relative to those on children. As a result, societies with limited human capital choose large families and invest little in each member; those with abundant human capital do the opposite (Becker, Murphy & Tamura, 1990, p. S35).

It is a strong assumption, which is hard to accept. One would have to have a strong proof that this assumption is correct but we do not have such a proof.

It is interesting that neither Becker, Murphy & Tamura (1990), nor Galor (2005a, 2011), nor Lagerlöf (2003b) tried to compare their model predictions with population data. Lagerlöf (2003b) came close to testing his model against data when he generated growth rates in his Monte Carlo simulations but we shall show that his model is in disagreement with data he was referring to in his publication.

Lagerlöf's model is an excellent example of convoluted models characterised by the abundance of parameters but models, which neither describe data nor explain the mechanism of growth. This model was also designed to reproduce the epoch of stagnation and the supposed transition from stagnation to growth at the time of the Industrial Revolution, all as specified by the prescribed instructions of the established knowledge in demography and in the economic research. Like Artzrouni & Komlos (1985), Lagerlöf was also on the verge of making an important discovery that the established knowledge in demography and in economic research is contradicted by science. Like Artzrouni & Komlos (1985), he was on the verge of proving that the epoch of Malthusian stagnation did not exist and that there was no transition from stagnation to growth. Like Artzrouni and Komlos (1985), he was on the verge of showing that simulations of Malthusian stagnation do not produce stagnation, that they do not fit data and that they do not explain the mechanism of growth. He missed making this important discovery because he did not take the final step normally expected in scientific investigations – he did not compare theory with data. Parameters and definitions used in Lagerlöf's model are listed in Table 1.

Table 1. *Parameters and definitions used in Lagerlöf's theory (Lagerlöf, 2003a, 2003b)*

Parameter	Description
t	Time interval or "period *t*" assumed to be 25 years, i.e. one generation
H_t	Human capital or "a component resulting from parental investment" (Lagerlöf, 2003b, p. 426) called also "human capital stock" (Lagerlöf, 2003a, p. 760)
L	The "units of skills" (Lagerlöf, 2003b, p. 426) endowed by nature to "every agent" (person)
$L + H_t$	The "productivity of a unit of time" (Lagerlöf, 2003b, p. 426)
v	"a fixed time cost of rearing one child" i.e. "the time required to nurse the child just enough to keep her alive" (Lagerlöf, 2003a, p. 759)
ρv	Assuming $0 < \rho < 1$, this product "measures the direct inheritance of human capital from one generation to the next" reflecting the assumption that less than 100% of the time invested in rearing (nursing) a child is converted into human capital.
h_t	The time invested in the education of each child
l_t	The "time input in the consumption good sector" (Lagerlöf, 2003a, p. 759) i.e. time spent on production or work
ω_t	The "mortality shocks" (Lagerlöf, 2003b, p. 426) "which can be interpreted as epidemics" (Lagerlöf, 2003a, p. 760), the function assumed to be described by the probability density function of a log-normal distribution.
P_t	"the (adult) population size" called also "population density" in the generation t (Lagerlöf, 2003a, p. 760). The fundamental assumption of OLG models is that people live only for two generations. All adults in the generation t are replaced by the children born during the generation t. This new generation will be completely replaced by the next generation.
$A(P_t)$	The productivity parameter, which enters into the equation of the time-dependence of human capital
B_t	"the number of born children (or births)" (Lagerlöf, 2003a, p. 759). It is the average number of children per capita of adult population born in the generation t, i.e. over the entire 25 years.
T_t	The "survival rate" (Lagerlöf, 2003a, p. 760). It is the average *fraction* of the number of individuals born during the 25 years of the generation t, who survive to the next generation $t + 1$.
$(v + h_t)B_t$	The total time invested in children per capita of the adult population calculated over the entire time of one generation, i.e. over the total time of 25 years
Y_t	The output of the consumption (production) of goods
C_t	The "adult consumption" (production) (Lagerlöf, 2003b, p. 426)
α	A parameter ($\alpha > 0$) used in the utility function

Assuming that each person (agent) is endowed with a unit of time, the time budget for each agent is given by

$$1 = l_t + (v + h_t)B_t \tag{36}$$

At any given time, each person (agent) is assumed either to work or to spend time with children.

Assuming a single economy (or non-interacting economies) and that children consume (produce) nothing, the output of the consumption (production) of goods is given by

$$Y_t = l_t(L + H_t) = C_t \tag{37}$$

It is simply the productivity per unit of time multiplied by the time spent at work.

The survival rate is given by

$$T_t = \frac{H_t}{S_t} \frac{1}{\omega_t + H_t / S_t} \tag{38}$$

In the absence of mortality shocks ($\omega_t = 0$), the survival rate $T_t = 1$.

The production of human capital is given by

$$H_{t+1} = A(P_t)[L + H_t](\rho v + h_t) \tag{39}$$

Human capital increases in proportion to the productivity per unit of time multiplied by the time spent with each child, with the part of this time corrected for the unproductive fraction of time when nursing a child. By including the parameter $0 < \rho < 1$ it is assumed that education is more profitable for the increasing of human capital than nursing.

Each agent is assumed to maximise a utility function describing personal preferences and is given by

$$U_t = \ln(C_t) + \alpha \ln(B_t T_t) + \alpha\delta \ln(L + H_{t+1}) \tag{40}$$

The first term of the utility function measures the utility (the preference) of consumption (production), the second measures the utility of surviving children given by $B_t T_t$ and the third the utility of human capital of the offspring.

By maximising the utility function, we get the following expression for the optimal (preferred) number of births

$$B_t = \left(\frac{\alpha}{1+\alpha}\right)\frac{1}{v+h_t} \tag{41}$$

The number of born children depends entirely on the time invested in each child corrected by a factor dependent on the parameter used in the utility function. The larger the invested time, the smaller is the number of children, or vice versa.

The annual crude birth and death rates ($B_{r,t}$ and $D_{r,t}$, respectively) are calculated using the following expressions:

$$B_{r,t} = 1000\left(B_t^{1/25} - 1\right) \tag{42}$$

$$D_{r,t} = 1000\left(1 - T_t^{1/25}\right) \tag{43}$$

Calculations become significantly more complicated if interacting countries are included. Thus, for instance, assuming that that a demographic shock in one country is also reflected in other countries, the survival rate can be expressed as

$$T_{i,t} = \frac{H_{i,t} + \sum_{j\neq i}^{N-1} k_{ij}H_{j,t}}{\omega_{i,t}P_{i,t} + \sum_{j\neq i}^{N-1} k_{ij}\omega_{j,t}P_{j,t} + \left[H_{i,t} + \sum_{j\neq i}^{N-1} k_{ij}H_{j,t}\right]} \tag{44}$$

If we look back at eqns (38), (42) and (43) we can see that when $\omega_t = 0$, then $T_t = 1$ and the death rate $D_{r,t} = 0$, which also means that if mortality shocks are low, i.e. if $\omega_t \approx 0$, the death rate is also approximately zero. If we assume that the time spent with each child remains approximately the same over time, or equivalently that the number of born children remains approximately the same, a dramatic decrease in mortality shocks should generate a prominent population explosion.

This mechanism is the essence of the Demographic Transition Theory, which claims that towards the end of the assumed first stage of human history, interpreted as the epoch of Malthusian stagnation, the death rate started to fall while the birth rate remained approximately the same, the process creating presumably population explosion, the explosion which in fact never happened.

This is also the essence of the three regimes of growth postulated by Galor & Weil (1999, 2000) but contradicted by the analysis of data (Nielsen, 2016f).

The mechanism of Malthusian stagnation followed by explosion is carefully incorporated in the Lagerlöf's model of growth. In particular, regarding the mortality shock function ω_t , Lagerlöf explains:

> To understand the mechanisms driving the results in the calibration later, it is useful to first think of economies where ω_t is constant over time: Either high or low. To replicate the Three Regimes of Galor & Weil (1999, 2000), discussed in the introduction, we shall rig the model so that a high-ω economy converges toward a locally stable (Malthusian) steady state, whereas a low-ω economy converges to a balanced growth path (Lagerlöf, 2003a, 763).

The three regimes Lagerlöf is writing about are the assumed Malthusian regime of stagnation, which was supposed to last for thousands of years but which never existed; the post-Malthusian regime marked presumably by the rapid increase of population and economy; and the modern growth regime or sustained growth regime, which presumably follows a little later but which also represents an imaginary stage of growth. We have already demonstrated that these three regimes of growth did not exist (Nielsen, 2016f).

Lagerlöf's theory is based on the scientifically contradicted fantasy and if he carried his research properly, if he compared his theory with empirical evidence, he would have soon discovered that he was guided by fiction. His hard and convoluted work was unnecessary because it has been known for a long time that the growth of population was hyperbolic (Kapitza,1992, 1996, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962, 2002; von Foerster, Mora, & Amiot, 1960; von Hoerner, 1975). Hyperbolic growth can be described by an exceptionally simple mathematical formula, which is just the reciprocal of a linear distribution. This type of growth is in contradiction of the concepts of stagnation and explosion.

Using his model and Monte Carlo simulations, Lagerlöf generated growth rate for the growth of population in England, France and Sweden (Lagerlöf, 2003b). His model produced *minor* fluctuations in the growth rate, which were interpreted by Lagerlöf as the proof of the existence of the regime of Malthusian stagnation. That was a serious mistake because even large fluctuations in the growth rate are not readily reflected in the growth of population (Nielsen, 2016i, 2016j), and we do not even have to carry out laborious calculations to see that fluctuations in the growth rate are not reflected as similar fluctuations in the growth of population. Data for Sweden are well known (Statistics Sweden, 1999). They are often used in defence of the Demographic Transition Theory without even realising that the they are in its contradiction. There, in the same document, for everyone to see, we have *graphs* showing fluctuating birth and death rates, and fluctuating annual population increase but also we have a graph of population growth with no signs of fluctuations. The usual practice of showing fluctuations in birth and death rates or in the growth rate and claiming that we have a proof of the existence of Malthusian stagnation is unjustified. These fluctuations are not reflected in the growth of population and consequently they have no impact on the mechanism of growth. They are, in this respect, irrelevant.

Figure 3 shows an example Lagerlöf's results for France. His model-generated growth of population was calculated using the numerical integration of the following differential equation:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = R_L(t), \tag{45}$$

where $R_L(t)$ is the Lagerlöf's, model-generated and *fluctuating* growth rate, precisely as published in his paper (Lagerlöf, 2003b).

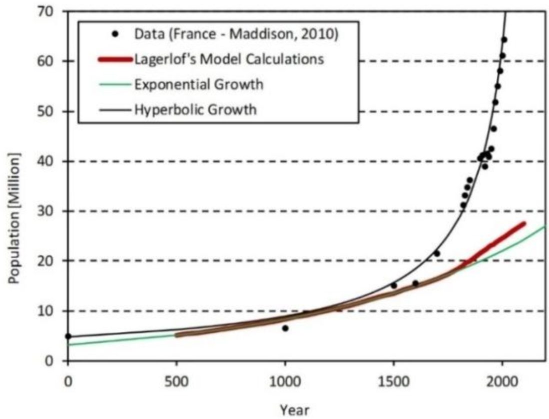

Figure 3. *The established knowledge in demography is contradicted by science. Simulations of Malthusian stagnation carried out by Lagerlöf (2003b) do not produce stagnation but a steadily increasing growth of population. Furthermore, his model calculations do not fit data (Maddison, 2010). Model-calculated distribution follows exponential trajectory because the growth rate was oscillating around a constant value. The claimed population explosion is just a small deviation from the exponential trajectory at its end. The growth of population in France was hyperbolic.*

In Figure 3, Lagerlöf's model-generated distribution is compared with the exponential distribution and with data. We also show the hyperbolic distribution fitting the data (Maddison, 2010). These data were not available to Lagerlöf but he had access to similar data (Maddison, 2001) published before the publication of his work.

Parameters describing hyperbolic distribution are: $C = 2.085 \times 10^{-1}$ and $k = 9.635 \times 10^{-5}$ [see eqn (1)]. The exponential distribution, which is so closely followed by Lagerlöf's model-generated results, is described by the following equation:

$$S(t) = C^{'} e^{rt} . \tag{46}$$

Its parameters are $C^{'} = 3.100 \times 10^{0}$ and $r = 9.780 \times 10^{-4}$.

The tiny, model-generated fluctuations in the growth rate presented in Lagerlöf's publication (Lagerlöf, 2003b) could not have possibly generated oscillations in the growth of population. Even large fluctuations are not readily reflected in distributions describing growth, such as growth of population or the GDP

(Nielsen, 2016i, 2016j). Lagerlöf could have seen it clearly if he looked at the data for Sweden (Statistics Sweden, 1999). He could have also known it if he studied the excellent data for England (Wrigley & Schofield, 1981). These results show clearly, even without carrying out any calculations, that even large fluctuations in birth and death rates and in the corresponding growth rate have no tangible effect on the growth of population and consequently that they have no effect on shaping the mechanism of growth. These data are clearly contradicting the established knowledge in demography but they are systematically ignored. The established knowledge in demography is also contradicted by results published over 50 years ago (von Foerster, Mora, & Amiot, 1960) but they are also systematically ignored.

The important contribution of Lagerlöf's Monte Carlo simulations is to show that the mechanism of Malthusian stagnation and population explosion does not work. Such a mechanism fails to produce the desired effects of stagnation and explosion and it fails to fit the data.

Results of Lagerlöf show that his model-generated distribution was exponential and that his claimed population explosion is just a minor deviation from the exponential trajectory. However, one might wonder why model-generated results follow exponential trajectory. Does it mean that the process of Malthusian stagnation generates exponential growth? No, it does not. Results depend on our assumptions about the way birth and death rates are fluctuating.

Lagerlöf *assumed* that crude birth rate was a non-zero constant and that crude death rate fluctuated around a non-zero *constant* value. Naturally, therefore, his growth rate also fluctuated around a non-zero constant, which in turn generated exponential growth. If Lagerlöf assumed that birth rate was zero and that the death rate fluctuated also around zero, he would have produced growth rate fluctuating around zero and thus he would have produced a constant size of population in his Monte Carlo calculations but he would still have not produced the required fluctuations in the size of population and his results would have been in a clear disagreement with data. The same applies to the calculations of Artzrouni & Komlos (1985). If they did not assume the modulated exponential growth during the postulated epoch of Malthusian stagnation [see the eqn (26)], they would have also produced a constant population without the so-called Malthusian oscillations.

If Lagerlöf took the final step and compared his model-generated distributions with data (Maddison, 2001), if he consulted the available to him literature (Statistics Sweden, 1999; Wrigley &

Schofield, 1981) he would have made an important discovery that the concept of Malthusian stagnation followed by explosion is incorrect, that it is contradicted by data and even by his own model. He could have then used his expertise to suggest new directions for the demographic and economic research.

The same applies to Galor. He uses Maddison's data but surprisingly he never attempts to analyse them. He prefers to distort them (Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002) to support the preconceived but erroneous ideas. He knows mathematics and he should be familiar with hyperbolic distributions. If he analysed data, the same data that he used in his publications, he would have soon discovered that the established knowledge in demography and in economic research is scientifically unsupported. He could have then also used his expertise to suggest new lines of research. These examples show how strongly the established knowledge is established and that even prominent researchers can be easily misled by the system of its doctrines.

Camouflaging the hyperbolic equation

Here is an example how the well-known differential equation describing hyperbolic growth was disguised as something new, which was supposed to explain the mechanism of growth based on the assumption that the growth of population is finely-tuned to the technological development. In its undisguised form, the differential equation (2) describes hyperbolic growth but does not explain its mechanism. It is just a mathematical equation, which when solved produces hyperbolic distribution. However, in its disguised form it seems to contain an explanation of the mechanism of growth. It seems to show that the growth of population is determined by the level of technology or knowledge.

This is a good example, which demonstrates that one should never be mesmerised by complicated mathematics. Mathematical formulations can be complicated and useful but just because they are complicated it does not mean that they are useful. Unified Growth Theory (Galor, 2005a, 2011), which is supposed to explain the mechanism of economic growth, is full of such complicated mathematical formulations. However, these complicated formulae do not explain anything. They just translate erroneous concepts into mathematical language. Data describing economic growth and the growth of population (Maddison, 2001) were used but they were never analysed to check the proposed theory. They were presented in a distorted way to make the impression that theory is confirmed by data. In the example presented here, the discussed mathematical equations are relatively simple and even attractive but they give a corrupted and mathematically unacceptable representation of the well-known differential equation [eqn (2)] describing hyperbolic growth.

Korotayev (2005) used the following differential equations to describe and explain the growth of population:

$$\frac{dS(t)}{dt} = a[bK(t) - S(t)]S(t), \tag{47}$$

$$\frac{dK(t)}{dt} = cS(t)K(t). \tag{48}$$

According to his interpretation "*K* is the level of technology/knowledge, *bK* corresponds to the number of people (*N*) [$S(t)$ in our notation], which the earth can support with the given level of technology (*K*)" (Korotayev, 2005, p. 81). Thus *bK* is interpreted as the carrying capacity of the planet.

To fit the population data, Korotayev carried out step-by-step calculations based on the eqns (47) and (48) but presented in a different form:

$$K_{i+1} = K_i + cS_iK_i, \tag{49}$$

$$S_{i+1} = S_i + a(bK_{i+1} - S_i)S_i. \tag{50}$$

There is absolutely no reason why $K(t)$ should represent technology or knowledge. We can call it whatever we want but just because we call it technology, knowledge or the carrying capacity it does not mean that it represents these imposed by us concepts. In the logistic model, which is similar to the eqn (47), it is a constant describing the limit to growth, which may or may not represent the carrying capacity. However, we shall show that in eqns (47) and (48), $K(t)$ has nothing to do even with the limit to growth. It is a variable that does not restrict growth in any way because $K(t)$ is in fact $S(t)$. It is simply the size of population or the size of any, hyperbolically-increasing quantity. Consequently, even if we use this set of differential equations and even if we fit data, we cannot claim that we have *explained* the growth of human population.

To show that $K(t)$ is in fact just $S(t)$, let us start with the differential equation for the hyperbolic growth [see eqn (2)]:

$$\frac{dS(t)}{dt} = kS^2(t). \tag{51}$$

It is the same equation as eqn (2) but it is now presented in s slightly different form. Let us now replace k by

$$k \equiv c \equiv a(b-1). \tag{52}$$

where *c*, *a* and *b* are constants. Mathematically, this modification is acceptable because k is a constant and we can always replace a constant by any combination of constants. Normally, we would not do it. We do it here to show that the eqns (47) and (48) represent a complicated representation of the eqn (51). However, we shall show that these equations represent also a corrupted form of the eqn (51).

Equation (51) can now be expressed as

$$\frac{dS(t)}{dt} = a[bS(t) - S(t)]S(t). \tag{53}$$

This equation is already almost the same as the eqn (47). But now let us corrupt this equation. Let us replace one $S(t)$ in the eqn (53) by $K(t)$, while keeping the other $S(t)$ unchanged. So now we have two equations:

$$\frac{dS(t)}{dt} = a[bK(t) - S(t)]S(t), \tag{54}$$

$$K(t) = S(t). \tag{55}$$

If $K(t) = S(t)$ then of course:

$$\frac{dK(t)}{dt} = \frac{dS(t)}{dt}. \tag{56}$$

However, according to the eqns (51), (52) and (55), and supported by the selective treatment of $S(t)$, we have

$$\frac{dS(t)}{dt} = kS^2(t) = cS^2(t) = cS(t)K(t). \tag{57}$$

So finally, we now have

$$\frac{dS(t)}{dt} = a[bK(t) - S(t)]S(t), \quad (58)$$

$$\frac{dK(t)}{dt} = cS(t)K(t). \quad (59)$$

These two equations are precisely the same as the eqns (47) and (48), and functionally the same as the eqn (51). However, now we have three constants, *a*, *b* and *c*, rather than just one constant *k*. We also have one $S(t)$ disguised as $K(t)$, while the other $S(t)$ retains its identity. *The variable* $K(t)$ *is just the size of the population.* It has nothing to do with technology, knowledge or carrying capacity. Korotayev's differential equations do not explain the mechanism of growth. They only describe the growth of human population using the well-known mathematical differential equation for the hyperbolic growth. They do not explain why the growth of population was hyperbolic.

We have repeated the calculations of Korotayev (2005) using his eqns (47) and (48) and his step-by-step procedure defined by eqns (49) and (50). Results are presented in Figure 4. They show that $K(t)$ is precisely the same as $S(t)$, $K(t) \equiv S(t)$. The two distributions are indistinguishable. $K(t)$ is not technology, knowledge or carrying capacity but the size of the hyperbolically increasing quantity, such as population or the GDP.

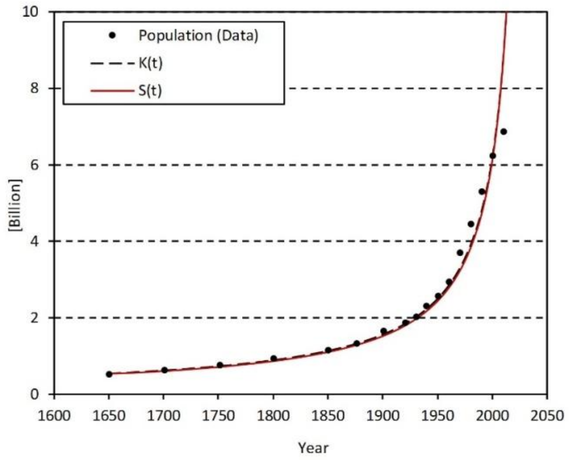

Figure 4. *Results of calculations carried out using eqns (47) and (48) and the step-by-step procedure defined by eqns (49) and (50). They confirm that $K(t) \equiv S(t)$. Eqns (47) and (48) represent a camouflaged eqn (51), which describes hyperbolic growth. The data represent the average values of the size of the world population calculated using the compilations of Manning (2008) and of the US Census Bureau (2016).*

Korotayev accepts now that he made a mistake: "I agree with what you wrote." (Korotayev, 2015). However, his model and his calculations have been published in a peer-reviewed journal and as far as we can tell they were never corrected.

This earlier attempt by Korotayev (2005) was followed by a new approach designed to link the growth of population with economic growth (Korotayev & Malkov, 2012; Korotayev, Malkov & Khaltourina, 2006a):

$$\frac{dS(t)}{dt} = aq(t)S(t), \tag{60}$$

$$\frac{dq(t)}{dt} = bq(t)S(t), \tag{61}$$

where $S(t)$ is again the size of human population, a and b are adjustable constant and $q(t)$ is claimed to be, again for no convincing reason, the surplus of the GDP per capita.

If we compare eqns (51) and (60), we can see that if the eqn (60) is supposed to describe hyperbolic growth of population or the GDP, then $q(t)$ cannot be anything else but $S(t)$, the size of the population or the GDP. The eqn (60) is the same as the eqn (51) except that, for no good reason, one $S(t)$ is now replaced by $q(t)$. However, this also means that $a = b$ and indeed the authors of these two equations have determined that $a = 1.04b$, which is as good as $a = b$. The two equations are *identical*. They are not two different equations but the same equation repeated twice, the same equations as eqn (51) but now one $S(t)$ is again disguised, this time as $q(t)$, which for absolutely no convincing reason is called the surplus of the GDP per capita.

We can replace S by any letter in the alphabet. We can call the replaced letter anything we want but in this context, it is nothing else but the size of population or the GDP or the size of any other hyperbolically increasing quantity. We are back to the original habit of corrupting the perfectly good and legitimate hyperbolic equation, but now we are not representing one of the $S(t)$ as $K(t)$, which for no good reason was called technology or knowledge. We are now representing one of the $S(t)$ as $q(t)$, which again for no convincing reason is called the surplus of the GDP per capita.

In the earlier mistake, the hyperbolic differential eqn (51) or (2) was disguised as two distinctly different equations. Now it is disguised as two similar equations, which are in fact identical. Previously, the growth of population was supposed to have been explained by technology, knowledge or the carrying capacity, which was incorrect and misleading, because the so-called technology or knowledge or the carrying capacity was nothing else but the size of the hyperbolically increasing quantity $S(t)$. Now, the growth of population is supposed to be explained by the surplus of the GDP per capita, which is again incorrect and misleading because the claimed surplus of the GDP per capita is just $S(t)$, which represents the size of the hyperbolically increasing quantity. They are making the same mistake as before. They have not introduced any new idea but present the same mistake in a different mathematical form.

The next step is to make it all even more mysteriously complicated. For obscure reasons, the growth of human population is now supposed to be described by a set of three differential equations (Khaltourina & Korotayev, 2007; Korotayev, Malkov & Khaltourina, 2006a, 2006b):

$$\frac{dS}{dt} = aqS\left(1-L\right), \qquad (63)$$

$$\frac{dq}{dt} = bqS, \qquad (64)$$

$$\frac{dL}{dt} = cqL\left(1-L\right), \qquad (65)$$

where *a*, *b* and *c* are adjustable constants and L is claimed, without any convincing justification, to represent the fraction of *literate* population, which implies that $1-L$ is the fraction of the illiterate population (Korotayev, 2015). In these equations, the time dependence is not explicitly displayed. So, $S \equiv S(t)$, $q \equiv q(t)$ and $L \equiv L(t)$

Again, if the eqn (63) is supposed to describe hyperbolically increasing distribution, such as population or the GDP, then $q(1-L) \equiv S$. We can replace one S in the hyperbolic differential equation (51) by whatever we want but functionally it will be still *S*.

A modified version of the three equations (63)-(65) are equations containing even more, spurious and meaningless parameters (Korotayev, Malkov & Khaltourina, 2006b):

$$\frac{dS}{dt} = aq^{\varphi_1} S^{\varphi_2} \left(1-L\right)^{\varphi_3}, \qquad (66)$$

$$\frac{dq}{dt} = bq^{\varphi_4} S^{\varphi_5}, \qquad (67)$$

$$\frac{dL}{dt} = cq^{\varphi_6} L^{\varphi_7} \left(1-L\right)^{\varphi_8}, \qquad (68)$$

where φ_i with $i = 1-8$ are arbitrary adjustable positive constants "not necessarily equal to one" (Korotayev, Malkov & Khaltourina, 2006b, p. 73). The interpretation of these additional parameters is also obscure.

Korotayev and his associates claim that they can generate hyperbolic growth with a transition to a new type of growth. However, they did not introduce any new concepts, which could justify this claim. They have just replaced two equations by three and one spurious variable by two. They follow the same idea as expressed in the eqns (47) and (48). In the original equations, a spurious variable $K(t)$, was introduced which for no good reason was called technology or knowledge or the limit to growth and which turned out to be just the size of population or some other hyperbolically increasing quantity. Now, the original two equations are replaced by three because two spurious variables are introduced, $q(t)$ and $L(t)$, which for no convincing reason are called the surplus of the GDP per capita and the fraction of literate population, respectively. The method of calculations is also the same, i.e. as outlined in the eqns (49) and (50).

Whatever is done is hidden in the obscure calculation procedure. As before, one would have to repeat their calculations to understand better the source of error or maybe to become convinced that whatever they are doing is correct. However, from a start, there is no convincing justification for claiming that q represents the surplus of the GDP per capita and that L represents the fraction of literate population, described also as "potential teachers" (Korotayev, Malkov & Khaltourina, 2006a, p. 26, 2006b, p. 73). There is also no convincing justification for claiming that the growth of population should be so vitally dependent on the surplus of the GDP per capita and on the number of potential teachers.

We could probably invent many other complicated formulae to replace the simple and working eqn (2) or (51). We could also label the new introduced variables or constants in whatever way we want but could we claim that we have contributed to a better understanding of the mechanism of hyperbolic growth?

Microscopic growth theory

The concept of Karev (2005a, 2005b, 2010) and Karev & Kareva (2014) is to see human population (or other biological systems) as being made of individuals, each characterised by a certain, unique parameter *a*. In a more general formulation of this theory, this uniquely defining parameter is a multi-dimensional vector $\vec{a} = (a_1, a_2 ... a_n)$ made of many characteristic components. In the extreme case, we could think that the components of the vector $\vec{a}$ are made of genes or even of the components of the whole genome. In such a case, the multidimensional vector would be made of 10^6-10^9 components (Karev, 2005b).

This theory is based on the advanced and aesthetically appealing mathematics. We shall explain the fundamental concepts of this theory. Once the fundamental ideas are understood, it will be easier for anyone to read the more advanced description presented by Karev (2005a, 2005b, 2010) and Karev & Kareva (2014). In our discussion, we shall replace vector $\vec{a}$ by constant *a*.

Rather than dealing with individuals characterised by the parameter *a*, it is assumed that the entire population is made of a certain number of groups of *a*-clones, each group characterised by the same parameter a and each group made of $n(t,a)$ number of members at a given time *t*. In order to calculate the growth of population we first calculate the growth of each group of *a*-clones. The differential equation describing the growth of *a*-clones is given by

$$\frac{1}{n(t,a)}\frac{dn(t,a)}{dt} = F(t,a)\,. \tag{69}$$

The function $F(t,a)$ is called "the per capita reproductive rate" (Karev & Kareva, 2014, p. 73) but the well-known and accepted definition of the net reproductive rate is the number of daughters born per woman in her lifetime. In the same publication, $F(t,a)$ appears also as $ag(N)$, where in our notation N represents $S(t)$, and $g(N)$ is interpreted as "some function, chosen depending on the specifics of the model" (Karev & Kareva, 2014., p. 69). Karev agrees (Karev, 2015) that it would be better to call $F(t,a)$ simply as a *growth factor*, which will depend on the model used in the calculations. However, if we use the concept of the general law of growth (Nielsen, 2016k), then this factor can be identified simply as the force of growth, which in the microscopic theory can have a variety of representations.

The factor $F(t,a)$ contains all the information about the mechanism of growth of each group of *a*-clones. The microscopic theory does not describe any single mechanism but gives a complete freedom to explore a variety of options. Each specifically chosen mathematical representation of the factor $F(t,a)$ will describe a certain mechanism of growth of each group of *a*-clones, but the mechanism will remain unknown until the chosen mathematical description of $F(t,a)$ is not only convincingly explained but also justified.

The additional complication in this theory is that the calculated size $S(t)$ of the population made of numerous groups of clones will depend on how their growth is *combined*. To understand the mechanism of growth of population it is necessary to explain not only the factor $F(t,a)$ but also to justify a specific mathematical way of combining the growth of all clones.

The growth rate of population is given by:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = E(t)F(t,a)\,, \tag{70}$$

where $E(t)$ is a function describing the mathematical way of combining the growth of population in all groups of clones. So now, the description of the mechanism of growth depends not only on $F(t,a)$ but also on $E(t)$. In order to explain this mechanism, it is not enough to explain and justify the factor $F(t,a)$ but also $E(t)$. The force of growth is given by the product of $F(t,a)$ and $E(t)$.

The calculation of $E(t)$ is based on the assumption that the populations of various groups of clones are distributed along a certain probability density function $p(t,a)$ defined as:

$$p(t,a) = \frac{n(t,a)}{S(t)}\,. \tag{71}$$

The function $p(t,a)$ describes the probability of having $n(t,a)$ number of individuals characterised by the unique parameter *a*, i.e. the probability distribution of the parameter *a*.

The definition of $E(t)$, based on the publication of Hofbauer and Sigmund (1998), is:

$$E(t) = \int_0^\infty a p(t,a)da\,. \tag{72}$$

To illustrate the application of this theory to the description of the growth of human population we shall use three models of growth presented by Karev (2005a) leading to three solutions.

Solution 1

This solution is based on the assumption that $F(t,a) = a$. Consequently,

$$\frac{1}{n(t,a)}\frac{dn(t,a)}{dt} = a\,. \tag{73}$$

In this model, it is assumed that each group of *a*-clones increases exponentially. The growth is prompted by a constant force generating a constant growth rate. This is the force of unknown nature. We do not know why this force should be constant. We just assume that it is. Thus, from the very beginning we cannot explain the mechanism of growth. Whatever we shall calculate will not help us to understand the growth of population. Maybe we shall be able to fit the data but we already know that the data can be fitted well (Nielsen, 2016a, 2016c) using the simple expression describing hyperbolic distribution [see eqn (1)]. The approach proposed by the microscopic theory will offer an alternative description but it is more complicated and there is no clear reason for preferring this approach.

The growth of human population as a whole is given now by:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = E(t)a\,. \tag{74}$$

Karev (2005a) gives the following expression for $E(t)a$, determined by his choice to describe mathematically the probability density function $p(t,a)$:

$$E(t)a = \eta + \frac{k}{s-t}\,, \tag{75}$$

where η, k and s are adjustable constants (s, k>0, $-\infty < \eta < \infty$). For $t = s$, this function escapes to infinity.

The differential equation for the growth of human population is now given by:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = \eta + \frac{k}{s-t}\,. \tag{76}$$

The right-hand side of this equation is again the force of growth of unknown origin and it is even less acceptable than the constant force because it is more complicated. If we had reservation about using a constant force of unknown origin to describe the growth of a group of clones [eqn (73)] our reservation is now even stronger because the force describing the growth of population is significantly more complicated and also unexplained. We can see that explaining the mechanism of growth is becoming progressively more difficult. We might only hope that perhaps our formula will give a better description of data but we shall soon see that it does not.

The solution of the eqn (76) presented by Karev (2005a) is

$$S(t) = S_0 \frac{e^{\eta t}}{(1 - t/s)^k}, \text{ for } t < s, \tag{77}$$

which is the exponentially modulated hyperbolic-like growth because it increases to infinity when $t = s$. It is not clear why we should want to use this distribution when we already have a simpler distribution given by the eqn (1) fitting the population data.

If $\eta = 0$, then

$$S(t) = S_0 \frac{1}{(1 - t/s)^k} \text{ for } t < s. \tag{78}$$

The size of population approaches singularity when time *t* approaches the parameter *s*. For $k = 1$ it is the first order hyperbolic growth given by the eqn (1). We can explain this formula but we cannot explain the mechanism of growth. We cannot explain why the growth should be expected to behave in this particular way.

Solution 2

Solution 2 is also based on the assumption of an exponential growth of each group of *a*-clones but now a different mathematical description is used for the probability density function $p(t,a)$, which gives different expression for $E(t)a$ used in the eqn (74):

$$E(t)a = \frac{1}{s - t} + \frac{c}{1 - e^{c(s-t)}}. \tag{79}$$

$E(t)a$ escapes to infinity when $t = s$.

The differential equation for the growth of human population is now given by:

$$\frac{1}{S(t)}\frac{dS(t)}{dt}=\frac{1}{s-t}+\frac{c}{1-e^{c(s-t)}}, \qquad (80)$$

and its solution by:

$$S(t)=S_0\frac{1-e^{c(t-s)}}{(1-t/s)(1-e^{-sc})}. \qquad (81)$$

Parameters used by Karev (2005a) are $c \approx 0.114$ and $s = 2026$. The corresponding product cs is large and the second term in the denominator can be neglected. The formula (81) can now be presented as

$$S(t) = S_0\frac{1 - e^{c(t-s)}}{(1 - t/s)}. \qquad (82)$$

This solution resembles the first-order hyperbolic growth because the denominator is a linear function of t, and if not for the function appearing in the numerator, the growth of the population would escape to infinity at $t = s$. However, when $t = s$, the numerator is also zero. Close examination of the eqn (82) shows that when t approaches s, this fraction approaches a constant value, which depends on parameters s and c. Furthermore, calculations show that for $t < s$, $S(t)$ increases approximately hyperbolically but for $t > s$, it increases approximately exponentially. Thus, the Solution 2 can be seen as being made of two parts: a hyperbolic growth to $t \approx s$ and an exponential growth from $t \approx s$ with an instantaneous discontinuity at precisely $t = s$.

Mathematically, this formula is interesting because it shows that by assuming a certain force of growth it might be possible to generate a trajectory, which would, at a certain stage, change from hyperbolic to a different type of growth. If we could explain the nature of this peculiar force and if we could reproduce data, we would make a huge progress in the understanding of the mechanism of growth. However, in this particular case we have no clue about the nature of this peculiar force and, as we shall soon see, the formula given by the eqn (82) does not fit the data.

Solution 3

Solution 3 is based on the assumption that $F(t,a)$, which in Solutions 1 and 2 was constant, is now represented by the modified logistic growth rate (Gilpin & Ayala, 1973).

$$F(t,a) = a\left[1-\left(\frac{S(t)}{K}\right)^k\right]. \quad (83)$$

Again, we do not know the nature of this force.

Under this assumption, the growth of each group of *a*-clones is given by

$$\frac{1}{n(t,a)}\frac{dn(t,a)}{dt} = a\left[1-\left(\frac{S(t)}{K}\right)^k\right], \quad (84)$$

where $k = const > 0$ and K is the limit to growth.

Unless $k = 1$, the driving force of growth for each group of clones decreases non-linearly with the size $S(t)$ of the whole population. The growth of each group of clones is no longer defined by the parameter *a* alone, which represents exclusive characteristics of any particular group of clones, but it also depends on the size of the *whole* population. The growth of each group of clones is somehow coupled to the growth of other clones.

The differential equation for the whole population is now given by:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = E(t)a\left[1-\left(\frac{S(t)}{K}\right)^k\right]. \quad (85)$$

Karev (2005a) uses the following expression for $E(t)a$:

$$E(t)a = \frac{1}{s-p(t)} + \frac{1}{1-e^{c[s-p(t)]}}, \quad (86)$$

where $p(t)$ is a solution of Cauchy problem:

$$\frac{dp(t)}{dt} = 1 - \left\{ S_0 \frac{s}{s - p(t)} \frac{1 - \exp\{c[p(t) - s]\}}{1 - Ke^{-sc}} \right\}^k . \qquad (87)$$

So, now, the differential equation describing the growth of human population is given by:

$$\frac{1}{S(t)} \frac{dS(t)}{dt} = \left[\frac{1}{s - p(t)} + \frac{1}{1 - e^{c[s - p(t)]}} \right] \left[1 - \left(\frac{S(t)}{K} \right)^k \right], \qquad (88)$$

and the size of population by

$$S(t) = S_0 \frac{s}{s - p(t)} \frac{1 - \exp\{c[p(t) - s]\}}{1 - e^{-sc}} . \qquad (89)$$

The description of the growth of human population is now significantly more complicated. Solutions given by eqns (78) and (82) were relatively simple because they were based on the assumption of the simplest type of growth of the individual groups of clones, growth of each clone prompted by a constant force. Even though we were not able to explain the mechanism of growth of the entire population made of various groups of clones we could at least explain the mathematical formulae describing growth. However, in the case of the growth described by the eqn (89) we cannot even understand this formula let alone to understand the mechanism of growth of the entire population. We do not understand why the growth of human population should follow this particular trajectory. Even if we could fit the theory to data precisely and over the entire range of time, we would be still unable to explain the mechanism of growth.

Comparing theory with data

Solutions 1-3 are shown in Figures 5 - 7. They are compared with data coming from a wide range of sources compiled by Manning (2008) and by the US Census Bureau (2016).

In Figure 5 we show the reciprocal values of data and the reciprocal values of Solutions 1-3. The advantage of using this display is that the decreasing linear trends identify uniquely hyperbolic distributions (Nielsen, 2014).

The common feature of all these solutions is that over the nearly entire range of time during the AD era they all follow hyperbolic trajectories. However, they reproduce data over a strongly limited range of time. Consequently, there is no advantage in using these solutions. The microscopic theory does not give a better description of data than the simple hyperbolic formula, which can reproduce data over the past 12,000 years (Nielsen, 2016a). Solutions 2 and 3 are indistinguishable in this display. Solution 1 is only slightly different. Differences between these three solutions can be observed only towards the end of the time scale, as shown in Figures 6 and 7.

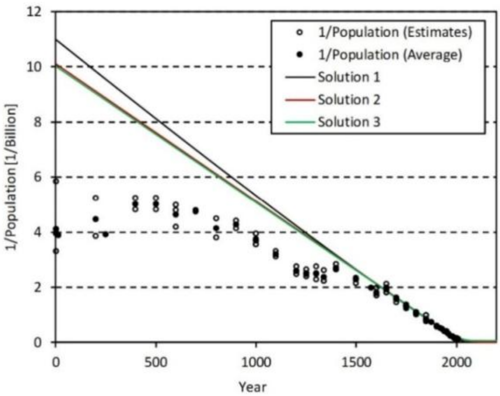

Figure 5. The decreasing straight lines of reciprocal values identify uniquely hyperbolic growth. *Reciprocal values of solutions 1, 2 and 3 [eqns (78), (82) and (89)] are compared with the reciprocal values of the world population data as compiled by Manning (2008) and by the US Census Bureau (2016).*

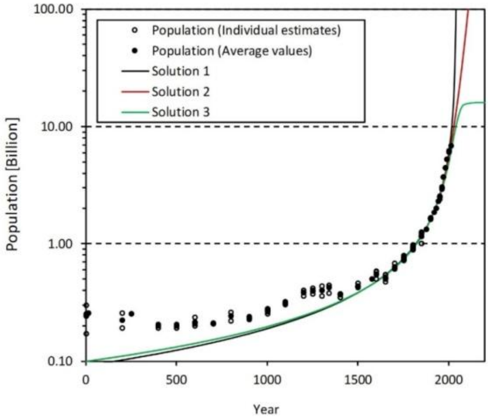

Figure 6. *Solutions 1, 2 and 3 [eqns (78), (82) and (89)] are compared with the world population data compiled by Manning (2008) and the by US Census Bureau (2016).*

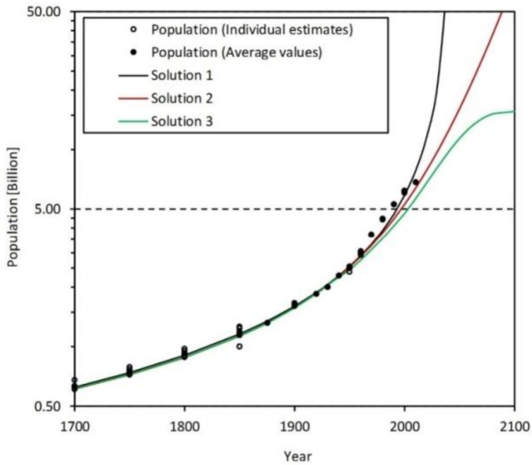

Figure 7. *Trajectories generated by Karev's Solutions 1, 2 and 3 (Karev, 2005a) in the region where they start to divert to different trajectories are shown together with population data using the compilations of Manning (2008) and of the US Census Bureau (2016). Solution 1 escapes to infinity. Solution 2 converts to an exponential growth, while Solution 3 converts into a logistic growth.*

While the concept of the microscopic approach to the study of the growth of population is interesting, it is not only extremely complicated but also it creates serious problems for the explanation of the mechanism of growth. Examples used by Karev (2005a) show that the complicated mathematical solutions generated by this theory imitate hyperbolic distributions, which can be represented by a much simpler equation [see eqn (1)]. Furthermore, these solutions reproduce only a very small range of data.

The problem with using this theory to explain the mechanism of growth is well illustrated by Solutions 1, 2 and 3 given by the eqns (78), (82) and (89) and by the accompanying expressions for $aE(t)$ given in the eqns (75), (79) and (86). While we can explain some of these expressions, we cannot use them to explain the mechanism of growth.

An interesting feature of this exercise is that a single force of growth can describe a trajectory, which at a certain stage can change from hyperbolic to some other type. If we could find a force that could reproduce data over the whole range of time and if we could explain the nature of this force, we would have made a huge progress in explaining the mechanism of growth. However, examples presented by Karev indicate that finding such a force of growth and explaining its origin is close to impossible.

Summary and conclusions

We have presented here a brief survey of attempts to understand hyperbolic distributions. The common characteristic of all these attempts is that they are not only complicated but that they are also unnecessarily complicated because a simple expression given by eqn (1) describes data exceptionally well (Nielsen, 2016a, 2016c). This simple formula describes not only the growth of population but also economic growth as expressed by the Gross Domestic Product (Nielsen, 2016b). Furthermore, by using this simple formula we can also easily describe income per capita and explain its puzzling features (Nielsen, 2015, 2016g).

Complicated methods used in the interpretations of hyperbolic growth did not yet result in explaining its mechanism. They also did not produce a better description of data than the descriptions given by the simple expression represented by the eqn (1).

When mathematical formulations become increasingly complicated it is usually a warning sign that we are on the wrong track, that we should stop, regroup and look for simpler descriptions and solutions. A simple formula [eqn (1)] describing population and economic growth suggests that there must be also a simple explanation of their mechanisms. Such a simpler explanation will be proposed in the next publication.

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3. Examination of Malthusian positive checks

Introduction

Malthus (1798) is well known for having his name associated with the erroneous concept of stagnation expressed in such phrases as Malthusian stagnation, Malthusian regime, epoch of Malthusian stagnation, Malthusian trap and escape from Malthusian trap, the concept he never proposed or advocated, the concept based on impressions, on a good dose of fantasy and on the suitable manipulation of data (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). The concept of stagnation and all other related concepts have been repeatedly and convincingly contradicted by data and by their mathematical analyses (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Lehmeyer, 2004; Livi-Bacci, 1997; Maddison, 2001; 2010; Mauritius, 2015; McEvedy & Jones, 1978; Nielsen, 2013a; 2013b; 2013c; 2014; 2015; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i; Podlazov, 2002; Shklovskii, 1962; 2002; Statistics Mauritius, 2014; Statistics Sweden, 1999; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960; Wrigley & Schofield, 1981).

Malthus (1798) proposed three fundamental mechanisms contributing to the growth of human population: (1) the devastating effects of positive checks, (2) the regenerating effects of positive checks and (3) the effects of preventative checks. He did not explain the mechanism of growth of population. He only claimed that "Population, when unchecked, increases in a geometrical ratio" (Malthus, 1978, p. 4). Now we know that this is not true. Population, when unchecked does not increase in a geometrical ratio (exponentially) but hyperbolically (Kapitza, 2006; Kremer, 1993; Nielsen, 2016b, 2016d; Podlazov, 2002; Shklovskii, 1962, 2002; von Hoerner, 1975, von Foerster, Mora & Amiot, 1960).

Malthus carried out an important pioneering work but it was for the future generations of researchers to explore these three proposed contributing mechanisms and to understand their impacts, if any, on the growth of population. Malthus did not have sufficient data to carry out such research. He described lethal effects of demographic catastrophes but he never claimed explicitly that they would create a lasting stagnation in the growth of human population, let alone that they would produce the epoch of stagnation as it is now erroneously claimed (Galor, 2005a, 2011). Indeed, his claim that population if unchecked increases exponentially seems to suggest that he did not imagine prolonged or lasting effects of such positive checks.

Now we know that the growth of the world population, for instance, might have been checked only once in the past 12,000 years by the unusual convergence of no fewer than *five* major demographic catastrophes, which introduced only a *minor disturbance* between AD 1200 and 1400 (Nielsen, 2016d). There is also no convincing evidence of the frequently occurring devastating effects of Malthusian positive checks in the growth of regional populations (Nielsen, 2016b). These surprising results could be perhaps explained in two ways: (1) that the relative impacts of demographic catastrophes were generally too small and (2) that the devastating effects of positive checks were to a certain degree compensated by their regenerating effects, which Malthus mentions in his book.

The aim of the current publication is to continue the work of Malthus and to investigate impacts of positive checks, the work Malthus could not do because he did not have relevant data. He described the devastating effects of positive checks but he also did not fail to notice and record their positive effects in stimulating growth.

We shall start from where Malthus was forced to stop and we shall investigate how the effects of positive checks are reflected in the growth of human population. We shall assume that the intensity of Malthusian positive checks can be measured by the level of deprivation. We shall first define this indicator. We shall then see how this indicator is reflected in the standard of living. Using this indicator, we shall then see how Malthusian positive checks are reflected in the destructive effects such as the increased death rates. We shall then investigate the other side of these positive checks and demonstrate how they are reflected in the process of regeneration, such as in the increased rate of natural increase, the increased growth rate and the increased total fertility rate. This study will allow us to extend the work of Malthus, which he published around 200 years ago, and to understand better the effects of his positive checks, the effects outlined only briefly in his book.

Measuring the intensity of Malthusian positive checks

Effects of Malthusian positive checks can be studied conveniently using the data compiled by the United Nations Development Program (UNDP, 2011). These data are linked with the three-dimensional Human Development Index (HDI) (UNDP, 2010), which is defined using the levels of health, education and income. Human Development Index varies between 0 and 1 and measures the level of human development or the level of prosperity. The HDI close to 1 is for prosperous countries.

We could use this index to describe *indirectly* the intensity of Malthusian positive checks but then in order to understand the studied correlations, which we are going to present, and to study the effects Malthusian positive checks, we would have to translate mentally the HDI into the levels of deprivation. In order to link the UNDP data *directly* to the level of deprivation and thus to the intensity of Malthusian positive checks, it is better to introduce just a slight modification of the HDI and define the Level of Deprivation Index (LDI) as:

$$LDI \equiv 1 - HDI \qquad (1)$$

This index varies from around 0 (for the low level of deprivation) to around 1 (for the high level of deprivation). At the high end of the spectrum, this index is linked with such conditions as poor health care, low income, severe poverty, inadequate access to sanitation facilities, inadequate access to pure water, hunger, inadequate housing, poor education, devastating effects of wars and military conflicts, high incidents of infectious diseases and all other conditions, which are usually identified as representing the intensity of Malthusian positive checks. As the level of deprivation decreases, the intensity of Malthusian positive checks also decreases.

It should be noted that for Malthus positive checks were not necessarily represented by the usually claimed great calamities such as wars, famines and pestilence. Furthermore, for him, positive checks did not have to apply to the whole country but to certain groups of people within a given country or even to individual families.

> Notwithstanding, then, the institution of the poor laws in England, I think it will be allowed that considering the state of the *lower classes* altogether, both in the towns and in the country, the *distresses* which they suffer from the want of *proper and sufficient food*, from *hard labour* and *unwholesome habitations*, must operate as a *constant check* to incipient population. (Malthus, 1798, p.31. Italics added.).
>
> Labour would be ill paid. Men would offer to work for a bare subsistence, and *the rearing of families would be checked by sickness and misery* (Malthus, 1798, p.64. Italics added.).

In all mathematical formulae and diagrams presented in this study, x is reserved exclusively for the LDI, which will be always used as an independent variable, while y will be used for any relevant dependent variable. It should be also noted that while the derived formulae can have a general application, the exact values of the constants apply only to the data published by UNDP (2011).

We shall first examine how the level of deprivation, i.e. how the intensity of Malthusian positive checks, is reflected in the standard of living represented by such indicators as the ecological footprint (EF), income per capita, the intensity of severe poverty, access to clean water and access to sanitation facilities. We shall then examine the devastating effects of Malthusian positive checks as reflected in the increased morality. Finally, we shall examine the regenerating effects by showing how growth-promoting indicators depend on the intensity of Malthusian positive checks.

Malthusian positive checks reflected in the standard of living

We shall now present three examples showing how the intensity of Malthusian positive checks as described by the Level Deprivation Index are correlated with the standard of living.

Malthusian positive checks reflected in the ecological footprint (EF)

Ecological footprint (EF) measures the level of consumption of natural resources and the level of the associated damage to the environment (Ewing, et al., 2010). The footprint is expressed in global hectares per person [gha/cap] of the biologically productive surface area: crops, grazing, fishing, forests for timber and firewood, forests for carbon dioxide absorption and land for human habitat.

The dependence of the ecological footprint (EF) on the Level of Deprivation Index (LDI), based on the UDDP data (UNDP, 2011), is shown in Table 1. Small ecological footprint is associated with a small consumption of natural resources and with a high intensity of Malthusian positive checks as reflected in the Level of Development Index (LDI). These data show that only a small fraction of human population is responsible for the excessively large ecological footprint.

Table 1. *The average Level of Deprivation Index (LDI) and the average ecological footprint (EF), expressed in gha/cap, for various levels of human development, based on the UNDP data (UNDP, 2011).*

Level of development	LDI	EF [gha/cap]	Population [Million]
Very high human development	0.111	5.8	1,130
High human development	0.259	2.5	973
Medium human development	0.370	1.7	3,546
Low human development	0.544	1.2	1,260

The average global ecological footprint in 2011 calculated using the UNDP data (UNDP, 2011) was 3 gha/cap. According to the data shown in Table 1, ecological footprint was higher than this average value for 16.5% of the world population. For this small groups of people, the average intensity of Malthusian positive checks was low (LDI = 0.111). In contrast, for 18.4% of global population, the intensity of Malthusian positive checks was approximately five times higher (LDI = 0.544). The dependence of the ecological footprint on the level of deprivation is shown in Figure 1. Each dot in this diagram (and in all other diagrams presented in this publication) represents one of the 187 countries listed in the UNDP compilation (UNDP, 2011).

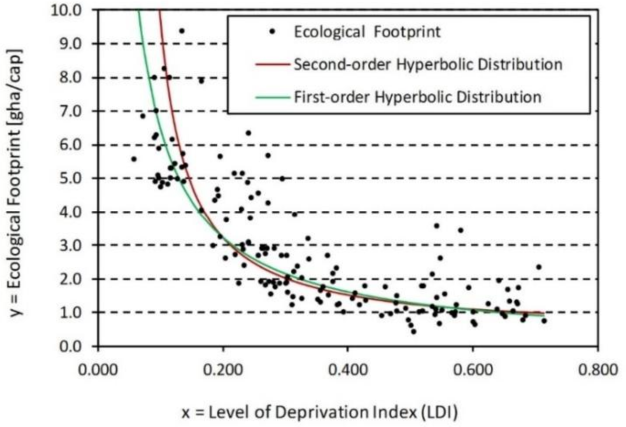

Figure 1. *Correlation between the intensity of Malthusian positive checks as measured by the LDI and the ecological footprint (EF). The data are from the United Nations Development Program (UNDP, 2011). They are compared with two best fits using hyperbolic distributions.*

The best fit to the data presented in Figure 1 is obtained by using the second-order hyperbolic distribution:

$$y = (a_0 + a_1 x + a_2 x^2)^{-1} \ , \qquad (2)$$

where x is the LDI and y is the ecological footprint. For this set of data (UNDP. 2011), $a_0 = 0.126$, $a_1 = 4.406$ and $a_2 = -1.139$
.

However, a satisfactory fit can be also obtained using a much simpler, first-order, hyperbolic distribution

$$y = ax^{-1}, \qquad (3)$$

where $a = 0.646$.

For the large values of the LDI, ecological footprint increases slowly with the decreasing level of deprivation. Thus, in the extreme case, for countries characterised by the high values of the LDI, i.e. by the high intensity of Malthusian positive checks, a large reduction in the level of deprivation, and thus in the intensity of Malthusian positive checks can be achieved by only a relatively small increase in the ecological footprint.

We shall show later how the intensity of growth of population increases with the intensity of the LDI, i.e. with the intensity of

Malthusian positive checks. Figure 1 suggests that large reductions in the intensity of growth of human population could be achieved by improving living conditions of poor countries through a relatively small increase in their ecological footprint.

In contrast, as we can see from the correlation presented in Figure 1, a large increase in the ecological footprint of rich countries (characterised by the low LDI values) results in only marginal improvement in their standard of living. Their standard of living is already so high that to improve it by only a small degree requires enormous increase in their consumption of natural resources, which is not only unfair for poor countries but also imprudent because a better distribution of wealth could contribute significantly to reducing the growth of human population and to the global security.

Malthusian positive checks reflected in the income per capita

The dependence of the Gross Domestic Product per person (GDP/cap) on the level of deprivation is shown if Figure 2.

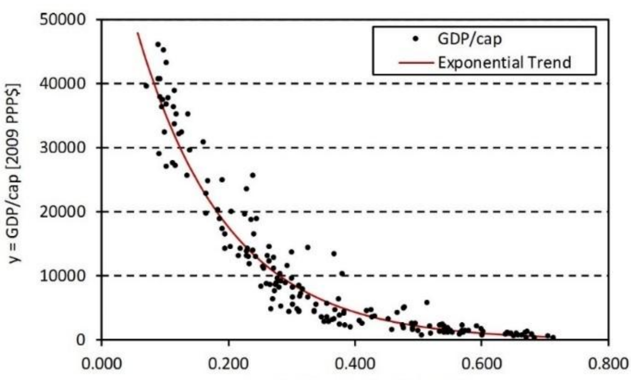

Figure 2. *Exponential dependence of the Gross Domestic Product per person (GDP/cap) on the Level of Deprivation Index (LDI). The GDP is in the purchasing power parity of 2009 international dollars*

The GDP/cap decreases exponentially with the increasing level of deprivation, i.e. with the intensity of Malthusian positive checks. The range of the GDP/cap is between $319 for the Demographic Republic of Congo and $91,379 for Qatar, with the US ($45,989) and Switzerland ($45,224) being located in the middle.

The best fit to the data is obtained using exponential function,

$$y = be^{rx},$$ (4)

where x is the LDI and y is the GDP/cap. For these particular set of data (UNDP, 2011), b = \$71,144 and $r = -6.97$.

Results presented in Figure 2 lead to the same conclusions as results shown in Figure1. At the far end of the LDI scale, a small increase in the GDP/cap by only around \$4,000, on average, would advance countries from the low to medium level of human development. In contrast, an increase by around \$20,000 would be needed to advance countries from high to very high level of human development. We can also look at it by comparing the increase in the GDP/cap needed to decrease the LDI by the same interval. For instance, to decrease the LDI from 0.500 to 0.400 one would need to increase, on average, the GDP/cap by only \$2,000. In contrast, to decrease the LDI from 0.300 to 0.200 one would need to increase the GDP/cap by around \$9,000.

Malthusian positive checks reflected in the severe poverty and in other related indicators

The dependence of the fraction of the population living in severe poverty on the intensity of Malthusian positive checks is shown in Figure 3.

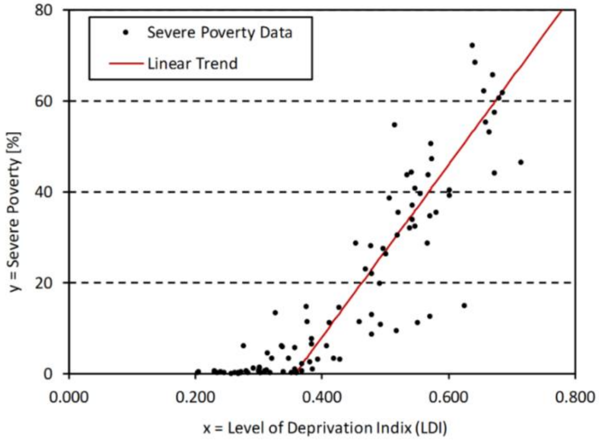

Figure 3. *The fraction of population living in severe poverty (y) represented as a function of the LDI (x), i.e. as a function of the intensity of the Malthusian positive checks.*

It is essential to notice two important features of the correlation presented in Figure 3. *First*, the correlation is linear. *Second*, the correlation is characterised by a certain threshold below which the fraction of the population living in severe poverty is on average zero. The intensity of Malthusian positive checks decreases linearly with the level of poverty. However, when the fraction of the population living in severe poverty reaches its zero value, the intensity of Malthusian positive checks reaches a certain threshold level. Any further decrease in the intensity of Malthusian positive checks is no longer correlated with the level of severe poverty, but it will continue to be correlated with the ecological footprint and with income per capita.

This linear correlation indicates that severe poverty can be reduced even to zero without trying to reduce the intensity of Malthusian positive checks to zero. The reduction in the level of severe poverty will come first. After that, there could be other improvements, which would be reflected in other parameters describing the standard of living.

Similar step-wise linear correlations apply also to the fraction of the population living below the poverty line and to the Multidimensional Poverty Index (MPI). The same type of correlations applies also to the size of the population with no access to clean water and to the size of the population with no access to sanitation facilities. All these data can be fitted using a simple mathematical expression:

$$y = m(x - n),\ y \geq 0, \tag{5}$$

where n is a threshold below which $y \approx 0$.

For the data presented in the UNDP report (UNDP, 2011) and for the fraction of the population living in severe poverty, m = 190 and n = 0.358. In terms of the ecological footprint and of the GDP/cap, this threshold corresponds to 1.7 gha/cap and \$5,858/cap, respectively. Above these thresholds, the fraction of the population living in severe poverty could be expected to be on average negligibly small.

For all indicators, mentioned earlier and characterised by such linear correlations, and for the data listed in the UNDP report (UNDP, 2011) the respective thresholds in the LDI vary between 0.293 and 0.371. In terms of the EF and the GDP/cap, they vary between 1.6 and 2.1 gha/cap for the EF, and between \$5,350 and \$9,218 for the GDP/cap. These figures suggest that a moderate improvement in the living conditions of poor countries could have an enormous impact on reducing the level of poverty and on improving access to clean water and to sanitation facilities. We shall see later that the added benefit of improving the standard of living in poor countries could be a significant reduction in the growth of population.

The lethal impacts of Malthusian positive checks

We can now take the next step and try to understand the destructive effects of Malthusian positive checks. An example of the dependence of mortality on the intensity of Malthusian positive checks is shown in Figure 4 for adult mortality.

The effect is quite remarkable: mortality increases exponentially. We can analyse other relevant data (UNDP, 2011) and we shall get consistently similar results. Such exponential increase applies to deaths due to polluted water, maternal mortality and under-five mortality. It would appear that the exponential dependence of mortality on the intensity of Malthusian positive checks could be expected to apply also to other forms of mortality.

Malthus did not study how mortality depends on the intensity of positive checks. He only pointed out that positive checks can be linked with the increased mortality. Now we know not only that mortality increases with the intensity of positive checks but also *how* it increases – it increases exponentially.

The exponential distribution shown in Figure 4 is described by the eqn (4) but now with the positive parameter r. For the set of data listed by UNDP (2011), b = 124.83 and r = 2.925. Adult mortality is on average 66% higher in countries characterised by low human development than in countries characterised by medium human development, nearly 130% higher than in countries characterised by high human development and 255% higher than in countries characterised by very high human development.

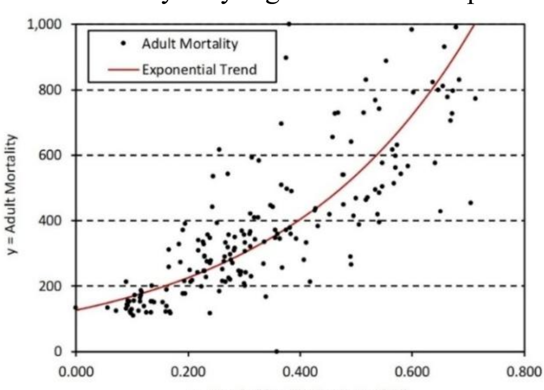

Figure 4. *Exponential dependence of adult mortality (per 1000 adult population) on the level of deprivation (LDI), i.e. on the intensity of Malthusian positive checks.*

The regenerating impacts of Malthusian positive checks

Intuitively, one might expect that high mortality should be reducing the size of population and thus that it should be suppressing growth. This is what Malthus expected. “These facts seem to shew that population increases exactly in the proportion that the two great checks to it, misery and vice, are removed, and that there is not a truer criterion of the happiness and innocence of a people than the rapidity of their increase” (Malthus, 1798, p. 34).

Thus, according to Malthus, the smaller is the intensity of misery and vice, the faster should be the growth of the population. Furthermore, his comment suggests that it should be a linear correlation.

If his interpretation of growth is correct, we should expect that the growth rate of population should be *decreasing* with the increasing intensity of Malthusian positive checks, i.e. with the increasing level of deprivation. Furthermore, the high growth rate could be used as an indicator of “the happiness and innocence” because “there is not a truer criterion of the happiness and innocence of a people than the rapidity of their increase.”

We are now going to show that Malthusian positive checks stimulate growth, which is hardly surprising because it is well known that poor countries are characterised by a rapid growth of population. Malthus observed this phenomenon of stress-induced growth but he did not follow his observation by a closer investigation perhaps because his access to relevant data was strongly limited. It is also obvious that rapid growth of population in poor countries does not contribute to their happiness.

Total fertility rate increases exponentially with the intensity of Malthusian positive checks

Total fertility rate is defined as the “number of children that would be born to each woman if she were to live to the end of her child-bearing years and bear children at each age in accordance with prevailing age-specific fertility rates” (UNDP, 2011, p. 142). The dependence of total fertility rate on the level of deprivation, i.e. on the intensity of Malthusian positive checks is shown in Figure 5.

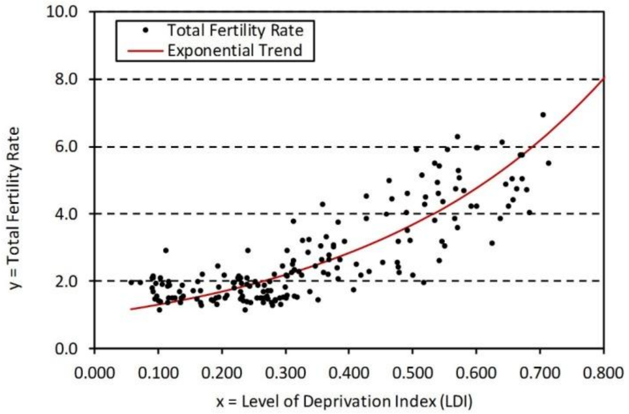

Figure 5. *Total fertility rate increases exponentially with the intensity of Malthusian positive checks, i.e. with the increasing level of deprivation (LDI).*

Analysis of the UNDP data (UNDP, 2011) leads to remarkable results. It shows that while morality increases exponentially with the intensity of Malthusian positive checks, total fertility rate also increases exponentially. This is the first and important indication that the growth of human population is not slowed down by the increased mortality.

Growth rate is directly proportional to the intensity of Malthusian positive checks

The correlation between the level of deprivation and the growth rate is shown in Figure 6. The data show that, on average, the annual growth rate is directly proportional to the level of deprivation, i.e. to the intensity of Malthusian positive checks. The larger is the intensity of Malthusian positive checks the larger is the growth rate.

Contrary to the intuitive expectations and contrary to the repeated claims of the existence of the mythical epoch of Malthusian stagnation, the growth of human population is not decreased by the Malthusian positive checks but increased. However, when the intensity of Malthusian positive checks is exceptionally high and when they continue over a long time, the growth of population might be temporarily slowed down. This effect was observed in the growth of the world population but even then the temporary disturbance in the growth of population was followed by their more accelerated growth (Nielsen, 2016d).

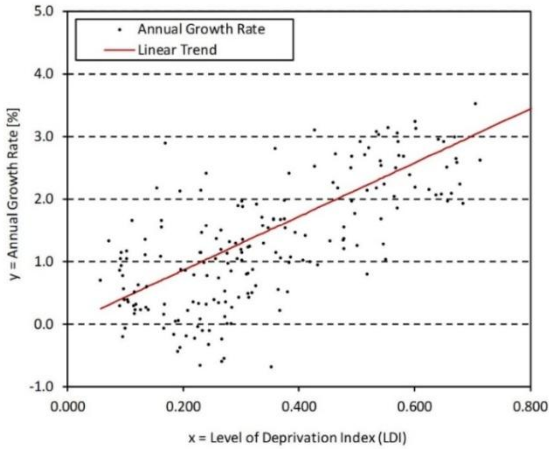

Figure 6. *The dependence of the annual growth rate on the level of deprivation, i.e. on the intensity of Malthusian positive checks.*

The straight line fitting the empirical growth rate data shown in Figure 6 is given by

$$y = m(x - n) , \tag{6}$$

where for this particular set of data (UNDP, 2011) $m = 4.3$ and $n = 0$.

In countries characterised by the low level of human development, and consequently experiencing high intensity of deprivation and of the associated mortality, the growth of population as given by the UNDP data (UNDP, 2011) was on average about 5 times faster than in countries characterised by the very high human development and experiencing the low level of deprivation and mortality.

Rate of natural increase is directly proportional to the intensity of Malthusian positive checks

The rate of natural increase is defined as the difference between the death and birth rates and thus excludes the immigration and emigration rates. The correlation between the rates of natural increase and the levels of deprivation can be studied by using the 2002 data for the rates of natural increase (US Census Bureau, 2002) and the 2002 data for the HDI extrapolated from the tabulated data for 2000 and 2005 (UNDP, 2011). Results are presented in Figure 7.

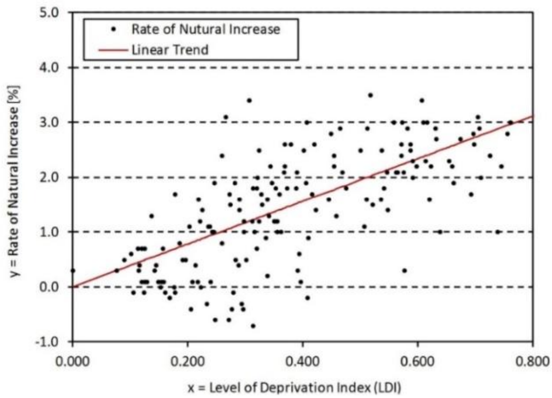

Figure 7. *The dependence of the rate of natural increase on the level of deprivation, i.e. on the intensity of Malthusian positive checks.*

The fitted straight line, represented by the eqn (6), corresponds to $m = 3.9$ and $n = 0$. These data show that the rate of natural increase is also directly proportional to the level of deprivation.

In principle, the rate of natural increase gives a better representation of the impacts of Malthusian positive checks because it is not obscured by contributions from immigrations and emigrations. However, by comparing Figures 6 and 7, we can see that the linear dependence applies to the growth rate and to the rate of natural increase suggesting that contributions from immigration and emigration are in general negligibly small. Individual points might be shifted but the general trend is the same. The gradients of the straight lines fitting the data are also similar, 4.3 for the growth rate and 3.9 for the rate of natural increase. It does not matter whether we are using growth rate or the rate of natural increase, results are the same: contrary to the widely accepted but erroneous doctrine of Malthusian stagnation, the intensified presence of Malthusian positive checks is correlated with the intensified growth of population.

Results presented in Figures 6 and 7 are most surprising. The regenerating impacts of Malthusian positive checks do not just keep the growth rate constant – they stimulate growth and make it even faster.

Devastating impacts of Malthusian positive checks are correlated with their regenerating impacts

The dichotomy of Malthusian positive checks can be also illustrated by the correlations between their opposite impacts, i.e. between destructive and regenerating effects. We could study how various forms of mortality (adult mortality, under-five mortality, maternal mortality or deaths by polluted water) are correlated with various forms of regenerating effects (total fertility rate, growth rate and the rate of natural increase). For the listed here effects we would have 12 such correlations. However, this study of multiple correlations can be reduced to a study of two types of correlations: (1) correlations between exponential distributions describing total fertility rate and various forms of mortality, and (2) correlations between exponential and linear distributions, with exponential distributions representing various forms of mortality while linear distributions representing growth rate or the rate of natural increase. As an example, we show two such correlations: (1) the correlation between adult mortality (exponential) and total fertility rate (exponential), displayed in Figure 8 and (2) the correlation between adult mortality (exponential) and the growth rate (linear), displayed in Figure 9.

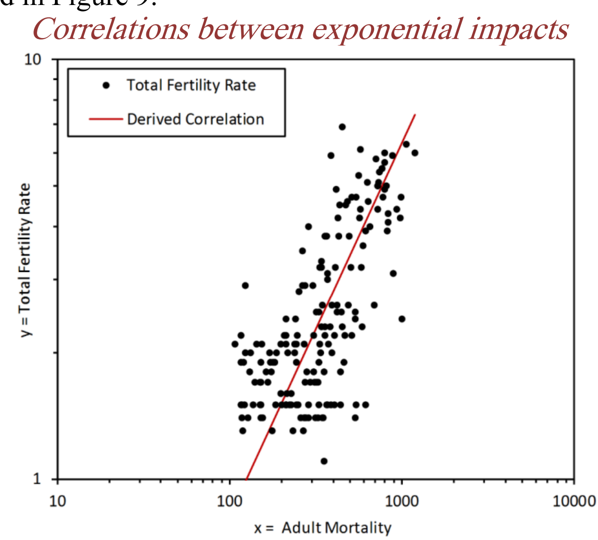

Figure 8. *The dichotomy of Malthusian positive checks: the increasing destructive impacts (mortality) are correlated with the increasing total fertility rate. Similar correlations exist also for under-five mortality, maternal mortality and deaths by polluted water. Parameters are given by the correlations of the LDI with adult mortality and with total fertility rate.*

As we have already seen, adult mortality and total fertility increase exponentially. If we represent mortality by y:

$$y = be^{rx} \tag{7}$$

and fertility by z:

$$z = b'e^{r'x},\tag{8}$$

then the correlation between y and z is given by

$$z = Ae^{B\ln y},\tag{9}$$

where

$$B = \frac{r'}{r}\tag{10}$$

and

$$A = b'e^{-B\ln b} \quad .\tag{11}$$

The eqn (9) can be also expressed as

$$\ln z = \ln A + B\ln y\tag{12}$$

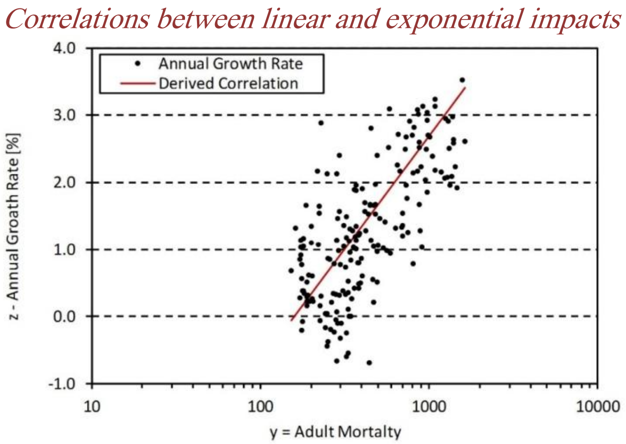

Figure 9. *The dichotomy of Malthusian positive checks: the increasing destructive impacts (mortality) are correlated with the increasing growth rate. Similar correlations exist also for under-five mortality, maternal mortality and deaths by polluted water as well as between the rate of natural increase and all these forms of mortality. Parameters are given by the correlations of the LDI with adult mortality and with the annual growth rate.*

Correlations between the growth rate or the rate of natural increase and various forms of mortality also illustrate the dichotomy of Malthusian positive checks. As we have seen, the rate of natural increase and the growth rate increase linearly with the intensity of Malthusian positive checks while mortality increases exponentially. If we follow similar procedure as outlined earlier for the correlations between mortality and fertility, we shall find that

$$z = m\left[\frac{\ln y}{r} - \left(n + \frac{\ln b}{r}\right)\right] \tag{13}$$

where z is the annual growth rate or the rate of natural increase and y is the mortality, which could be adult mortality, under-five mortality, maternal mortality or deaths by polluted water. For the UNDP data (UNDP, 2011), $n = 0$ and this formula is reduced to

$$z = \frac{m}{r}\left[\ln y - \ln b\right] \tag{14}$$

An example of these correlations is shown in Figure 9. As the intensity of the destructive effects (mortality) of Malthusian positive checks increases, the intensity of the regenerating effects also increases and is reflected in the increased growth rate or in the increased rate of natural increase.

It is generally believed that Malthusian positive checks cause stagnation in the growth of population and in the associated economic growth (see for instance Artzrouni & Komlos, 1985; Desment & Parente, 2012; Galor, 2005a, 2011; Galor & Weil, 1999; 2000; Guest & Almgren, 2001; Hansen & Prescott, 2002; Komlos, 1989; 2000; Komlos & Baten, 2003; Lagerlöf, 2003a; 2001b; Lee, 1997; Leibenstein, 1957; McKeown, 1983; 2009; Nelson, 1956; Olshansky & Ault, 1986; Omran, 1971; 1983; 1998; 2005; Robine, 2001; van de Kaa, 2010; Vollrath, 2011; Wang, 2005; Warf, 2010; Weisdorf, 2004). The concept of Malthusian stagnation is at the root of the established knowledge in demography and in economic research, the knowledge, which is largely based on conjectures, impressions and even on distorted presentations of data (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

As discussed elsewhere (Nielsen, 2016j), the currently established knowledge in demography and in economic research, based on the concept of Malthusian stagnation, is scientifically unacceptable. There is now an overwhelming evidence that the so-called Malthusian stagnation never existed in the growth of population and in the economic growth (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Lehmeyer, 2004; Livi-Bacci, 1997; Maddison, 2001; 2010; Mauritius, 2015; McEvedy & Jones, 1978; Nielsen, 2013a; 2013b; 2013c; 2014; 2015; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; 2016i; Podlazov, 2002; Shklovskii, 1962; 2002; Statistics Mauritius, 2014; Statistics Sweden, 1999; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994; United Nations, 1973; 1999; 2013; von Hoerner, 1975; von Foerster, Mora & Amiot, 1960; Wrigley & Schofield, 1981). The investigation of the UNDP data (2011) contributes to the explanation why there was no stagnation. One of the contributing factors was the dichotomy of Malthusian positive checks. As originally noticed by Malthus and as now confirmed by the study of the UNDP data (UNDP, 2011) the destructive action of Malthusian positive checks is accompanied by their regenerating impacts. Destruction induces regeneration.

Summary of the observed correlations

Summary of the observed correlations between the Level of Deprivation Index (LDI) representing the intensity of Malthusian positive checks and a series of indicators illustrating the standard of living, the destructive impact of Malthusian positive checks and their regenerating impacts is presented in Table A1 (in the Appendix). This table includes also correlations between the destructive and regenerating impacts of Malthusian positive checks.

Hunger and famines are correlated with the intensified growth of population

The dichotomy of Malthusian positive checks can be also studied by investigating impacts of hunger. As with other, specifically mentioned indicators describing levels of deprivation, hunger is just the reflection of the whole spectrum of Malthusian positive checks. "Natural disasters, climatic shocks, conflict, and insecurity are major causes of hunger. But hunger's root causes are tied to a lack of access by individuals to the resources they need to produce, sell, and buy food" (Sheeran, 2008, p. 180). "The tragic fact is that, although our planet produces enough food for everyone, one person in seven still goes to bed hungry each night. 25,000 people die every day – including one child every 5 seconds

– from hunger-related causes" (Sheeran, 2008, p. 180). "The overall finding is that 3.1 million children younger than 5 years die every year from undernutrition; that is a staggering 45% of total child deaths in 2011" (Horton & Lo, 2013, p. 371).

Hunger appears to be one of the leading causes of death in the world. "Every year over 10 million people die of hunger and hunger-related diseases. Nearly six million of these are children under the age of five; that is one child's death approximately every six seconds." (Gibson, 2012, p. 18). This should be compared with other leading causes of death in the world in 2012: ischaemic heart disease, 7.4 million deaths per year; stroke, 6.7 million; COPD, 3.1 million; lower inspiratory infections, 3.1 million; trachea bronchus lung cancers, 1.6 million; HIV/AIDS, 1.5 million; diarrhoeal diseases, 1.5 million; diabetes mellitus, 1.5 million; road injury, 1.3 million; hypertensive heart disease, 1.1 million (WHO, 2014).

Again, it is repeatedly but erroneously claimed that lethal effects of hunger and famines suppress the growth of human population and create a stagnant state of growth. We shall demonstrate that such is not the case. These popular and widely-accepted interpretations are incorrect. They are based on scientifically unsupported dogmas (Nielsen, 2016j).

Evidence from Africa

Table A2 (in the Appendix) and Figure 10 present a series of growth-related indicators for two groups of African countries, one group where hunger stress is $\geq 35\%$ and another where hunger stress is less than 5%.

Data presented in Table A2 and Figure 10 are based on the examination of three sources of reference (PRB, 2010; UNDP, 2011; WFP, 2010). In Figure 10, birth and death rates are expressed in percent while infant mortality rate in per cent of live births.

Table A2 and Figure 10 show that on average, and for these set of data, countries exposed to high level of hunger stress experience 71% higher intensity of Malthusian positive checks as expressed by the LDI and have strongly reduced access to natural resources, as reflected in their ecological footprint, when compared with countries experiencing low hunger stress. Countries with high hunger stress experience 39% higher death rate and a massive 120% higher infant mortality rate. However, for these countries, total fertility rate is 47% higher, birth rate is 35% higher, the rate of natural increase is also 35% higher and the population increase factor is 26% higher, all these indicators showing that the natural response to the lethal Malthusian checks is the increased rate of procreation and the intensified process of regeneration. Thus, contrary to the generally accepted interpretations, hunger does not reduce the growth of human population but is associated with a faster growth.

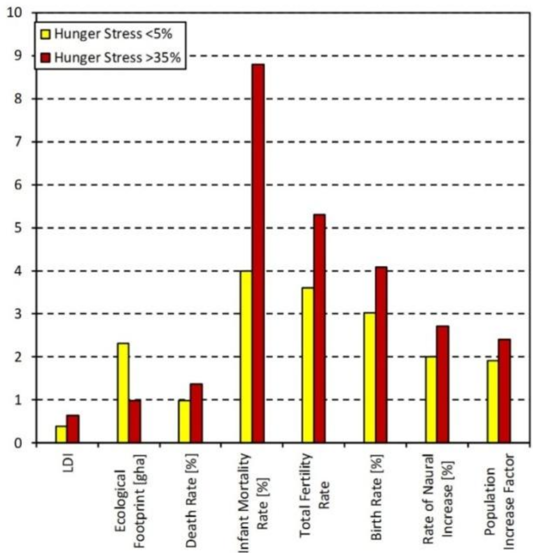

Figure 10. *The dichotomy of Malthusian positive checks as reflected in the intensity of hunger stress. Populations suffering high hunger stress experience a higher death rate and a higher rate of infant mortality than populations experiencing a small hunger stress. However, populations suffering high hunger stress are also characterised by a higher total fertility rate, higher birth rate, higher rate of natural increase and higher population increase factors.*

It is, of course, impossible to isolate hunger as a single stress factor. Malthusian positive checks are interconnected. However, if for instance, hunger stress increases the susceptibility to infectious diseases, then the primary stress factor is still hunger.

Data make it clear that in countries suffering high hunger stress population growth is *faster* than in countries experiencing significantly lower hunger stress. Clearly, Malthusian positive checks bring not only the destruction as reflected in the increased death tolls but also the regeneration. Furthermore, the regeneration process is more powerful than the process of destruction because the growth of population is not just at the same level as in countries experiencing low hunger stress but faster.

Evidence from China

A prominent example of devastating impacts of Malthusian positive checks associated with famines is China. "Between the years 108 B.C. and 1911 AD, there were 1828 famines or one nearly every year in some of the provinces. Untold millions have died of starvation. In fact, the normal death rate may be said to contain a constant famine factor" (Mallory, 1926, p. 1). However, China is also an excellent example of regenerating impacts of Malthusian positive checks. "In spite of the tremendously high death rate, particularly of infants, due to lack of modern medical knowledge, in spite of the depopulating effect of terrible famines, and in spite of the immense loss of life caused by civil wars we find today a denser population on the plains than ever before; and since there has been no appreciable influx from other countries we much ascribe the present conditions to the excessive birth rate" (Mallory, 1926, p. 87).

The dichotomy observed by Malthus and in nature

While Malthus is well known for suggesting lethal effects of positive checks, he is not so well known for being aware of the existence of a competing mechanism, the mechanism of spontaneous regeneration and preservation of identity.

"The absolute population at any one period, in proportion to the extent of territory, could never be great, on account of the unproductive nature of some of the regions occupied; but there appears to have been a most rapid succession of human beings, and *as fast as some were mowed down by the scythe of war or of famine, others rose in increased numbers to supply their place.* Among these bold and improvident Barbarians, population was probably but little checked, as in modern states, from a fear of future difficulties (Malthus, 1798, p. 15. Italics added).

If Malthus had access to the data available to us he would have probably presented a more appropriate description of people exposed to lethal effects of positive checks. We would be reluctant to describe people living in poor countries as "bold and improvident Barbarians." We also would not like to use the same description to parents of the post-war baby boomers.

Apart from this general observation, Malthus presents also data, which suggest that high-intensity positive checks are linked with the intensified process of regeneration (Malthus, 1798, pp. 36-40). He has noticed, for instance "that the greatest proportion of births to burials, was in the five years after the great pestilence" (Malthus, 1798, p. 37). He concludes that "Great and astonishing as this difference is, we ought not to be so wonder-struck at it as to attribute it to the miraculous interposition of heaven. The causes of it are not remote, latent and mysterious; but near us, round about us, and open to the investigation of every inquiring mind"

(Malthus, 1798, p. 40). He was convinced that these incidents of intensified growth, after the episodes of epidemics, were not only the manifestation of the natural law of growth but also that they should be closely examined. It is, therefore, disappointing that numerous scholars who refer to the work of Malthus overlooked his suggestion that the effects of regeneration should be further examined. Malthus also lists many examples of successful regeneration in such places as Flanders, Palestine, London, Turkey, Egypt, China, Naples and Lisbon (Malthus, 1798, p. 35).

While emphasising the importance of food in supporting the growth of human population, he did not fail to notice that people can "live almost upon the smallest possible quantity of food" (Malthus, 1798, p. 41), which implies that hunger or famines should not be immediately identified as factors controlling the growth of population. It is incorrect to suggest that "Malthusian positive checks (mortality crises) maintained a long-run equilibrium between population size and the food supply" (Komlos, 1989, p. 194). It is incorrect to claim that "the food-controlled homeostatic equilibrium had prevailed since time immemorial" (Komlos, 2000, p. 320). It is not immediately obvious the "Throughout human history, epidemics, wars and famines have shaped the growth path of population" (Lagerlöf, 2003b, p. 435).

Malthus uses China as an example where "the lower classes of people are in the habit of living almost upon the smallest possible quantity of food and are glad to get any putrid offals that European labourers would rather starve than eat" (Malthus, 1798, p. 41). He also cautions against using food as a factor controlling the growth of population. He points out twice in his book that in some cases population may "permanently increase without a proportional increase in the means of subsistence" (Malthus, 1798, pp. 41, 43). Furthermore, he points out that there could be "some variations in the proportion between the number of inhabitants and the quantity of food consumed, arising from the different habits of living that prevail in each state" (Malthus, 1798, p. 42). Food consumption is not proportional to the size of population. The relation between food consumption and the growth of population is not immediately obvious.

Malthus placed a significant emphasis on the role of positive checks but he also made an attempt to present a balanced interpretation of growth, a balanced view which is conspicuously missing in the numerous publications referring to Malthus and describing erroneously the effects of Malthusian positive checks as Malthusian stagnation. While making attempts to praise Malthus for his work such publications are in fact diminishing the importance of his contribution.

His early observations, combined with the vast body of data available now to us, help to understand the mechanism of growth of human population. It is a process, which can be influenced by the devastating impacts of Malthusian positive checks but also a process, which is influenced by their regenerating impacts.

The phenomenon of regeneration noticed and recorded by Malthus is similar to the well-known process observed in nature. It is the natural and spontaneous process of self-preservation of living organisms triggered by stressful conditions. It is the resilience of ecological systems (Holling, 1973). There are numerous examples and definitions of this omnipresent phenomenon.

According to Cumming et al. (2005, p. 976), resilience is "the ability of the system to maintain its identity in the face of internal change and external shocks and disturbances." The definition proposed by the National Research Council (NRC) is "the continued ability of a person, group, or system to adapt to stress – such, as any sort of disturbance – so that it may continue to function, or quickly recover its ability to function, during and after stress" (NRC, 2011, pp. 13, 14). "Resilience is the ability to handle stresses or recover from disturbances or shocks" (Bapna, McGray, Mock & Withey, 2009, p. 3). "In general, resilience refers to a system's capacity to deal with change and to continue to develop" (Boyd, et al., 2008, p. 391). There are also many other definitions of resilience, all describing either the ability of a quick and efficient recovery or the ability to cope with stress.

Malthus noticed the existence of this mechanism of regeneration. This process is also well known in science but for reasons, which are hard to understand, it is overlooked in publications based on the erroneous assumption of the existence of the epoch of Malthusian stagnation. While repeatedly describing the lethal effects of Malthusian positive checks, a balanced interpretation suggested originally by Malthus is missing.

Summary and conclusions

Using the UNDP data (UNDP, 2011), we have investigated impacts of Malthusian positive checks. We have assumed that a convenient way of measuring the intensity of Malthusian positive checks is to use the Level of Deprivation Index (LDI), which we defined using the well-known Human Development Index (HDI). This approach allows not only for studying impacts of Malthusian positive checks but also for describing them mathematically. Our empirical formulae are simple but mathematical formulae do not have to be complicated to be useful.

First, we have investigated how the intensity of Malthusian positive checks is reflected in the standard of living as represented by the ecological footprint (EF), income per capita, (GDP/cap), levels of severe poverty, access to clean water and access to sanitation facilities. We have found that the ecological footprint (EF) decreases hyperbolically with the intensity of Malthusian positive checks while the GDP/cap decreases exponentially. We have also found that severe poverty, inadequate access to clean water and to sanitation facilities depend linearly on the intensity of Malthusian positive checks. However, we have also found that these linear correlations are characterised by certain thresholds.

Thus, our analysis indicates that severe poverty can be eliminated without the necessity of reducing the intensity of Malthusian positive checks or equivalently the levels of deprivation to zero or close to zero. The level of the severe poverty reaches its zero value at a certain threshold of the intensity of Malthusian positive checks. A significant reduction in the level of severe poverty can be achieved by only a relatively small increase in the average income per capita in poor countries.

We have investigated the lethal effects of Malthusian positive checks and we have found that *mortality* increases *exponentially* with the increasing level of deprivation, i.e. with the increasing intensity of Malthusian positive checks. However, we have found that *total fertility rate* also increases *exponentially* with the increasing intensity of Malthusian positive checks.

One of the important results of our analysis is that the growth rate *increases* with the intensity of Malthusian positive checks. The rate of natural increase also increases. The larger is the intensity of the destructive impacts of Malthusian positive checks, the faster is the growth of population. The destructive impacts of Malthusian positive checks are not just balanced by their regenerating process – they stimulate an even faster growth. These results suggest that the essential step in controlling the growth of population is to reduce the levels of severe poverty. Helping poor countries to help themselves is not an option.

We have also investigated correlations between destructive and regenerating impacts of Malthusian positive checks and again we have derived simple mathematical formulae describing these correlations. We have demonstrated that the intensity of the regenerating process increases with the increasing intensity of the destructive process of Malthusian positive checks. We have derived a general formula showing how the total fertility rate increases with the increasing mortality. We have also derived a simple mathematical formula showing how the rate of natural increase and the growth rate increase with the increasing mortality. We have presented diagrams for the adult mortality but the same formulae apply also to other forms of mortality such as maternal mortality, under-five mortality and the mortality caused by polluted water.

Our investigation shows that contrary to the interpretations based largely on intuition and impressions, growth of population is not controlled by the increased mortality. On the contrary, the increased mortality stimulates growth. Our study suggests that in order to have better control of the growth of human population, levels of deprivation experienced by poor countries should be significantly reduced. The first and the essential step is to improve the economic status of these countries. However, helping poor countries to increase their income per capita is only a partial solution. This step should be accompanied by making a wider range of *accessible* options such as options for education and employment available to people living in poor countries. The improvement of economic status should also go hand in hand with the improvement in gender equality, which will facilitate better family planning. Only by improving the living conditions of poor countries we can hope to have a better, long-term, control of the growth of human population and of its stabilization. Successful control of the growth of human population is essential for controlling our ever-increasing ecological footprint (Ewing, et al., 2010; WWF, 2010) and for finding at least some solutions to the current critical trends shaping the future of our planet (Nielsen, 2006).

Appendix

Table A1. *Mathematical dependence of listed indicators (y) on x = LDI (the intensity of Malthusian positive checks) with the corresponding parameters describing the UNDP data (UNDP, 2011). The table includes also two types of correlations between destructive and regenerating impacts.*

Indicator	Formula	Parameters
Standard of living		
Ecological footprint (EF)	$y = ax^{-1}$	$a = 0.646$
GDP/cap	$y = be^{rx}$	$b = \$71,144$, $r = -6.97$
Population in severe poverty	$y = m(x - n)$	$m = 190$, $n = 0.358$
Population below the poverty line	$y = m(x - n)$	$m = 195$, $n = 0.308$
Multidimensional Poverty Index	$y = m(x - n)$	$m = 1.23$, $n = 0.293$
Pop. with no access to clean water	$y = m(x - n)$	$m = 175$, $n = 0.371$
Pop. with no access to san. facilities	$y = m(x - n)$	$m = 220$, $n = 0.327$
Lethal impacts of positive checks		
Deaths due to polluted water	$y = be^{rx}$	$b = 5$, $r = 9$
Maternal mortality	$y = be^{rx}$	$b = 3.58$, $r = 8.59$
Under-five mortality	$y = be^{rx}$	$b = 2.4$, $r = 6.7$
Adult mortality	$y = be^{rx}$	$b = 62.42$, $r = 2.93$
Regeneratingimpacts		
Total fertility rate	$y = be^{rx}$	$b = 1$, $r = 2.6$
Growth rate	$y = m(x - n)$	$m = 4.3$, $n = 0$
Rate of natural increase (RNI)	$y = m(x - n)$	$m = 3.9$, $n = 0$
Regenerating v lethal impacts		
Total fertility rate (z) v mortality (y)	$\ln z = \ln A + B \ln y$	eqns (7) – (12)
Growth rate or RNI (z) v mortality (y)	$z = \frac{m}{r}[\ln y - \ln b]$	eqn (13)

Table A2. *The dichotomy of Malthusian positive checks reflected in the contrasting levels of hunger stress*

		Destruction			Regeneration		
Country	**LDI**	**DR**	**IM**	**TFR**	**BR**	**RNI**	**PIF**
Hunger Stress: ≥ 35%							
Angola	0.514	15	102	2.7	43	2.8	2.4
Burundi	0.684	10	66	6.4	42	3.2	2.7
Chad	0.672	16	125	6.0	45	2.9	2.4
CARent. Afr.	0.657	15	102	4.7	37	2.1	2.1
DRCongo	0.714	17	111	6.1	45	2.8	2.2
Eritrea	0.651	8	43	4.7	34	2.6	1.9
Ethiopia	0.637	10	77	5.3	37	2.7	2.0
Malawi	0.600	15	84	5.7	42	2.7	2.4
Mozambique	0.678	14	86	5.6	41	2.8	2.6
Sierra Leone	0.674	15	89	5.0	37	2.2	2.5
Zambia	0.570	15	84	6.3	46	3.1	3.3
Average Values	**0.641**	**13.6**	**88.1**	**5.3**	**40.8**	**2.7**	**2.4**
Hunger Stress: < 5%							
Algeria	0.602	5	22	2.3	19	1.5	1.3
Egypt	0.356	5	23	2.9	25	2.0	1.5
Gabon	0.326	9	45	3.4	27	1.8	1.8
Libya	0.240	4	14	2.5	22	1.8	1.4
Morocco	0.418	6	30	2.2	19	1.3	1.2
South Africa	0.381	14	48	2.4	21	0.6	1.1
Tunisia	0.302	6	18	2.1	18	1.2	1.2
Average Values	**0.375**	**9.8**	**40.0**	**3.6**	**30.2**	**2.0**	**1.9**
Ratio	**1.71**	**1.39**	**2.20**	**1.47**	**1.35**	**1.35**	**1.26**

LDI – Level of Deprivation Index; DR – Death Rate; IMR – Infant Mortality Rate; TFR – Total Fertility Rate; BR – Birth Rate; RNI – Rate of Natural Increase; PIF – Population Increase Factor; CAR – Central African Republic; DRC – Demographic Republic of Congo; Ratio – High stress/Low stress.

Death and birth rates are per 1000 of the population. Infant Mortality Rate is defined as "The annual number of deaths of infants under age 1 per 1,000 live births" (PRB, 2010). The Rate of Natural Increase is in percent. Population Increase Factor gives the projected population in 2050 as a multiple of the population in 2011.

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4. Demographic catastrophes did not shape the growth of human population

Introduction

Demographic catastrophes were supposed to have shaped the growth of human population and indirectly also the economic growth because as indicated by Maddison's data (Maddison, 2001; 2006; 2010) these two processes are strongly correlated. Demographic catastrophes were supposed to have been responsible for creating the supposed, but non-existent, epoch of Malthusian stagnation in the growth of population and in the associated economic growth. This concept, which was accepted for decades in the demographic and economic research, has been recently reinforced by Galor and his associates by the deliberately distorted presentation of data (Ashraf, 2009; Galor, 2005a; 2005b; 2007; 2008a; 2008b; 2008c; 2010; 2011; 2012a; 2012b; 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008). We have discussed these issues in earlier publications and we have shown that *precisely the same data*, which were used in their distorted way by Galor and his associates to support their preconceived but erroneous ideas, *are in fact in the direct contradiction of the concept of Malthusian stagnation* (Nielsen, 2014; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h).

The erroneous concept of Malthusian stagnation and takeoffs from the supposed but non-existent Malthusian trap in the demographic and economic growth is based on the incorrect interpretations of hyperbolic distributions. They are indeed slow over a long time and fast over a short time but they increase *monotonically* and there is no place on them where they change suddenly from being slow to being fast. In order to explain the mechanism of hyperbolic growth, hyperbolic distributions have to be treated as a whole. They cannot be divided into two or three different regimes of growth as incorrectly imagined by Galor (2005a; 2011) and by many other researchers.

In the discussion presented here we shall extend our earlier discussions of the growth of human population by concentrating our attention on the possible impacts of demographic catastrophes. We have already explained (Nielsen, 2016i) that in harmony with the observation published by Malthus (1798), his positive checks (demographic catastrophes and harsh living conditions) have a dichotomous effect on the growth of population: they are destructive by increasing the death toll but they are also constructive by triggering the process of regeneration. In the discussion presented here we are going to demonstrate that there is also another reason why demographic catastrophes did not shape the growth of human population: they were generally too weak to have any tangible impact. They might have been strong enough to upset the growth of some local populations but with only one exception discussed earlier (Nielsen, 2016j) when there was an unusual convergence of *five* remarkably strong demographic catastrophes, they had no effect on the growth of global population, or even on the growth of regional populations (Nielsen, 2016d).

The supposed age of pestilence and famine

In one of his publications, Lagerlöf stated that "Throughout human history, epidemics, wars and famines have shaped the growth path of population" (Lagerlöf, 2003a, p. 435). He studied the growth of population in England, France and Sweden using his model of growth, which incorporated the concept of Malthusian stagnation. Similar calculations were carried out earlier by Artzrouni & Komlos (1985) for the world population. These two studies are most interesting because when closely examined they show that the mechanism of stagnation does not produce expected results (Nielsen, 2016k). They did not produce a stagnant state of growth. Lagerlöf missed the opportunity of seeing it because he did not compare his model calculations with data. Artzrouni & Komlos (1985) produced a distribution for the growth of the world population but did not notice that their model generated exponential growth with no signs of stagnation. Furthermore, their results are contradicted by data (Nielsen, 2016k) because the world growth of population was never exponential (Nielsen, 2016d; 2016j).

Lagerlöf carried out Monte-Carlo calculations, which were supposed to confirm the existence of the epoch of Malthusian stagnation in the growth of population supposedly caused by the effects of demographic catastrophes, such as epidemics, wars and famines. He incorporated explicitly the mechanism of stagnation in his model. Consequently, his model should have been expected to produce the process of stagnation but *it did not*. Before the publication of Lagerlöf's paper, data for the United Kingdom, France and Sweden were already available (Maddison, 2001) but unfortunately Lagerlöf did not compare his model-generated calculations with these most essential data.

These data are shown in Figure 1. Their analysis demonstrates that data for the UK and France follow hyperbolic trajectories. For Sweden, there was a change from a hyperbolic distribution to exponential growth. All these data and their analysis demonstrate that there was *no stagnation* in the growth of population and that contrary to the original assumption of Lagerlöf, "epidemics, wars and famines" *did not shape* "the growth path of population". The past growth may have been slow but it was not stagnant. It was slow because it was hyperbolic. It then became fast because it was hyperbolic. Only in Sweden it was diverted to a faster new trajectory but it was not a transition from stagnation to growth but a transition from growth to growth, from a hyperbolic growth to an exponential growth.

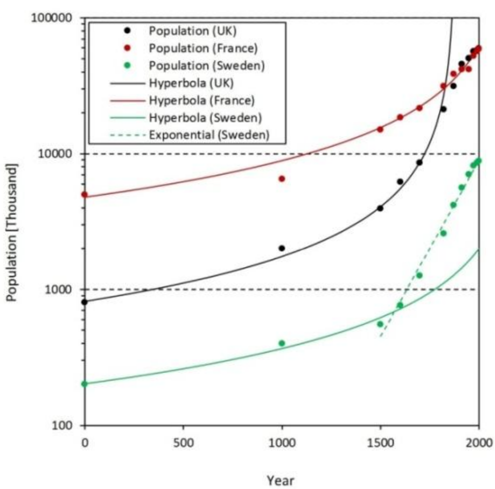

Figure 1. *Population growth in the United Kingdom, France and Sweden. Data (Maddison, 2001) are compared with hyperbolic distributions. Population growth was increasing monotonically. It was slow in the past because it was hyperbolic. For Sweden, there was a change from a slow hyperbolic growth to a faster exponential growth around AD 1600. There was no stagnation. The growth of population was not shaped by demographic catastrophes, as claimed by Lagerlöf (2003a).*

For France, the growth of population was following closely hyperbolic distribution at least until around 2000. For the United Kingdom, the growth was hyperbolic until around 1820, when it started to be diverted to a *slower* trajectory. According to the generally accepted interpretations of the mechanism of the growth of human population, we should expect a significant boosting (takeoff or explosion) around the time of the Industrial Revolution, 1760 and 1840, (Floud & McCloskey, 1994). This takeoff should be clearly indicated in the United Kingdom, the very centre of this revolution, where its impacts should have been most clearly demonstrated. The takeoff did not happen. On the contrary, in the direct contradiction of these usually claimed expectations, the growth of population in the UK started to be diverted to a *slower* trajectory at around 1820, right at the time when it was supposed to have been boosted.

In Sweden, the growth of population was boosted but it was boosted at a wrong time, around AD 1600, i.e. well before the Industrial Revolution. The boosted growth follows an exponential trajectory, as indicated by the straight line in this semi logarithmic display.

Hyperbolic distributions displayed in Figure 1 are described by the following simple equation:

$$S(t) = \frac{1}{a - kt}, \tag{1}$$

where $S(t)$ is the size of the population, t is time, while a and k are the parameters determined by fitting hyperbolic distributions to data.

For the hyperbolic distributions displayed in Figure 1, parameters are: $a = 1.221 \times 10^{-3}$ and $k = 6.511 \times 10^{-7}$ for the UK, $a = 2.085 \times 10^{-4}$ and $k = 9.635 \times 10^{-8}$ for France and $a = 4.935 \times 10^{-3}$ and $k = 2.221 \times 10^{-6}$ for Sweden.

Exponential distribution describing the growth of population in Sweden from AD 1600 is given by the following equation:

$$S(t) = be^{rt}, \tag{2}$$

with parameters $b = 5.798 \times 10^{-2}$ and $r = 5.973 \times 10^{-3}$. From AD 1600, population in Sweden was increasing at an approximately constant rate of 0.6%. Just before the transition to this new trend, the growth rate for the preceding hyperbolic distribution was only 0.16%. The new exponential trajectory was approximately 3.7 times faster than the preceding hyperbolic trajectory at the time of the transition.

Hyperbolic growth is often described as "faster-than-exponential" or "hyper-exponential". Such descriptions should be avoided. They are inaccurate and misleading. The concept of the faster-than-exponential growth was introduced, or at least strongly promoted, by Bartlett (1993). However, he has readily admitted that he was wrong: "Thanks for your thoughtful analysis of my writing about faster and slower than exponential. You are right! My wording is unclear and confusing and wrong. I have used these terms for years and you are the first person to point out this error to me" (Bartlett, 2011). If only the erroneous concepts adopted in the economic and demographic research could be so readily corrected, we would see progress in these two fields of study, rather than the existing and long-lasting stagnation.

In the example presented in Figure 1, exponential growth in Sweden after around AD 1600, is faster than the preceding hyperbolic growth and thus, in this case, hyperbolic growth (the so-called "faster-than-exponential" growth) is in fact slower than exponential.

We can only compare *specific* distributions and see which of them are faster or slower. Faster-than-exponential distributions do not exist because we can always design exponential growth, which over a certain time will be faster than some other incorrectly claimed faster-than-exponential growth.

If we have to use the expression "faster-than-exponential" we have to be specific. We have to describe clearly, which specific distributions are being compared and over specifically what range of the independent variable. Thus, for instance, for the distributions shown in Figure 1 for Sweden we could say that over the range of the displayed time, the exponential growth, which commenced in 1600 was faster than the preceding hyperbolic growth. However, we obviously cannot claim that the hyperbolic growth before 1600 was "faster-than-exponential" or "hyper-exponential" because we have already demonstrated that in this particular case this so called "faster-then-exponential" or "hyper-exponential" growth was obviously slower than the exponential growth, which replaced this hyperbolic growth.

Lagerlöf did not invent the concept of Malthusian stagnation, which is supposed to be caused by the lethal effects of demographic catastrophes. He just accepted it without any criticism maybe because going with the flow increases the chance of publishing new results. When in one of his papers, published also in 2003, Lagerlöf was associating the hypothetical epoch of

Malthusian stagnation with "epidemic shocks" he was quickly corrected by a referee for missing the effects of wars: "As suggested by a referee, this process could possibly be interpreted in terms of wars, instead of epidemics" (Lagerlöf, 2003b, p. 766).

Both, Lagerlöf and his referee were wrong. The process of Malthusian stagnation cannot be interpreted "in terms of wars, instead of epidemics" because as shown by data presented in Figure 1, Malthusian stagnation did not exist. However, neither Lagerlöf nor his referee cared to consult the relevant data. Data appear to be of lesser importance than the mantra of stagnation.

Unfortunately, this mantra is repeated without any convincing justification in the economic and demographic research, and every effort is made to make sure that it is repeated faithfully and as required. As mentioned earlier, Lagerlöf did not compare his model-calculations with data, but his referee was also misguided because the doctrine of Malthusian stagnation is repeatedly contradicted by data (Nielsen, 2016a; 2016d; 2016j).

According to the established knowledge in demography and in economic research, "The age of pestilence and famines lasted until 1875" (Rogers & Hackenberg, 1987, p. 234) when there was supposed to have been a transition from stagnation to a fast growth, the transition described usually as a takeoff or explosion. It is unclear how this precise date was determined but it might have been suggested by the generally accepted but erroneous notion of the supposed transition from stagnation to growth around the Industrial Revolution, 1760 and1840 (Floud & McCloskey, 1994). Analysis of data shows convincingly that there was no stagnation in the growth of population and in the economic growth, and that there was no transition, which could be described as a takeoff or explosion. What is interpreted as an explosion is just the natural continuation of hyperbolic growth (Nielsen, 2016a; 2016d; 2016j).

The mythical age of pestilence and famines was supposed to have been characterised by what is known as Malthusian oscillations. According to this doctrine "…periodic epidemics of plague, cholera, typhoid and other infectious diseases would in one or two years wipe out the gains made over decades. Over long periods of time there would, consequently, be almost no population growth at all" (van de Kaa, 2010, p. 87). "The pattern of growth [of human population] until about 1650 is cyclic" (Omran, 1971, Table 4, p. 533). Here we have a different date for the termination of the age of pestilence and famine, which is hardly surprising because these dates are based on impressions combined with a good dose of imagination. The age of pestilence and famine, with its assumed strong effects on the growth of population and on the economic growth, did not exist.

The supposed, but non-existent, transition from stagnation to growth was supposed to have been associated with the transitions in the birth and death rates. It is interesting, however, that while Omran shows examples of the claimed transitions in birth and death rates, his examples (for Sweden, England, Japan, Ceylon and Chile) show clearly and convincingly that these transitions had absolutely no impact on the growth of population (Omran, 2005).

Changes in birth and death rates are not necessarily reflected in changes in the growth of population. The growth of population is not determined by the birth and death rates alone but the *average difference*, i.e. by the *average gap*, between these two quantities. Birth and death rates might be changing from high to low but such changes will not be reflected in the growth of population unless the average difference between them is also changing. Furthermore, small changes in the difference between birth and death rates are also not reflected as the associated changes in the growth of population (Nielsen, 2016l).

Birth and death rates might be decreasing but if they are decreasing in such a way that the average difference between them is approximately constant, the growth of population will be approximately exponential. If the difference increases systematically, then the growth of population will be described by a non-exponential trajectory. For instance, if the difference increases, on average, hyperbolically, then the growth of population will be hyperbolic.

To produce stagnation, the average difference between birth and death rates has to be approximately *zero*. To produce a stagnant but slowly increasing population, the average difference between birth and death rates would have to be changing in a very specific and complicated way. It would have to be on average zero over a long time but then it would have to be on average non-zero to generate growth. Then again it would have to revert back to zero to produce stagnation. This process would have to be repeated over a long time for thousands of years. We do not have data to demonstrate that such a process ever existed. We do not have data for birth and death rates extending over thousands of years. The claim that birth and death rates were high and that they were producing stagnation is *unscientific* because we do not have data to prove it. However, we have a large body of data describing the growth of population and we can show that the growth of population was in general hyperbolic, which by inference means that the average difference between birth and death rates was increasing hyperbolically. *There was no stagnation*.

To demonstrate a dramatic change from stagnation to growth in the growth of population we would have to demonstrate that there was a dramatic change in the average difference between birth and death rates from zero to a clearly and systematically larger value, and it does not matter whether birth and death rates were decreasing or increasing. What determines the growth of population is the average difference between birth and death rates.

If we want to claim some kind of transitions in birth and death rates, there is nothing to stop us from doing it. However, we should remember that, in general, such studies will not help us to understand the mechanism of growth of population. They might be interesting and stimulating for another reason but unless we pay close attention to *how* the difference between these two quantities is changing, how large or how systematic are these changes, and how these changes can be explained, we shall not explain the mechanism of growth of human population. We also should remember that only *significant* changes in the difference between birth and death rates are reflected as changes in growth trajectories.

The frequently used example in support of the concept of stagnation followed by explosion is the growth of population in Sweden between around AD 1750 and 2000. It shows changes in the difference between birth and death rates but no-one seems to have noticed that these changes are relatively small. Furthermore, no-one seems to have noticed that these small changes are not reflected in the growth of population, even though data for the birth and death rates and for the growth of population come *from exactly the same source* (Statistics Sweden, 1999). These data are selectively and consistently ignored in order to preserve the perfect intonation of the mantra of Malthusian stagnation.

Small changes in the average values of birth and death rates are repeatedly but incorrectly interpreted as a proof of the existence of the epoch of Malthusian stagnation and of a transition from stagnation to growth while data presented *in the same primary source* show clearly that the growth of population in Sweden was *increasing monotonically without any signs of stagnation* and without any sign of a transition from stagnation to growth. These issues were discussed earlier (Nielsen, 2016l).

Demographic research concentrating on the study of birth and death rates might be important but it is incorrect to think that such a research can be necessarily useful for explaining the mechanism of growth of human population. The two mechanisms are related only via the average difference between birth and death rates.

There might be strong fluctuations in birth and death rates but these fluctuations are generally not reflected in the growth of population. They might be reflected only as minor variations in the growth trajectory describing the growth of population.

In conformity with the established knowledge, Komlos claimed that "Malthusian positive checks (mortality crises) maintained a long-run equilibrium between population size and the food supply" (Komlos, 1989, p. 194). Here we have a hinted link to the specific type of demographic catastrophes: famines. He also claimed that "the food-controlled homeostatic equilibrium had prevailed since time immemorial" (Komlos, 2000, p. 320). Komlos appears to have been guided by the generally accepted consensus. However, science never relies on any generally accepted consensus. It is not unusual in science to show that the generally accepted consensus is scientifically unacceptable.

The postulate of Malthusian stagnation in the economic growth and in the growth of human population, as well as all other related postulates, are scientifically unacceptable. because they are systematically contradicted by data (Nielsen, 2014; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f; 2016g; 2016h; von Foerster, Mora & Amiot, 1960). Growth of population, global or regional, was hyperbolic, (Nielsen, 2016d; 2016j; von Foerster, Mora & Amiot, 1960). Economic growth was also hyperbolic (Nielsen, 2016a).

In the case of the growth of human population we can extend our study to 10,000 BC. It is remarkable, that over the past 12,000 years the growth of population was not only hyperbolic but also exceptionally stable (Nielsen, 2016j) because over this long time there was *only one major* transition around AD 1 from a fast to a slow hyperbolic trajectory. There was also another but only *minor* transition around AD 1300 from a slower to a slightly faster hyperbolic growth. Currently we are experiencing a new transition to a yet unknown trajectory but the growth is still close to the historical hyperbolic trajectory.

We can extend the analysis of the growth of population even further, over the past 2,000,000 years and show that the growth was hyperbolic (Nielsen, 2017). There is nothing in the data to support the claim that "Throughout human history, epidemics, wars and famines have shaped the growth path of population" (Lagerlöf, 2003a, p. 435).

We already know that Malthusian positive checks, which include demographic catastrophes, trigger the process of regeneration (Malthus, 1798; Nielsen, 2016i). This process alone, explains the remarkable stability of the growth of human population. However, in order to understand even better why demographic catastrophes had generally no impact on the growth of population we shall now investigate their relative intensity and other parameters defining their possible impact.

Preliminary remarks

Impacts of demographic catastrophes depend on the *death toll,* their *duration* and on the *size* of population. Death toll for a given demographic event might be high but to understand its impact we have to express it as *the relative impact* by comparing the death toll with the size of population, which could be the size of local population directly affected by a demographic crisis or it could be the size of a regional or global population, depending on whether we are interested in the study of local, regional or global impacts.

Impacts of demographic catastrophes depend also on the historical time. In the distant past, when the population was small, local impacts of demographic catastrophes could be large. However, people were living in greater isolation so the global or even regional impacts could have been small. Likewise, at the other end of the historical time scale, when the population increased to a certain large size, relative impacts were small even if the number of people killed by a given demographic catastrophe was large. It can be, therefore, expected that there is only a relatively small window of time, mainly during the AD era until around 1800, when the global population reached its first billion, or maybe until around 1900, that the demographic catastrophes could have had a noticeable impact on the growth of population. However, the study of human population shows that in general they had no damaging impact, with the exception of the already mentioned minor disturbance around AD 1300 (Nielsen, 2016j).

The further we go back in time with our investigation the less we know about the intensity of demographic catastrophes but we have enough information for the AD era to assess their possible impacts.

In order to understand human population dynamics, it is essential to identify the *main* and the most obvious driving force of growth and add to it any other force or forces only if the assumed main force cannot explain growth. The fundamental force of growth of human population is obviously the force of procreation expressed as the difference between the *biologically-controlled* force of sex drive and the *biologically-controlled* process of aging and dying. This force cannot be dismissed and it turns out that this force alone explains why the spontaneous and unconstrained growth of human population is hyperbolic and why for the most part of the past human history it was hyperbolic (Nielsen, 2016m).

In the past 12,000 years, other forces were playing a significant role only during the major demographic transition around AD 1 and during the minor transition around AD 1300. They are also strong and active during the current transition. With the exception of these rare events, the growth was hyperbolic in the past 12,000 years. Furthermore, with the exception of the minor disturbance around AD 1300, there is no evidence that demographic catastrophes were ever shaping the growth of human population (Nielsen, 2016d; 2016j).

It should be also noted that the recorded impacts of demographic catastrophes are likely to be exaggerated. Recorded death rates "are largest when the supporting evidence is skimpiest. When data are better, the death rates are usually lower and the percentage increases less" (Watkins & Menken, 1985, p. 651). For instance, both Durand (1960) and Fitzgerald (1936; 1947) claim that impact of the An Lu-Shan Rebellion (AD 756-763) is probably exaggerated. Likewise, Russel (1968) and Twigg (1984) believe that the number of casualties caused by the Justinian Plague (AD 541-542) is also grossly overestimated.

Another example is the Antonine Plague (AD 166-270), which was first estimated to have killed about 50% of the population of the Roman Empire (Seeck, 1921). However, this estimate was later downgraded to 1-2%, or to the total number of casualties of 500,000-1,000,000 (Gilliam, 1961) and then upgraded to 7-10% or to a maximum of 5 million (Littman & Littman, 1973), the last estimate being still significantly smaller than the original estimate. It appears that the further back in time we go the larger is the possibility of exaggerated claims of the number of casualties.

We shall describe demographic catastrophes in the way they are reported in the literature. However, labelling them with just a single cause might not be accurate. For instance, a war considered as the main cause of a crisis might include famine but famine might be linked with pestilence. For example, during the Madras famine in the 1870s, about 40% of casualties were caused by smallpox and cholera (Lardinois, 1985). The Justinian Plague was also accompanied by smallpox, diphtheria, cholera and influenza (Shrewsbury, 1970) and was "perhaps aided by wars, famines, floods and earthquakes" Scott & Duncan (2001, p. 5). Likewise, "a number of epidemics in France were preceded by famine, sometimes in conjunction with bad weather conditions" (Scott & Duncan, 2001, p. 105) whereas "frequent and virulent outbreaks in France during 1520-1600 were accompanied by food shortages, famines, flooding, peasant uprisings and religious wars" (Scott & Duncan, 2001, p. 291).

While drawing from primary sources about the frequency and intensity of demographic catastrophes, the presented here survey has been also assisted by some useful compilations (Austin Alchon, 2003; Kohn, 1995; Spignesi, 2002; White, 2011).

Examples of prominent demographic catastrophes

One of the earliest recorded devastating plagues was the Asiatic disease identified now as *tularaemia*, a bacterial disease caused by *Francisella tularensis*, first recorded around the early 1700s BC. It spread over a large area between Cyprus and Iraq and between Palestine and Syria. This disease appears to be also the first recorded example of the use of biological weapon when it was introduced deliberately to Anatolia (Trevisanato, 2004; 2007). The same disease has been also probably recorded in the Bible as causing a great number of deaths among Philistines in the city of Ashdod, the event dated either to around 1000 BC (Khan, 2004) or to 1320 BC (Cunha & Cunha, 2006).

Early recorded plagues include also a viral haemorrhagic fever in Egypt between 1500-1350 BC (Duncan & Scott, 2005) but it might have been the same disease as recorded earlier in Egypt and the same plague that decimated Philistines. Incidentally, Duncan & Scott (2005) claim that Black Death was not a bubonic plague caused by bacterium *Yersinia pestis*, as traditionally claimed, but rather that is was a viral haemorrhagic fever, which according to them includes also the plagues of Mesopotamia (700-350 BC), the Plague of Athens (430-427 BC), the Plague of Justinian (AD 541-542), Plagues of Islam (AD 627-744), plagues in Asia minor (1345-1348), and the plague of Denmark and Sweden (1710-1711).

The epidemic of Athens (460-399 BC) is claimed to have killed 25% of Athenian army and a great number of civilians (Austin Alchon, 2003). It created a turning point in the history of Greece (Ross, 2008). It is also claimed that this plague killed 50% of the army of Pericles and 50% of the navy coming to the rescue from Piraeus (Beran, 2008). The plague was triggered by the overcrowding of Athens when Spartan's attacks prompted rural population to seek shelter in that city, which was already housing a relatively large number of people, an estimated 300,000 citizens and around 3 million slaves.

The earliest large demographic catastrophe in the AD era appears to have been associated with the Red Eyebrows Revolt, which commenced around AD 2. The estimated size of Chinese population at that time is claimed to have been 59.6 million but it might have been reduced to 21 million in AD 57 (Durand, 1960). However, Durand also discusses possible inaccuracies in these estimates and presents corrected numbers of 74 million in AD 2 and 45 million in AD 88, for the entire Chinese Empire. He also estimates 71 million and 43 million, respectively, for the China proper (Durand, 1960, p. 221). By China proper he means the current 18 provinces. He uses this estimate in his graph (Durand, 1960, p. 247). In both cases, the relative death toll is approximately 39% of the original population in China but only a maximum of 12% of the global population, too weak to produce any noticeable impact.

The Red Eyebrows Revolt and the associated dramatic decrease in the size of population in China was in the middle of a massive demographic transition, one and only major demographic transition in the past 12,000, a transition from a fast to a slow hyperbolic trajectory, the transition which lasted for approximately 1000 years. This transition is shown in Figure 2. The dramatic event in China had no impact on the growth trajectory of the world population.

Durand points out also that estimates of the size of the population at the time of demographic catastrophes might be inaccurate. “Even if such huge loss were conceivable, it would be naïve to suppose that accurate count of the survivors could have been carried out in the midst of the ensuing chaos” (Durand, 1960, p. 224). White (2011) attributes only 10 million of casualties to the Red Eyebrows Revolt. However, to estimate the impact of this demographic catastrophe we shall use the revised estimate of Durand (1960) representing the total death toll of 29 million over 87 years.

Similar uncertainty in the estimated death toll applies also to the An Lu-Shan Rebellion (AD 756-763). Acceptable records appear to show the death toll of 36 million but White (2011) attributes only 13 million.

> Between A.D. 705 and 755 to all appearances the census machinery functioned much more effectively; but after 755 it broke down again. The recorded number of persons dropped from nearly 53 millions in the year 755 to only 17 millions in 760. During this time, China was torn by revolts which were suppressed with bloody force, including the notorious rebellion of An Lu-Shan. Many historians have affirmed that 36 million lives were lost as a result of these violent events, but Fitzgerald and others have shown that this is incredible (Durand, 1960, p. 223; Fitzgerald, 1936, 1947).

In order to maximise the possible impact of this demographic crisis, we shall assume that the death toll was 36 million.

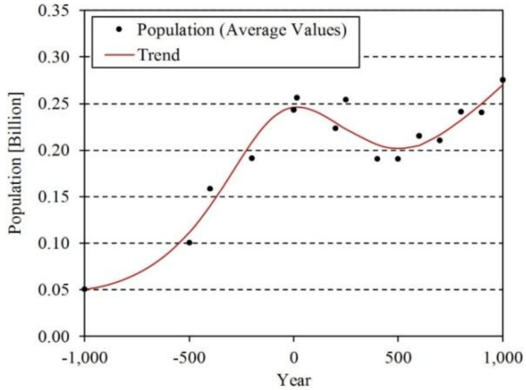

Figure 2. *Demographic transition in the growth of global population around AD 1. The transition can be described by the monotonically changing distribution (Nielsen, 2016j; 2017). Rebellion of An Lu-Shan, which caused a massive reduction in the size of population in China, had no impact on the growth of global population. For the reference to the sources of data and for the description of their analysis see Nielsen (2016j; 2017).*

The impact of the Plague of Justinian is hard to estimate because of the incomplete information combined with conflicting claims. The plague is claimed to have reduced the population of Constantinople by 40% between AD 541 and 542 (Austin Alchon, 2003). Cunha and Cunha (2006) estimated a 30% reduction of the population of the Roman Empire between AD 542 and 590, or a maximum of about 14 million out of the total of 48 million (Maddison, 2006; Seeck, 1921). "The plague so weakened the Roman Empire that not long after the plague had passed, Roman borders were overrun by Huns, Goths, Moors, and other 'barbarians'" (Cunha & Cunha, 2006). Rosen (2007) estimates that this plague killed 25 million people in a short time of only between AD 541 and 542. In around AD 549 the same plague emerged also in Britain (Carmichael, 2009). It is also claimed that "The Plague of Justinian recurred in discernible cycles of about nine to twelve years" (Dols, 1974, p. 373).

There is also one claim, which is distinctly different than all other estimates. Assisted by the Eurasian Silk Road, this plague was supposed to have spread to China in around AD 610, (Ross, 2008) continuing its devastation until around AD 700 (Duncan & Scott, 2005) and killing probably a maximum of about 100 million people (Ross, 2008), which would represent a 50% reduction in the *world* population. Even if we consider the regenerating effects of Malthusian positive checks (Malthus, 1798; Nielsen, 2016i), such a huge reduction should be reflected in the growth trajectory but it is not. By AD 500, the growth of the world population was at the end of its transition (see Figure 2) and commenced its new hyperbolic trend. In AD 500, the estimated size of the world population was only 190 million (Nielsen, 2016j and references therein). The claimed massive death toll of 100 million was supposed to have occurred between AD 610 and 700, i.e. when the growth of the world population settled already along a new hyperbolic trajectory, but we see no sign of such a disturbance. This claim of such a large death toll is almost certainly incorrect.

In our survey, in order to maximise the evidence *in favour* of the concept of Malthusian stagnation, we are considering the strongest impacts, which for the Plague of Justinian appears to be the death toll of 25 million in a very short time, between AD 541 and 542. We shall see later that, under this assumption, this plague had the strongest *overall* impact of all demographic catastrophes ever recorded, as manifested by *four out of five indicators*, and yet it caused no noticeable disturbance in the growth of the world population (see Figure 2).

Black Death (1343-1351) is another example of a massive demographic catastrophe and is claimed to have killed over 60% of the urban population in Asia, about 30% of the population of the Middle East and 30-60% of the population of Europe (Hawas, 2008). Beran (2008) claims that in many cities the death toll was over 90%, creating a severe hardship for the surviving population and adding to the total death toll caused also by the lack of food and lack of access to safe drinking water. The decaying corpses were also reducing the chance of survival. About 20% of the population of England died between AD 1348 and 1350 and a total of 50% by AD 1400 (Gilliam, 1961). Depending on the affected area, mortality rates varied between 25% and 70% (Cunha & Cunha, 2006). In terms of the total and relative death toll, Black Death was the greatest single demographic catastrophe ever recorded.

As mentioned earlier, plagues were also used as biological weapons by employing a gruesome practice of catapulting infected corpses at the walls of fortifications or hurling them over the walls by using trebuchets. This ghastly method was used by Greeks, Romans and other attackers between 300 BC and AD 1100, and by Tartars in AD 1346 against the residents of Genoa (Cunha & Cunha, 2006; Khan, 2004).

Other examples of large local casualties caused by demographic catastrophes include smallpox in Japan (AD 812-814), killing about half of the population of that country (Austin Alchon, 2003);

the 1696 famine killing between 25% and 30% of the population of Finland (Jutikkala, 1955); the 1770 famine in Bengal, killing about 30% of the population (or a total of 10 million) and the 1376 famine in Italy, killing 60% of the population (Ghose 2002; Keys, *et al.,* 1950; Walford, 1878).

According to Mallory (1926), 18 provinces of China experienced 1015 draughts between AD 620 and 1619, or about one per year. However, they were unevenly distributed, illustrating that while the number of casualties and impacts of demographic catastrophes might be high in small and isolated regions, their effects could be much less severe when averaged over a larger number of population.

There was a total of 443 draughts in the Northern Division, 352 in the Central Division and 220 in the Southern Division. However, even within the same division, the number of draughts varied significantly between various districts. For instance, in the Northern Division, Honan District experienced a total of 112 draughts but Kansu Division only 4. In the Central Division, the largest number of draughts (113) was in the Chekiang District and the smallest (28) in the Anhwei District. In the Southern Division, the number of draughts varied between 4 and 59 per district.

The list of significant lethal events in China includes: 60-70% of troops killed during a single military engagement in AD 16; 70% of Mongolians killed by hunger in AD 46; 30-40% of troops killed in AD 162; about 70% of troops killed in a single military engagement and by famine and epidemic; close to 100% killed by locusts and famine in AD 312 in the northern and central China; over 30% killed in Shantung in AD 762; over 50% in Chekiang in AD 806; 30-40% in Hupeh, Kinagsu and Anhui in AD 891; 90% in Hopei in 1331; 50% of troops between 1351-1352; over 70% in Shansi in AD 135; 60-70% in Hupeh in 1354, and 100% in various towns and villages in Hunan in 1484 (Austin Alchon, 2003; McNeill, 1976)

It is claimed that in Mexico, 25-50% of the population died of smallpox (1520-1521), 60-90% probably of typhus (1531-1532), and over 50% of either the bubonic plague or typhus between 1576 and 1581 (Austin Alchon, 2003; Motolinía aka Fray Toribio de Benavente o Motolinía, 1971; del Paso y Troncoso, 1940; Prem, 1992).

The estimated death toll in the Andes between 1524 and 1591 includes 30-50% by smallpox (1524-1527), 25-30% by measles or bubonic plague (1531-1533), 15-20% by influenza, measles and smallpox (1558-1559), and about 50% by influenza, measles, smallpox and typhus between 1585 and 1591 (Cook, 1981;

Dobyns, 1963). Dobyns (1993) gives also many examples of large death tolls, sometimes as high as 98% but most often close to 80-90%, caused by diseases among Native American population.

So, it appears that humans always lived with the threats and with deadly effects of demographic catastrophes strong enough to reduce often substantially the size of local populations. We shall now investigate their potential impact on the growth of the world population.

Indicators of impact

In order to study the potential impacts of demographic catastrophes we have to introduce a few useful gauge indicators. Their definition is assisted by the diagram presented in Figure 3.

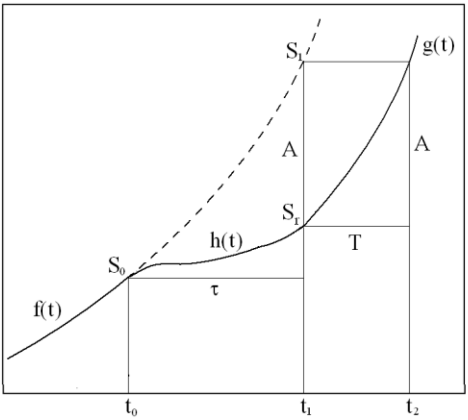

Figure 3. *Schematic diagram describing the composition of a demographic catastrophe. The leading parameters are: τ – the duration of demographic catastrophe; T – the recovery time; A – the death toll.*

Before the onset of a demographic catastrophe, the population increases along the trajectory $f(t)$. It reaches the size S_0 at the time t_0, which marks the beginning of the demographic catastrophe. The demographic crisis lasts for τ number of years, between the time t_0 and t_1. Depending on the intensity of the demographic catastrophe and on the efficiency of the process of regeneration (Malthus, 1798; Nielsen, 2016i), the growth of the population during the demographic crisis may be diverted to a new trajectory $h(t)$, which might be still increasing, remain constant or decreasing. At the end of crisis, the size of the population is S_r, which might be larger than, equal to, or smaller than the original size S_0. S_1, is the size of the population, which would have been reached if the crisis did not occur

When the crisis is over, the growth of population continues along a new trajectory $g(t)$ The quantity A is the death toll and T is the recovery time, i.e. the time required for the population to reach the size S_1.

Recovery time depends on the growth rate, R, during the time of crisis. Over a relatively small span of time associated with demographic catastrophes we can use linear approximations of the relevant trajectories.

$$R \equiv \frac{1}{S_r}\frac{dg(t)}{dt} \approx \frac{1}{S_r}\frac{A}{T} \quad , \tag{3}$$

which gives

$$T = \frac{a}{R}, \tag{4}$$

where

$$a \equiv \frac{A}{S_r} \approx \frac{A}{S_0} \tag{5}$$

is *the relative impact*, i.e. the number of people killed by the demographic catastrophe as compared with the size of the population at the onset of crisis.

The growth rate R can be estimated by examining the population data around the time of crisis while the quantity a can be easily calculated using the reported number of people A killed by the crisis and the estimated size of the population at the beginning of crisis. Using these readily accessible quantities we can then calculate the recovery time T, which together with a (the relative number of people killed during demographic crisis) will help to gauge the intensity of the demographic catastrophe.

Another way of calculating the recovery time T is to use the exponential rather than linear approximation for the function $g(t)$. Under this assumption and using the well-known expression for the exponential function [eqn (2)] we can easily show that

$$T = \frac{1}{R}\ln\left(\frac{S_1}{S_r}\right) = \frac{1}{R}\ln(1 + a) \quad (6)$$

This is a general formula that does not have to be related to a demographic crisis. It is simply a formula for calculating the time needed for the exponential growth to increase from S_r to S_1, which happens to be precisely what we want to use to calculate the recovery time. For small *a*, the recovery time calculated using eqn (6) is virtually the same as by using the eqn (4). Thus, for instance, for $a = 20\%$, the recovery time calculated using eqn (6) is only 10% smaller than using eqn (4). For lower values of *a*, the discrepancy is even smaller. It increases to 23% for $a = 50\%$. As we shall soon see, in our survey of demographic catastrophes we shall be dealing with *a* values of up to only 20%.

If we use the hyperbolic approximation, then referring to the eqn (1), the recovery time is given by

$$T = \frac{A}{(S_r + A)kS_r} . \quad (7)$$

If we want to use an approximate expression incorporating the relative impact *a*, then using the eqn (1) and (5) we get

$$T = \frac{a^2}{A(1+a)k} . \quad (8)$$

So now rather than using just the parameter *A* (the total death toll) we have two additional gauge indicators, *a* and *T*, the parameters that give us additional information about the intensity of crisis.

However, we can also introduce yet another useful gauge indicator, which compares the recovery time with the duration of the demographic catastrophe. We shall call it the *intensity indicator (I)* and we shall define it simply as

$$I \equiv \frac{T}{\tau} . \quad (9)$$

If the recovery time T is large when compared with the duration of crisis, then we are dealing with a potentially strong demographic catastrophe. The larger was the recovery time compared with the duration of crisis the stronger was the devastating impact of crisis.

However, there is also another hidden information in this indicator. Using the diagram presented in Figure 3 and assuming that gradients of functions $f(t)$ and $g(t)$ are approximately the same, we can see that

$$I \approx \frac{A}{S_1 - S_0}. \tag{10}$$

If $A < S_1 - S_0$, then $I < 1$, which then indicates that population size continued to increase during crisis. If $A \approx S_1 - S_0$, then $I \approx 1$, which then indicates that the size of population remained approximately constant during crisis. If $A > S_1 - S_0$, then $I > 1$, which indicates that the size of population was decreasing during crisis.

Thus, by looking at the I indicator we can tell not only whether the crisis was weak or strong but also whether the population was still increasing, remained constant or decreasing during the crisis. However, even if $I > 1$ it does not mean that we a dealing with a potentially strong crisis, because depending on the duration of crisis the size of the population could still remain approximately constant. A potentially strong demographic crisis will be characterized by $I >> 1$. A guide to the intensity indicator is presented in Table 1.

Finally, we can also introduce two other indicators: *the per annum relative impact* (α) and *the per annum intensity indicator* (β).

$$\alpha \equiv \frac{a}{\tau}, \tag{11}$$

$$\beta \equiv \frac{I}{\tau}. \tag{12}$$

The complete list of indicators used to evaluate the effects of demographic catastrophes is presented in Table 2.

Table 1. *A guide to the interpretation of the intensity indicator (I)*

$I < 1$	Weak crisis. Population continues to increase during crisis.
$I \approx 1$	Moderate crisis. Population size remains approximately constant during crisis.
$I > 1$	Moderate or potentially strong crisis depending on the value of *I*. Population size decreases during crisis.
$I >> 1$	Potentially strong crisis

Table 2. *Indicators used to process information about demographic catastrophes*

Symbol/Definition	Description	Unit
t_0	The onset of crisis	*year*
A	Death toll	10^6
τ	Duration of crisis	*year*
$a = A / S_0$	The fraction of the population killed by crisis	%
$\alpha = a / \tau$	Per annum fraction of population killed by crisis	%/*year*
T	Recovery time	*year*
t_2	The year of the full recovery	*year*
$I = T / \tau$	Intensity indicator	
$\beta = I / \tau$	The per annum intensity indicator	$year^{-1}$

Note: Demographic catastrophe may be considered potentially strong if indicators $a = A / S_0$, $\alpha = a / \tau$, $T, I = T / \tau$ and $\beta = I / \tau$ are large.

Evaluation of impacts of demographic catastrophes

The survey of demographic catastrophes and their impacts is presented in Table 3. The summary of all impacts is shown in Table 4. In order to maximise evidence in favour of the postulate of Malthusian stagnation we have considered only the most significant demographic catastrophes characterised by the death toll of $A \geq 1$ million. Had we included smaller demographic catastrophes, the fraction of potentially strong catastrophes, which could have had noticeable impact on the growth of the world population, would have been significantly reduced.

The remarkable feature of this survey is that, in general and as revealed by the values of the introduced gauge indicators, even large catastrophic events had much smaller impact on the growth of the world population than it might have been expected by looking just at the death toll or at their reported local impacts. Indeed, we only have a few events that might have had a tangible impact, and they are all clustered around the early years of the AD era when the estimates of the total number of casualties were probably grossly exaggerated (Durand, 1960; Fitzgerald, 1936; 1947; Gilliam, 1961; Littman & Littman, 1973; Russel, 1968; Twigg, 1984, Watkins & Menken, 1985).

Table 3. *Survey of major demographic catastrophes AD 1-1900. The most significant values of gauge indicators are indicated by bold characters and moderately significant by italics. (Symbols are explained in Table 2.)*

Event	t_0	t_2	A	τ	a	α	T	I	β
Red Eyebrows Revolt	2	245	29.0	87	*11.5*	0.13	**157.5**	*1.8*	0.02
Antonine Plague	166	214	5.0	15	2.2	0.15	*34.1*	*2.3*	0.15
Plague of Justinian	541	756	25.0	2	*12.5*	**6.23**	**214.2**	**107.1**	**53.54**
An Lu-Shan Rebellion	756	845	36.0	8	**15.4**	*1.93*	**227.7**	*28.5*	*3.56*
N. Egypt Earthquake	1201	1206	1.5	1	0.5	0.46	4.8	*4.8*	*4.80*
Mongolian Conquest	1260	1405	40.0	35	*11.3*	0.32	**110.6**	*3.2*	0.09
Great European Famine	1315	1336	7.5	3	2.0	0.68	*19.1*	*6.4*	*2.13*
Famine in China	1333	1369	9.0	15	2.4	0.16	*21.8*	*1.5*	0.10
Black Death	1343	1530	75.0	9	19.7	*2.19*	**178.7**	*19.9*	*2.21*
Fall of the Yuan Dynasty	1351	1385	7.5	18	1.9	0.11	*17.4*	1.0	0.05
Sweating Sickness	1485	1556	3.0	67	0.6	0.01	4.6	0.1	0.00
Mexico Smallpox Epidemic	1520	1527	4.0	2	0.8	0.42	6.0	*3.0*	*1.50*
French Wars of Religion	1562	1602	3.0	37	0.6	0.02	3.7	0.1	0.00
Russia's Time of Trouble	1598	1619	5.0	16	0.9	0.06	5.6	0.3	0.02
Fall of the Ming Dynasty	1618	1669	25.0	27	4.3	0.16	*25.2*	0.9	0.03
Thirty Years War	1618	1655	7.0	31	1.2	0.04	7.0	0.2	0.01
Deccan Famine in India	1630	1633	2.0	2	0.3	0.17	2.0	1.0	0.50
Famine in France	1693	1696	2.0	2	0.3	0.15	1.5	0.8	0.38
Bengal Famine	1769	1778	10.0	5	1.2	0.23	4.7	0.9	0.19
Napoleonic Wars	1803	1816	4.0	13	0.4	0.03	1.4	0.1	0.01
Famines in China	1810	1819	22.5	2	2.3	1.13	7.9	*3.9*	*1.96*
Great Irish Famine	1845	1850	1.0	6	0.1	0.01	0.2	0.0	0.01
Famine in China	1846	1849	11.3	1	1.0	0.96	2.8	*2.8*	*2.83*
Taiping Rebellion	1850	1868	20.0	15	1.6	0.11	4.5	0.3	0.02
Famine in India	1866	1866	1.0	1	0.1	0.08	0.2	0.2	0.20
Famine in Rajputana	1869	1869	1.5	1	0.1	0.11	0.3	0.3	0.29
Famine in Persia	1870	1871	2.0	2	0.1	0.07	0.4	0.2	0.10
Famine in N. China	1876	1880	13.0	3	0.9	0.31	2.3	0.8	0.25
British India Famine	1876	1903	17.0	25	1.1	0.05	2.6	0.1	0.00
Yellow River Flood	1887	1887	2.0	1	0.1	0.13	0.3	0.3	0.31
Famine in India	1896	1902	8.3	6	0.5	0.08	1.1	0.2	0.03

Table 4. *Summary of impacts of demographic catastrophes.*

Indicator	Impact	Number of Events	Fraction of Total [%]	Insignificant [%]
Relative impact (a)	Strong	2	6	
	Moderate	3	10	
	Negligible	26	84	94
Per annum relative impact (α)	Strong	1	3	
	Moderate	2	6	
	Negligible	28	91	97
Recovery time (T)	Strong	5	16	
	Moderate	5	16	
	Negligible	21	68	84
Intensity indicator (I)	Strong	1	3	
	Moderate	11	35	
	Negligible	19	62	97
Per annum intensity indicator (β)	Strong	1	3	
	Moderate	7	23	
	Negligible	23	74	97
The average of all five	Strong	2.0	6.5	
	Moderate	5.6	18.1	
	Negligible	23.4	75.4	93.5

Note: The attribute described as *strong* should be interpreted as *potentially strong* or *the strongest* of all impacts. This attribute does not identify impacts, which had a strong impact on the growth of population but only impacts, which were potentially strong enough to have a noticeable effect.

The leading indicator is the relative impact a because it gives the direct information about how the growth trajectory might have been affected by a given individual demographic catastrophe. Events for which a is less than or equal to around 10% can be ignored, because such displacements would be hardly noticeable on the trajectories describing the growth of population. The corresponding demographic catastrophes could be described as negligible. Even events with a up to around 20% could be expected to have only relatively small effect. However, in this survey we have two events (An Lu-Shan Rebellion and Black Death), with the relative impact of 15.4% and 19.7%, which we shall describe as having a potentially strong impact. They account for only 6% of all impacts. Thus 94% of all large demographic catastrophes, i.e. catastrophes with $A \geq 1$ million, were individually too weak to have a significant impact on the growth of the world population.

We should remember, however, that we are ignoring the spontaneous process of regeneration (Malthus, 1798; Nielsen, 2016i). By describing a crisis as strong we are only distinguishing it from other catastrophes. A strong crisis is only relatively strong or potentially strong. It is a crisis, which could have been reflected in the growth of population but considering the ever-present mechanism of regeneration its impact is likely to be significantly reduced.

If we consider the per annum impact measured by the indicator α, we can see that there was possibly only one event (Plague of Justinian) that might have had a relatively strong impact on the growth of the population and two (An Lu-Shan Rebellion and Black Death) that might have had a marginal impact. Thus, when measured by this indicator, 97% of all large demographic catastrophes had insignificant effect on the growth of the world population.

The recovery time (T) shows five significant events (Red Eyebrow Revolt, Plague of Justinian, An Lu-Shan Rebellion, Mongolian Conquest and Black Death). For all of them, the estimated recovery time was between around 100 and 200 years. They represent 16% of all demographic catastrophes, the largest fraction in this survey. However, even for this indicator, the fraction of negligible events is high, 84%. The majority of all large critical events could have potentially inflicted only negligible impact on the growth of population.

The intensity indicator (I) suggests only one prominent event (Plague of Justinian) and possibly 11 moderately strong events. This indicator, therefore, shows that 97% of all large demographic catastrophes could have had, at best, only small impact on the growth of the world population. For the per annum intensity indicator (β), the fraction of insignificant impacts is the same, 97%.

If we consider the average values of all five indicators we can see that only 6.5% of all demographic catastrophes with the death toll larger or equal to 1 million might have had a tangible impact on the growth of the world population. The remaining 93.5% were too weak to have any significant impact. It is, therefore, clear that demographic catastrophes were too weak to shape the trajectory of growth of the world population, particularly if we consider that demographic catastrophes trigger also a strong process of regeneration (Malthus, 1798; Nielsen, 2016i).

The generally large percentage of insignificant impacts is an overwhelming evidence contradicting the concept of Malthusian stagnation but confirming conclusions based on the analysis of distributions describing the growth of population and the economic growth (Nielsen, 2016a; 2016d; 2016j), the analysis showing the absence of convincing evidence of frequent impacts of demographic catastrophes.

It is also useful to notice certain correlations between gauge indicators because such correlations could give a closer insight into the process of demographic catastrophes. They can reveal what was happening during a given crisis. Thus, for instance, the intensity indicator *(I)* for the Mongolian Conquest shows that population was decreasing during this crisis but the per annum intensity indicator shows that the population was approximately constant. The intensity indicator was also not excessively large. The size of the population was decreasing but slowly. However, the recovery time was exceptionally high. We can explain it by noticing that the duration of the crisis was long.

Our survey shows also a unique convergence of *five* demographic catastrophes. They were: the Mongolian Conquest (1260-1295) with the total estimated death toll of 40 million; Great European Famine (1315-1318), 7.5 million; the 15-year Famine in China (1333-1348), 9 million; Black Death (1343-1352), 75 million; and the Fall of Yuan Dynasty (1351-1369), 7.5 million. Their combined maximum death toll was 139 million. The estimated size of the world population in AD 1250 was around 380 million. The combined maximum relative impact of these five catastrophes was, therefore, around 37%. Such a strong impact should be reflected in the growth of the world population and indeed it was but not as strongly as we could have expected (Nielsen, 2016j). It caused only a minor disturbance. During this crisis, the population was decreasing but very slowly to reach 360 million at the termination of these five catastrophes, illustrating the efficient process of regeneration even during this combined crisis. This crisis was followed by a faster growth and the lost time was soon recovered, the faster growth illustrating again the efficient process of spontaneous regeneration (Malthus, 1798; Nielsen, 2016i).

Before the crisis, the growth of population was following hyperbolic trajectory characterised by $k = 3.448 \times 10^{-3}$. If continued undisturbed, it would have reached the size $S_1 = 470$ million in around AD 1400. However, the actual size, S_r , at that time was 360 million. If the growth of population after the crisis continued along the same hyperbolic trajectory as before the crisis, then the recovery time, calculated using the eqn (7), would have been 224 years. However, after the crisis, the growth of population was following a faster trajectory, characterised by $k = 4.478 \times 10^{-3}$. So, if we use the eqn (7) again we can calculate that the corresponding recovery time for this faster trajectory was 173 years. The actual recovery time, as recorded by data, was around 165 years, which is in good agreement with the calculated value. The process of regenerations decreased the recovery time by 50-60 years.

Summary and conclusions

The study presented here adds to the explanation why demographic catastrophes did not shape the growth of population and the associated economic growth.

The currently accepted interpretation of the historical growth of population is succinctly summarised in the following statement: "Throughout human history, epidemics, wars and famines have shaped the growth path of population" (Lagerlöf, 2003a, p. 435). If such is the case we should have no problem with showing many examples of this mechanism but we cannot find them. We can analyse data going as far back as 2,000,000 years ago and we can see that with the exception of just one *minor* disturbance around AD 1300 there is no evidence of such effects (Nielsen, 2016j; 2017). We also see no evidence in the distributions describing regional growth of population (Nielsen, 2016d).

This imagined, but never proven mechanism, was supposed to have been responsible for creating an endless epoch of Malthusian stagnation characterised by irregular and generally stagnant state of growth of population and of economic growth, but data are in clear contradiction of this doctrine (Nielsen, 2016a; 2016d; 2016j; 2017;

von Foerster, Mora & Amiot, 1960). It is a doctrine, which is based on the incorrect interpretation of hyperbolic growth.

The growth of population and economic growth were hyperbolic. It is a *monotonically* increasing growth. It is slow over a long time and fast over a short time but there is no stagnation and no takeoff or explosion at any time. Stagnation and explosion are just illusions, which readily disappear when we use the method of reciprocal values (Nielsen, 2014) to analyse data. What we see as a stagnation is just a monotonically increasing growth and what we see as an explosion is just the natural continuation of hyperbolic growth.

We have demonstrated that with the exception of just one event in the past 12,000 years (Nielsen, 2016j), and indeed in the past 2,000,000 years (Nielsen, 2017), there is no evidence that demographic catastrophes were ever shaping the growth of the world population. This unique event occurred around AD 1300 and coincides with *five* strong demographic catastrophes: the Mongolian Conquest (1260-1295) with the total estimated death toll of 40 million; Great European Famine (1315-1318), 7.5 million; the 15-year Famine in China (1333-1348), 9 million; Black Death (1343-1352), 75 million; and the Fall of Yuan Dynasty (1351-1369), 7.5 million. The combined death toll caused by them is estimated at a maximum of 139 million. At the onset of this unique event the world population was only about 380 million, so the relative impact should have been strong. This combined crisis lasted for about 280 years but it caused only a minor disturbance in the growth of population. At the end of this crisis, the size of population was reduced to only 360 million. There is also no convincing evidence that demographic catastrophes were shaping the growth of regional populations (Nielsen, 2016d). Likewise, there is no convincing evidence that they had any tangible impact on the economic growth, global or regional (Nielsen, 2016a).

We have already explained why demographic catastrophes did not shape the growth of population. We have demonstrated (Nielsen, 2016i) that, as first observed by Malthus (1798), his so-called positive checks (demographic catastrophes and many forms of harsh living conditions) are responsible not only for increasing the death toll but also for triggering the process of regeneration, reflecting the well-known phenomenon observed commonly in nature. Thus, the destructive action of even strong demographic catastrophes is quickly compensated by this process, which is likely to produce even faster growth than before.

We can now understand why a combination of five strong demographic catastrophes were needed to cause only minor and relatively short-lasting disturbance in the growth of population around AD 1300. This was one and only example in the past 2,000,000 years (Nielsen, 2017) when we can see a correlation between the growth of population and demographic catastrophes. Now we have added to this explanation by showing that individually, demographic catastrophes were generally too weak to have a tangible impact on the growth of population. On rare occasions, when they were strong enough to cause some minor damage, their action was quickly counteracted by the spontaneous and efficient process of regeneration (Malthus, 1798; Nielsen, 2016i).

We have defined a series of gauge indicators allowing for a study of impacts of demographic catastrophes. We have also concentrated our attention on the strongest catastrophes, thus maximising the fraction of potentially destructive impacts. Even then, this fraction turned out to be small. On average, only 6.5% of all major demographic catastrophes could have had a certain impact but as demonstrated by the analysis of relevant data (Nielsen, 2016d; 2016j; 2017) they had no impact. They were only relatively strong but even if they were stronger, such isolated actions could have been hardly expected to cause lasting disturbances in the growth trajectory, particularly if we consider the apparently ever-present process of regeneration (Malthus, 1798; Nielsen, 2016i).

Any negative impact on the growth of population could be expected to be reflected also in the economic growth but the analysis of data shows that the economic growth remained also undisturbed (Nielsen, 2016a). The growth of population and economic growth were exceptionally stable and generally uninterrupted. Demographic catastrophes *did not* shape the economic growth or the growth of population.

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5. Mathematical analysis of income per capita in the United Kingdom

Introduction

According to the generally accepted interpretations, the supposed long epoch of the so-called Malthusian stagnation in the economic growth and in the growth of human population was followed by a rapid increase, which is claimed to have been caused by modern progress reflected in and coinciding with the Industrial Revolution. The United Kingdom was at the centre of the Industrial Revolution and consequently, this postulated transition from stagnation to growth should be most clearly demonstrated in this country.

We have already shown that the currently accepted interpretations are incorrect (Nielsen, 2014, 2015a, 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g, 2016h, 2016i, 2016j, 2016k). Within the range of analysable data, epoch of Malthusian stagnation did not exist in the economic growth and in the growth of population, global or regional. Likewise, the Industrial Revolution, 1760-1840 (Floud & McCloskey, 1994) did not boost the growth trajectories.

In particular, we have demonstrated (Nielsen, 2016k) that even in the United Kingdom, where the effects of the Industrial Revolution should be most clearly demonstrated, there was no boosting in the economic growth and in the growth of population. On the contrary, shortly after the Industrial Revolution, economic growth and the growth of population started to be diverted to slower trajectories. We have also demonstrated that within the range of the mathematically analysable data, the mythical epoch of Malthusian stagnation did not exist in the United Kingdom.

Our analysis and conclusions are supported by data and by earlier investigations (Biraben, 1980; Clark,1968; Cook,1960; Durand, 1974; Gallant, 1990; Haub, 1995; Kapitza, 2006; Kremer, 1993; Lehmeyer, 2004; Livi-Bacci, 1997; Maddison, 2001, 2010; Mauritius, 2015; McEvedy & Jones, 1978; Podlazov, 2002; Shklovskii, 1962, 2002; Statistics Mauritius, 2014; Statistics Sweden, 1999; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973, 1999, 2013; von Hoerner, 1975, von Foerster, Mora & Amiot, 1960; Wrigley & Schofield, 1981). The only way to accept the doctrine of Malthusian stagnation and the concept of the boosting effects of the Industrial Revolution is to ignore data or to manipulate them in such a way as to make them appear to support the erroneous ideas (Ashraf, 2009; Galor, 2005a, 2005c, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

Our aim now is to investigate the new trend of income per capita in the UK, the trend which commenced relatively recently when the economic growth started to be diverted from its historical, linearly modulated hyperbolic trajectory (For its definition see Nielsen, 2015a). We shall first show how to describe growth by using the differential equation defining the growth rate. This part of the analysis will show that contrary to the generally expected outcomes, fluctuations and long-term variations in the growth rate have negligible effect on the growth trajectories. We shall then study the future growth of income per capita.

Overview

Distributions describing the growth of population and economic growth in the UK based on using Maddison's data (Maddison, 2010) are shown in Figure 1-3.

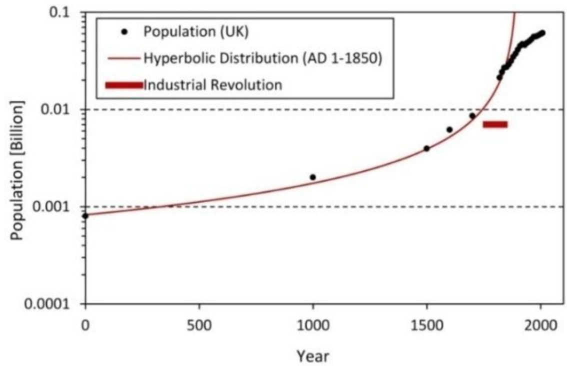

Figure 1. *Growth of population in the UK between AD 1 and 2008. Growth was hyperbolic between AD 1 and 1850. From around 1850, towards the end of the Industrial Revolution, the growth of population started to be diverted to a slower trajectory. Industrial Revolution had no impact on shaping the growth trajectory.*

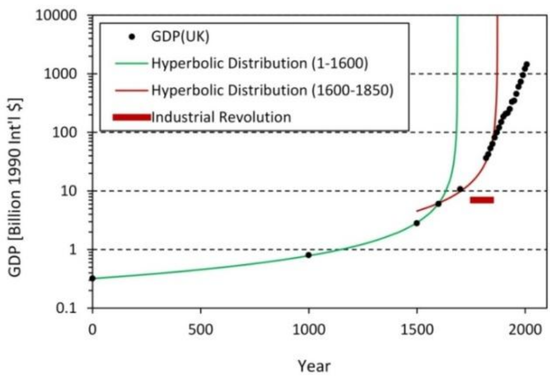

Figure 2. *Economic growth (as described by the GDP) in the UK. The growth was hyperbolic between AD 1 and 1600 and again (but a little slower) between AD 1600 and 1850. From around 1850, the growth started to be diverted to a slower trajectory. Within the range of analysable data, i.e. from AD 1, the mythical epoch of stagnation did not exist. Economic growth was steadily increasing. Industrial Revolution did not boost the economic growth. There was no escape from the Malthusian trap because there was no trap.*

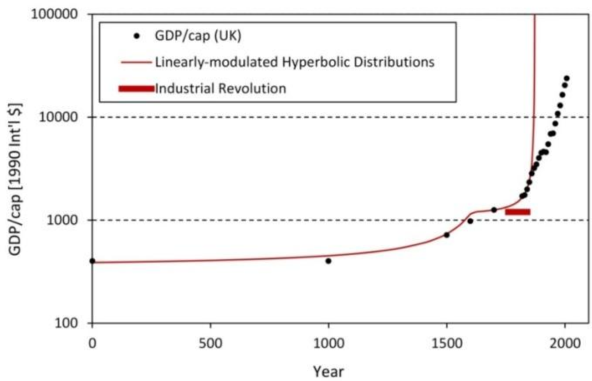

Figure 3. *Growth of income per capita (GDP/cap) in the UK between AD 1 and 2008. The GDP data follow closely the empirically-determined linearly-modulated hyperbolic distributions (defined in* Nielsen, 2015a*). Industrial Revolution did not change the growth trajectory. From around 1850, the growth of the GDP/cap started to be diverted to a slower trajectory.*

Population and the Gross Domestic Product (GDP) were increasing hyperbolically. Contrary to the currently accepted interpretations, there was no Malthusian stagnation and the Industrial Revolution had no impact on the economic growth and on the growth of population even in the United Kingdom, the very centre of this revolution.

We should notice that at the time of the Industrial Revolution, economic growth and the growth of population in the United Kingdom were close to escaping to infinity. It was most fortunate that natural processes did not comply with the imagined interpretations of the growth mechanism. Any boosting by the Industrial Revolution would have been catastrophic.

While the growth of population was following a single hyperbolic trajectory, the growth of the GDP experienced a transition around AD 1600 from a fast to a slower hyperbolic trajectory. This transition is reflected in the income per capita (GDP/cap) shown in Figure 3. Industrial Revolution did not boost the growth of population or the economic growth. From around 1850, economic growth and the growth of population started to be diverted to a slower, non-hyperbolic, trajectory. This simultaneous transition in the growth of population and in the growth of the GDP is reflected in a clear transition in the income per capita (GDP/cap). It is the purpose of this publication to investigate this new trajectory.

Mathematical method

Our analysis is based on the examination of the growth rate (Nielsen, 2015b):

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = R_e(t) \qquad (1)$$

where $S(t)$ is the size of the growing entity and $R_e(t)$ is the empirically-determined growth rate. In the case discussed here, the size of the growing entity is the GDP/cap.

There are two ways of solving this equation: numerical or analytical. If the empirically-determined growth rate $R_e(t)$ can be described analytically, then

$$S(t) = \exp\left[\int f(t)dt\right] \qquad (2)$$

where $f(t)$ is the analytical representation of $R_e(t)$.

If $R_e(t)$ cannot be represented by a simple mathematical function, as in the case of randomly-fluctuating growth rate, then the eqn (1) has to be solved numerically. We also have to solve the eqn (2) numerically if the integration of the function $f(t)$ leads to computational problems, such as when $S(t)$ has to be expressed by an infinite series. We shall now use both of these methods, analytical and numerical to describe the growth of income per capita in the UK and to predict growth.

Mathematical analysis

Four representations of the growth rate of income per capita (GDP/cap) in the UK between AD 1830 and 2008 are shown in Figure 4. They are (1) *R*(*Direct*) calculated directly from the GDP/cap data; (2) *R*(*Refined*) calculated using the GDP/cap data and interpolated gradients; (3) calculated using the best polynomial fit to *R*(*Refined*) represented in this case by a sixth-order polynomial; and (4) calculated by using linear fit to *R*(*Direct*). Virtually the same linear distribution was obtained by fitting *R*(*Refined*).

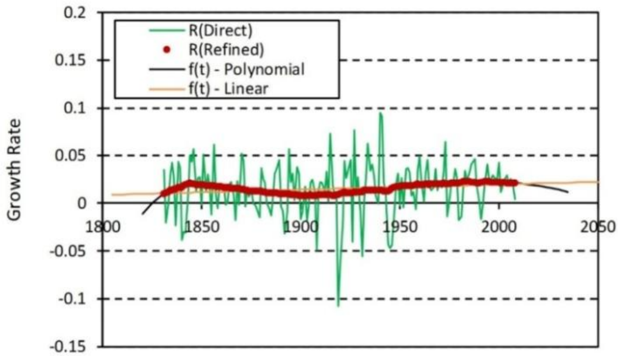

Figure 4. *Four representations of the growth rate of income per capita (GDP/cap) in the United Kingdom between AD 1830 and 2008. R(Direct) is the growth rate calculated directly from the GDP/cap data. R(Refined) is the growth rate calculated using the GDP/cap data and interpolated gradients.* $f(t)$ *- Polynomial is the sixth-order polynomial fitted to R(Refined) and* $f(t)$ *- Linear is the linear fit to R(Direct).*

We shall now use all these four representations of the growth rate to describe the GDP/cap distribution. We shall present two numerical solutions of the eqn (1) by using $R_e(t) = R(Direct)$ and $R_e(t) = R(Refined)$. We shall also present two analytical solutions $f(t)$ represented by a six-order polynomial or by the linear function. Finally, we shall use the linear representation of the growth rate to predict growth of income per capita.

Describing the growth trajectory

Two numerical solutions of the eqn (1) are presented in Figure 5. They are so close to the data that in order to see the difference between them we have to look at a magnified section (Figure 6) in the region of large fluctuations of $R(Direct)$ (see Figure 4).

Results presented in Figures 5 and 6 show that the two numerical integrations of the eqn (1) give excellent description of data. However, while the numerical integration using $R_e(t) = R(Refined)$ reproduces the general trend of the GDP/cap distribution, the calculation based on using $R_e(t) = R(Direct)$ reproduces the fine structure.

We can now understand the origin of the fine structure, which can be seen in Figure 5, and even more clearly in Figure 6. These small ripples are caused by strong fluctuations in the growth rate.

However, it is important to notice that even strong fluctuation in the growth rate do not change the growth trajectory.

It is incorrect to claim that fluctuations in the growth rate represent evidence of the existence of Malthusian stagnation. They do not. Whatever might be their origin, they have no tangible effect on the growth trajectories and consequently on the mechanism of growth. Fluctuations in the growth rate can be neglected when trying to understand the mechanism of growth.

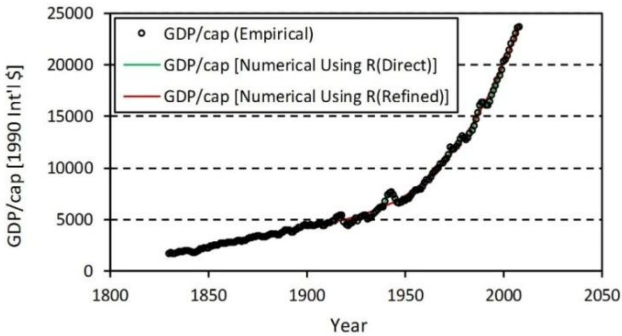

Figure 5. *The growth of income per capita (GDP/cap) in the UK between 1830 and 2008. Data of Maddison (2010), are reproduced by carrying out numerical integration of the eqn (1) using* $R_e(t) = R(Direct)$ *or* $R_e(t) = R(Refined)$*, both growth rates displayed in Figure 4. Both numerical calculations give good representation of data.*

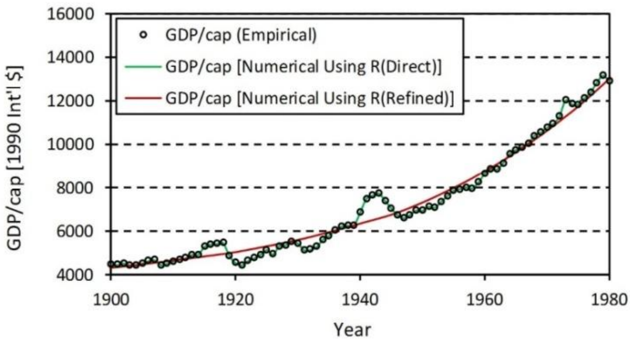

Figure 6. *Magnified section of the GDP/cap distributions showing the difference between the results of two numerical integrations of the eqn (1). These calculations explain the origin of the fine structure of GDP/cap distribution.*

We shall now turn our attention to the analytical solutions of the eqn (1) given by the eqn (2). In Figure 7 we show two such solutions, using $f(t)$ representing either the best, sixth-order polynomial, fit to *R*(*Refined*) or the best linear fit to *R*(*Direct*). In order to examine the differences between these two solutions, we are displaying data every 10 years.

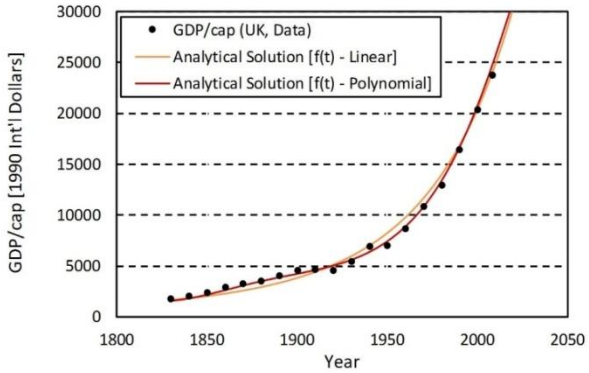

Figure 7. *Two analytical solutions of the differential eqn (1) compared with the GDP/cap data (Maddison, 2010). While the solution obtained using $f(t)$ represented by linear function reproduces the general trend of the GDP/cap distribution, the solution corresponding to $f(t)$ represented by the sixth-order polynomial reproduces the gentle oscillations around the general trend.*

Results presented in Figure 7 show that the solution based on using the $f(t)$ represented by a linear function reproduces the general trend of the GDP/cap distribution but the calculations based on using the sixth-order polynomial, which reproduces the oscillating behaviour of *R*(*Refined*), reproduces also the gentle oscillations of the GDP/cap around the general trend.

We can now understand the origin of these gentle oscillations in the GDP/cap distribution: they are generated by the long-term oscillations of the growth rate. These oscillations are present in *R*(*Direct*) but they are obscured by strong fluctuations. However, they are revealed in *R*(*Refined*), which is calculated using interpolated gradient.

Thus, in summary, combining results presented in Figures 5-7, we can see that small ripples in the time-dependent distributions describing growth, if present, reflect strong fluctuations in the growth rate, while gentle oscillations around the prevailing trend reflect the long-term oscillations in the growth rate.

We have now shown how the interpretations of the mechanism of growth can be simplified. We do not have to worry about the strong fluctuations or about the long-term oscillations in the growth rate. We can concentrate our attention on the general trend of the time-dependent distributions and on simple mathematical representations of the growth rate.

Of course, if we want to go a step further and to try to explain the origin of minor forces, which have no impact on the general trend, we would have to study the oscillations or minor ripples in the time-dependent distributions. Maybe such studies could lead to some interesting discoveries but they would have no impact on explaining the prevailing mechanism of growth.

Predicting growth

We can now use the GDP/cap data between AD 1830 and 2008 to predict economic growth. We can see that the growth is not exponential because the best linear fit to the growth rate is not constant. The linear fit,

$$f(t) = a_0 + a_1 t \, , \tag{3}$$

shown in Figure 4 is described by parameters $a_0 = -8.964 \times 10^{-2}$ and $a_1 = 5.549 \times 10^{-5}$. The gradient is small but positive, which means that the growth rate is steadily increasing. The growth rate around AD 1830 was about 1% but by 2000 it increased to around 2%. By 2050, it is projected to increase to 2.2% and by 2100, to 2.5%.

The predicted growth is faster than the corresponding exponential growth fitting the same data. Any exponential growth becomes unsustainable after a certain time but the growth of income per capita in the UK is going to become unsustainable even faster than the corresponding exponential growth. The predicted growth is shown in Figure 8 and in Table 1.

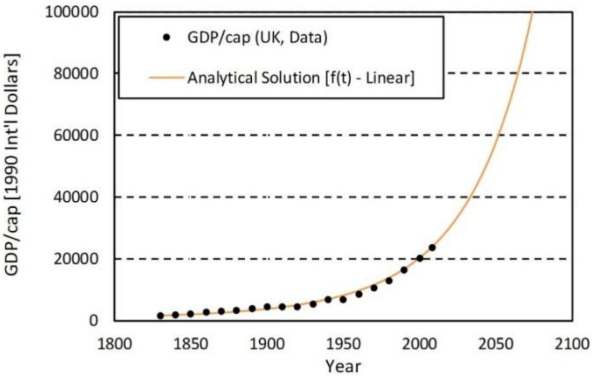

Figure 8. *The projected growth of income per capita in the United Kingdom.*

Sustainability of economic growth is defined not only by the availability of natural resources but also by the associated stress to maintain a given growth. We have defined the relative stress factor for the growth of the GDP (Nielsen, 2015c). We can use the same definition for the growth of income per capita. Thus, the relative stress factor σ for the growth of income per capita can be defined as

$$\sigma \equiv \frac{(GDP/cap)_t}{(GDP/cap)_{t_0}} \quad (4)$$

where $(GDP/cap)_t$ is the income per capita at a certain time t and $(GDP/cap)_{t_0}$ is the income per capita at a certain, fixed time t_0 .

Table 1. *Projected growth of income per capita in the United Kingdom and the associated relative stress factor.*

Year	GDP/cap	*R*	σ	Year	GDP/cap	*R*	σ
2000	22,031	1.95	1.00	2080	125,268	2.39	5.69
2015	29,717	2.04	1.35	2090	159,536	2.45	7.24
2020	32,924	2.06	1.49	2100	204,291	2.50	9.27
2030	40,579	2.12	1.84	2110	263,033	2.55	11.94
2040	50,289	2.17	2.28	2120	340,520	2.61	15.46
2050	62,662	2.23	2.84	2130	443,246	2.66	20.12
2060	78,508	2.28	3.56	2140	580,121	2.72	26.33
2070	98,899	2.34	4.49	2150	763,419	2.77	34.65

GDP/cap in the 1990 International Dollars; *R* – the growth rate of the GDP/cap, in per cent; σ - the relative stress factor, in per cent.

The relative stress factor in 2015 was only 35% higher than in 2000. A 35% greater effort was required to keep the economy growing along this new trajectory. By 2050, the stress factor is projected to increase to 2.84. Economic output per year will have to be almost three times as high as in the year 2000 to keep the economy growing along the same trajectory. Such a large stress might be already hard to tolerate. By the end of the current century, the annual economic output per year will have to be about 9 times as high as in the year 2000 and by 2150 it would have to be about 35 times as high. Even with unlimited natural resources, there will come a time when such a large economic output will be physically impossible to achieve and the economic growth will either have to be diverted to a new trajectory or it will collapse.

The general drive everywhere, not only in the UK but also in other countries, is to keep the economic growth rate increasing or constant. This is a serious mistake. Even with a constant growth rate, which describes exponential growth, such economic growth will become, at a certain stage, impossible to maintain, even if we had unlimited natural resources. To make the economic growth safe and secure, the growth rate should be now slowly decreasing, not only in the UK but also globally (Nielsen, 2015c).

Summary and conclusions

We have carried out the analysis of Maddison's data (Maddison, 2010) describing income per capita (GDP/cap) in the United Kingdom between 1830 and 2008. Our analysis is based on solving differential equation describing the growth rate. We have presented two numerical and two analytical solutions of this equation. We have explained the origins of various features of the time-dependent GDP/cap distribution.

We have demonstrated that strong fluctuations in the growth rate do not change the growth trajectory. They can, at best, be reflected only as just small ripples along the prevailing trend. It is incorrect to interpret even strong fluctuations in the growth rate as the evidence of the existence of Malthusian stagnation because these fluctuations have no impact on shaping growth trajectories.

Long-term oscillations in the growth rate can be reflected as small oscillations of the growth trajectory. They are also unlikely to affect the general trend of growth. The mechanism of growth is determined by the prevailing trend of the growth trajectory.

In order to study the mechanism of growth or to predict its future there is no need to worry about reproducing mathematically the details of the corresponding growth rate. Random fluctuations and long-term oscillations in the growth rate can be neglected and the growth rate can be reproduced by a simplest function. Often, it is possible to do it by using linear functions. Indeed, using the simplest descriptions of the growth rate is most acceptable.

Our analysis demonstrated that the current economic growth in the UK is unsustainable even if supported by unlimited natural resources, because after a certain time it will be impossible to maintain the ever-increasing output. At a certain time in the future, economic growth will have to start to be diverted to a slower trajectory or it will be likely to collapse.

The same problem applies globally (Nielsen, 2016c). Global economic growth should now, or soon, be characterised by a *slowly decreasing* growth rate. The example of the economic growth in Greece shows that rapid decrease or increase in the growth rate can lead to catastrophic results (Nielsen, 2015d).

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6. The Law of Growth

Introduction

The aim of this publication is to formulate a simple law of growth, which could be used to study, interpret and understand any type of growth including economic growth described by the Gross Domestic Product or by income per capita. This comment will make it clear why this discussion is applicable to the study of economic growth. This law will link directly the trajectory of growth with the driving force. The aim here is to facility an easy and transparent way for studying the mechanism of growth because the mechanism of growth is defined by the associated driving force.

If we can link the driving force with the growth trajectory, we can then easily check our interpretation of the mechanism of growth. We can use various types of forces to test whether proposed mechanism is in agreement with the empirical evidence. We can extend our study to predict growth assuming that the mechanism of growth is going to be unchanged, but we can also predict growth by assuming a different mechanism of growth. In such a case, we can also use the general law of growth but with a new, suitably defined driving force.

The law of growth

The definition

The well-known principle in scientific investigations is: *Entia non sunt multiplicanda praeter necessitatem.* Before looking for complicated explanations or formulations, it is always advisable to adopt the simplest possible approach. Complicated explanations might be impressive and in some cases even unavoidable but the simplest solutions are always more attractive.

It is well-known and generally accepted that any growth can, and usually is, described by the *growth rate*. Once we know the growth rate, we can immediately understand whether the growth is fast or slow. We can even have a certain degree of understanding of its possible future, whether it can be sustained or not, whether it is too slow and should be speeded up, if possible, or maybe that it is too fast and should be slowed down.

Thus, for instance, economic growth is routinely described using the *percentage* of the annual increase or decline. The growth of human population is also characterised in the same way.

Often, in order to understand growth, the growth rate is converted into the *doubling time*. The equivalent quantity for radioactive isotopes is the half-lifetime calculated using the decay rate. If we have a radioactive material, which decays within seconds, we do not have to worry too much about its harmful effects. However, if we have radioactive contamination containing substantial amount of radioactive isotopes with half-lifetime of millions or billions of years, we can be sure that we have a serious problem. Likewise, if the doubling time for a spread of certain infectious diseases is millions of years, we do not need to be worried but if the doubling time is measured in days, then again we have a serious problem. We do not have to carry out any laborious calculation. A simple calculation assuming a constant doubling time or a constant growth rate can lead easily to an approximately correct answer, which in many cased is quite acceptable.

Even though the simple formula for calculating the doubling time by dividing 70 or 69.3 by the growth rate expressed in per cent is applicable only to the exponential growth, and should never be applied to any other type of growth, such calculations are still carried out for other types of growth because we know that if we calculate the growth rate or the corresponding doubling time we can have a little better understanding of a given process.

It seems to be obvious that the growth rate reflects the mechanism of growth and that there must be a close connection between the growth rate and the driving force of growth. The *simplest* way of describing this close connection is to assume that the growth rate is directly proportional to the driving force:

$$G = \kappa F \,, \tag{1}$$

where G is the growth rate, F is the driving force, and κ is a constant, which we could call *the growth promoting factor*, or *the* *compliance*, because the larger is the parameter κ, the faster is the growth.

Growth rate is defined as:

$$G(t) \equiv \frac{1}{S(t)} \frac{dS(t)}{dt}, \tag{2}$$

where $S(t)$ is the size of the growing entity, and t is time.

This quantity is sometimes labelled unnecessarily and confusingly as *the relative growth rate* to distinguish it from another redundant and confusing term *the absolute growth rate*, which describes just the change in the size of the growing entity per unit of time, i.e. dS/dt. To make it even more confusing, the term *absolute growth rate* is sometimes replaced by *the growth rate* (e.g. Karev & Kareva, 2014) or by *the exponential growth rate*.

This needless confusion could be easily avoided by leaving the well-known *growth rate*, as defined by the eqn (2), alone. It is a widely-used quantity applicable not only to the exponential growth but also to any other type of growth. All descriptions of growth in terms of per cent of the increase or in terms of the doubling time use the growth rate defined by the eqn (2), so the use of the term: *the absolute growth rate*, for this well-known growth rate represents an unnecessary and confusing aberration. We should always use the term *growth rate* only for the quantity defined by the eqn (2).

If we insist on using dS/dt to describe growth, we should never create confusion by associating it with the term *growth rate* but we should simply call it the *absolute change*. We do not create science by introducing complicated and confusing terms.

The eqn (1) represents the simplest, general *law of growth*. There could be many other ways of linking the growth rate with the driving force but we have assumed the simplest relation. We call the eqn (1) the law of growth rather than the model of growth because this equation can be used to formulate a variety of models of growth, some of them already well known, but many of them yet unknown. Rather than using the existing models, such as exponential or logistic, even if their application could be questionable, we can *tailor* the models of growth to the studied processes and by doing so we can then try to explain the mechanisms of growth described by the relevant driving force.

The eqn (1) can be rewritten as

$$F = rG\,, \tag{3}$$

where $r = \kappa^{-1}$. This is another representation of the general law of growth.

Similarities

In the form given by the eqn (3), the law of growth is similar to the Newton's second law of motion:

$$F = ma\,, \tag{4}$$

where m is the mass of a physical object and a is the acceleration.

Newton's law describes the *dynamics of physical objects*. If the driving force is zero, the acceleration is zero, which means that the physical object is either stationary or that it moves along a straight line with a constant velocity.

The law of growth describes the *dynamics of growing entities*. If the driving force is zero, the growth rate is zero and the size of the growing entity remains constant.

In the Newton's law, m is the mass of the physical object. The larger is m the larger force has to be used to have the same acceleration. The equivalent parameter in the law of growth is r, which can be interpreted as the *resistance* to growth. The larger is r the larger must be the driving force to have the same intensity of growth.

Acceleration is a well-known quantity and because of it, Newton's law can be used easily to understand the dynamics of physical objects. Growth rate is also a well-known quantity and because of it, the general law of growth can be also used easily to understand the dynamics of growing entities.

In its explicit form, Newton's second law of motion can be expressed as

$$F(t) = m\frac{d^2 s(t)}{dt^2}\,, \tag{5}$$

where $s(t)$ is the trajectory of the moving object. The dynamics of the moving object is explained by linking the trajectory $s(t)$ with the driving force $F(t)$.

Likewise, in its explicit form, the law of growth can be expressed as

$$F(t) = r\frac{1}{S(t)}\frac{dS(t)}{dt}. \qquad (6)$$

The dynamics of the growing entity (the mechanism of growth) is explained by linking the size $S(t)$ of the growing entity with the driving force $F(t)$.

For physical objects, the driving force, i.e. the mechanism of motion, is *reflected* in the acceleration $a(t)$ and in the corresponding trajectory $s(t)$. If the driving force (the mechanism) is known, we can use it to calculate the corresponding trajectory of a moving object. However, if the trajectory is known but the driving force (the mechanism) is unknown, we can assume a driving force (a mechanism) and calculate the corresponding trajectory $s(t)$. If our calculations agree with relevant data, we can then claim that we have *explained* the mechanism of the moving object.

For growing entities, the driving force, i.e. the mechanism of growth, is *reflected* in the growth rate $G(t)$ and in the corresponding trajectory describing the size $S(t)$ of the growing entity. If the driving force (the mechanism) is known, we can use it to calculate the corresponding trajectory of a growing entity. However, if the trajectory is known but the driving force (the mechanism of growth) is unknown, we can assume a driving force (a mechanism of growth) and calculate the corresponding trajectory $S(t)$. If our calculations agree with relevant data, we can then claim that we have *explained* the mechanism of growth.

We can calculate the trajectory $s(t)$ of a physical object directly from the acceleration without using the Newton's law. However, to understand why a moving object follows a certain trajectory we have to understand the driving force, and the link between the driving force and the trajectory is given conveniently by the Newton's law of motion.

Likewise, we can calculate the trajectory $S(t)$ of the growing entity directly from the growth rate without using the law of growth. However, to understand why the growth follows a certain trajectory we have to understand the driving force, and the link between the driving force and the trajectory is given by the law of growth.

The difference between the Newton's law of motion and the law of growth is that while Newton's law is a three-dimensional vector, the law of growth is a scalar, which makes the description of growth much simpler than the description of the dynamics of physical objects.

In Newton's law, mass m represents an *intrinsic* property of a physical object. For the law of growth, resistance to growth, r, might have a broader interpretation. It might represent an intrinsic property of a growing entity but it might also depend on exogenous conditions. In this respect, there is a close similarity between the law of growth and other similar simple and well-known laws listed in Table 1.

Table 1. *Examples of similar laws*

Name	Law	Explanation
Newton's law	$F = ma$	F – driving force; m – mass; a – acceleration
Ohm's law	$U = RI$	U – potential; R – resistance; I – current
Hagen–Poiseuille law	$\Delta P = rV$	ΔP – pressure difference; r – resistance; V – volume velocity
Darcy's law	$\nabla P = rV_F$	∇P – pressure gradient; r – resistance to flow; V_F – volumetric flux
Fourier's law	$\nabla T = rH$	∇T – temperature gradient; r – thermal resistivity; H – heat flux
Law of growth	$F = rG$	F – driving force; r – resistance to growth; G – growth rate

For instance, the law of growth is similar to Ohm's law, $U = RI$, describing the flow of electricity. The electrical potential, U, plays here the role of the driving force [*cf* eqn. (3)] and R is the resistance to flow. The parameter κ in the law of growth given by the eqn (1) plays similar role as the *conductance*, $1/R$, in the Ohm's law. Resistance, R, is determined by the intrinsic property of the conducting material (*electrical resistivity*) but it also depends on the geometrical dimensions of the conducting medium (its length and the cross-section area). Furthermore, while resistivity characterises an *intrinsic* property of the conducting medium, it also depends on the temperature.

The law of growth is also similar to the Hagen–Poiseuille law describing the flow of fluids through cylindrical conduits. In this law, pressure difference plays the role of the driving force. Resistance to flow depends not only on the intrinsic property of a given liquid (*viscosity*) but also on the geometrical dimensions of the cylindrical conduit (its length and its radius). However, viscosity depends also on the temperature.

The law of Hagen–Poiseuille is usually expressed using pressure difference and *volume velocity* but it can be also presented using *pressure gradient* and *volumetric flux* (volume velocity per unit area). In this from it resembles the Darcy's law describing the flow of fluids through porous medium where the resistance to flow is given by the ratio of *viscosity* and *permeability* both depending on the temperature.

The law of growth is also similar to the Fourier's law describing the conductive heat transfer, where *heat flux* (energy transferred per units of time and area) is given by the product of *conductivity* and the temperature gradient, in the same way as the growth rate is given by the product of κ and the driving force in the eqn (1). Temperature gradient plays the role of the driving force while thermal conductivity is equivalent to the parameter κ. The inverse value of thermal conductivity is *thermal resistivity*. This quantity characterises the *intrinsic* property of the heat transferring medium but it also depends on the temperature.

Examples of applications of the law of growth

We shall now give a few simple examples how the law of growth can be used in the study of the mechanism of growth. We shall show how we can tailor our interpretations of growth to understand better its mechanism. We do not have to be restricted to using just a certain, limited range of models of growth. We can design and use our own models. We can explore a wide range of mechanisms of growth and check, which of them gives the best description of data. In general, we might have to solve the relevant differential equations numerically but, in many cases, we might have a convenient analytical solution.

Exponential growth

We might assume, for instance, that the driving force is constant,

$$F(t) = c . \tag{7}$$

It is the simplest force of growth. By being constant it, obviously, does not depend on time or on the size of the growing entity. This comment might sound trivial but it is important to understand that for other types of growth the driving force can depend not only on time but also on the size of the growing entity, or on the combination of time and size, and that all such options will describe the multitude of possible *models* of growth.

If we use this force in the eqn (1) we shall get

$$\frac{1}{S(t)}\frac{dS(t)}{dt}=k\,, \tag{8}$$

where $k \equiv c/r = c\kappa$.

The solution of this differential equation is the exponential function,

$$S(t)=Ce^{kt} \tag{9}$$

where C is related to the constant of integration.

Now, we can understand this growth a little better because we know where it belongs. It belongs to a specific class or the type of growth, for which the driving force is constant. For the same intensity c of the driving force, the smaller is the resistance r or the larger is the compliance (or growth promoting factor) κ, the larger is the growth rate k and the faster is the exponential growth.

The extension of the exponential growth

Let us now use a more general example when the driving force is not constant but depends on time,

$$F(t)=f(t)\,. \tag{10}$$

If we use this force in the eqn (1) we shall have

$$\frac{1}{S(t)}\frac{dS(t)}{dt}=\kappa f(t)\,. \tag{11}$$

The solution to this equation is similar to the solution for the eqn (8):

$$S(t)=C\exp\left[\kappa\int f(t)dt\right]. \tag{12}$$

Here we have a large variety of models of growth with one of them being the *exponential model* of growth characterised by $f(t)=c$. We might represent $f(t)$ by a polynomial function or by any other function of our choice.

The logistic model

We might assume that the driving force of growth *decreases with the size* of the growing entity. An example could be the growth of a tree. A tree does not grow indefinitely. It might be growing fast at the beginning but then it reaches a certain average height and does not grow. Growth of an individual person can be also a good example. Initially the growth is fast but eventually a given person reaches a certain height and stops growing. In the simplest case, we might assume that the driving force decreases *linearly* with the size of a growing entity:

$$F(t) = a - bS(t), \tag{13}$$

where a and b are positive constants.

Using the eqn (1) we then have

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = \kappa\left[a - bS(t)\right]. \tag{14}$$

This equation can be expressed as

$$\frac{dS(t)}{\left[A - BS(t)\right]S(t)} = dt, \tag{15}$$

where $A \equiv \kappa a$ and $B \equiv \kappa b$.

The left-hand side of this equation can be easily integrated if we split it into a sum of two fractions:

$$\frac{B}{A}\frac{dS(t)}{\left[A - BS(t)\right]} + \frac{1}{A}\frac{dS(t)}{S(t)} = dt. \tag{16}$$

From now on, the integration is easy.

Alternatively, we can solve the eqn (15) by using the general integration formula we have derived earlier (Nielsen, 2015):

$$\int \frac{dx}{u \cdot v} = \frac{1}{\Delta}\ln\frac{v}{u}, \tag{17}$$

where $u = a + bx$, $v = c + dx$ and $\Delta = ad - bc$.

The solution of the eqn (14) is represented by the sigmoid function:

$$S(t) = \left[\frac{b}{a} + \left(\frac{1}{S_0} - \frac{b}{a} \right) e^{-\kappa a t} \right]^{-1}, \tag{18}$$

where $S_0 = S(t = 0)$.

We can see that

$$S(t \rightarrow \infty) \Rightarrow \frac{a}{b} \equiv K, \tag{19}$$

where K defines the limit of growth.

The extension of the logistic growth

In the logistic model, the driving force decreases *linearly* to a certain limit *K*. However, we might have many other possibilities. One of them is the modified logistic model introduced by Gilpin & Ayala (1973). Some of the variations to the logistic growth are shown in Figure 1.

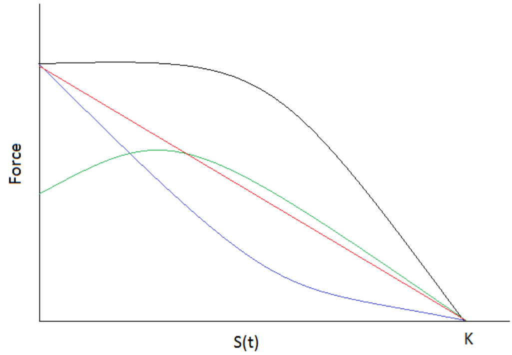

Figure 1. *Examples of the extensions to the conventional logistic growth.*

In Figure 1, the driving force represented by a decreasing straight line to a certain limit K represents the well-known, conventional logistic model of growth. However, we might have a force that is initially approximately constant but then is changing gradually to a linearly decreasing. Such a force would describe the initially approximate exponential growth changing seamlessly into a logistic growth, which would be approaching asymptotically the limit *K*.

We could also have a force, which could be initially decreasing rapidly with the size of the growing entity but after a certain time it would start to follow a gently decreasing trajectory. It would be an approximately fast logistic growth changing seamlessly into a slow logistic growth.

Another alternative shown in Figure 1 is a force, which is initially increasing with the size of the growing entity, reaches a certain maximum and then starts to decrease to a certain limit *K*. This type of growth could, for instance, follow an approximately pseudo-hyperbolic trajectory (Nielsen, 2015). However, it would not increase to infinity but it would change seamlessly to an approximately logistic growth, approaching asymptotically the limit *K*.

Possibilities are endless and each of them could be tried to fit data and find their best mathematical representation. However, if we introduce complicated descriptions of the driving force we might have a problem with explaining why we use a complicated description. For instance, if we can see that the growth is indeed initially exponential but then gradually levels off and approaches a certain limit *K*, we could easily describe such a growth mathematically by using a constant driving force changing gradually into the linearly decreasing force shown in Figure 1 but we would still have to explain why the force changed in such a way and why the growth changed from exponential to logistic.

Further extensions

Even though the general principle in scientific investigations is to use the simplest interpretations, in certain cases it might be necessary to try more complicated solutions and the law of growth offers an easy definition of such more complicated models. The ultimate extension would be to assume that the force of growth and the resistance to growth depend not only on time but also on the size of the growing entity. Such an assumption will probably never be used but it shows that we can have a practically unlimited number of models of growth.

The general principle of investigation

Even though the described here general law of growth opens virtually unlimited possibilities for defining and using a wide variety of models of growth, the general principle of scientific investigation is to use the simplest descriptions. In the study of the mechanism of growth the general principle is to use the simplest mechanism of growth as represented by the simplest driving force.

Using complicated mathematical expressions without understanding why we use them and without convincingly justifying their use makes absolutely no sense. Even if complicated expressions lead to a good description of data we have learned nothing about the mechanism of growth unless we can explain why such complicated mathematical descriptions are necessary.

The initial and important step in the study of growth is to identify the *type* of growth. For instance, if we can show that the growth is not exponential but hyperbolic, we can then focus our attention on a limited range of forces or maybe even on using just an obvious single force to explain the mechanism of growth. Complicated mathematical descriptions might look impressive, they might create an aura of science, but simple descriptions are always preferable.

Summary and conclusion

Using the simplest possible assumption, we have formulated a simple general law of growth. We have shown that this law is analogous to many other simple but useful laws, one of them being the Newton's law of motion. Using a few examples, we have shown how this simple law of growth can be used to define a multiplicity of models of growth, which in turn can be used to study the mechanism of growth. Even though this general law of growth allows for the introduction of a wide variety of models of growth, the general recommendation is to use the simplest descriptions of driving forces to describe and explain the observed phenomena.

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7. Mechanism of hyperbolic growth explained

Introduction

Hyperbolic growth gives a remarkably good description of population and economic data (Nielsen, 2014, 2016a, 2016b, 2016c). It describes historical growth of the Gross Domestic Product (GDP) and of population, global and regional and even in individual countries. This conclusion is based on the analysis of the extensive data published by Maddison (2010). They describe the economic growth and the growth of population during the AD era, starting from AD 1 and extending to 2008. Hyperbolic growth describes also remarkably well the growth of global population in the past 12,000 years (Nielsen, 2016a). This analysis is supported by population data coming from a wide range of sources (Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973, 1999, 2013).

Hyperbolic growth of population was first noticed by von Foerster, Mora & Amiot (1960) close to 60 years ago and it was soon confirmed and accepted by other authors (Kapitza, 2006; Kremer, 1993; Podlazov, 2002; Shklovskii, 1962, 2002; von Hoerner, 1975). Hyperbolic growth turns out to be exceptionally stable and generally undisturbed. Many driving forces might be considered as influencing growth. For the growth of human population, and as pointed out by Kapitza (2006), all these forces can be arranged in such categories as industrial, economic, cultural, social and biological. However, he also pointed out that the simple formula describing the growth of the world population suggests that many of these forces must have been "suppressed by the process of averaging" (Kapitza 2006, p. 77).

Economic growth is also described by the simple hyperbolic formula and in general it has been also stable over a long time in the past suggesting a simple explanation of the mechanism of growth and indicating that the growth must have been also controlled by single net force. It is the aim of this publication to identify these dominating forces of growth and to explain the mechanism of the historical hyperbolic growth of population and of the GDP.

Mechanism of growth

Mechanism of the historical economic growth

Gross profit may depend on many factors but it obviously depends on the size of investment. "Money makes money. And the money that makes money makes more money" (Benjamin Franklin). Economic growth is directly related to the size of our investments. With the sufficiently high investment, we can build more retail stores, or larger retail outlets, we can buy more goods for sale, employ more people in our business, buy more tools and machinery, invest in a better equipment to increase production, build more houses either for sale or for rent, build more factories, improve agriculture, improve our services, pay for advertising, pay for the transportation and distribution of goods and support all other necessary activities aimed at generating profit. According to the well-known theory of Cobb & Douglas (1928), production yield can be described by the following simple equation:

$$Y = aL^{\alpha}K^{\beta}. \quad (1)$$

Where Y is the production yield, a is the so-called total factor productivity, L is labour expressed as person-hours during a given time, e.g. during one year, K is capital input (the money invested in the equipment, buildings or anything else to support production), α and β are constants, $\alpha + \beta = 1$, $0 < \alpha < 1$ and $0 < \beta < 1$.

In this equation, wealth generates wealth or money makes money not only through the investment K, which could be passed from one year to another, but also through the ongoing costs of labour.

In essence, therefore, the right-hand side of the eqn (1) represents the investment of a certain amount of money to produce profit. The left-hand side does not represent the total wealth but the increase in wealth, which could be the annual increase. This increase is proportional to the money locked as K and to the annual investment of money expressed as L. We need money to make money. We need wealth to generate wealth.

In order to explain the mechanism of economic growth we shall look at it from the point of view of a driving force, because driving force represents the mechanism of growth. For the economic growth, it is the net market force. We can have many market forces but in order to explain the mechanism of growth it is best to start with the simplest assumption and make it complicated only if necessary. This is the fundamental principle in scientific research, known as the Occam's razor or the law of parsimony: *Entia non sunt multiplicanda praeter necessitatem.*

The simplest way to describe mathematically the driving force of economic growth is to assume that it is directly proportional to the invested wealth. The larger is the circulated wealth, the greater wealth can be produced.

$$F = cW, \tag{2}$$

where W is the total existing wealth and c is a constant.

It is essential to understand that we are dealing here with average quantities. In explaining economic growth of a country or region or of the world we are not dealing with individual economic units but with the whole assembly of these units. The eqn (2) describes the *average* force of economic growth. The quantity W represents the total wealth of a country, a group of countries or of the whole world, expressed usually as the GDP and c could represent the average fraction of this wealth used to drive economic growth. The larger is the already generated wealth, the larger is the driving force of economic growth when this wealth is invested to produce more wealth. Wealth generates wealth. This principle and this process appears to be well known and universally accepted. However, this principle has been never expressed in mathematical form, which could be compared directly with data.It was never used to describe economic growth trajectories. It was never used to describe and explain the mechanism of the historical growth of the GDP.

In our earlier publication (Nielsen, 2016d), we have formulated a general law of growth:

$$F = rG, \tag{3}$$

where G is the growth rate and r is the resistance to growth.

The advantage of using this simple law of growth is that it links the force of growth with trajectories of a growing entity. The force of growth represents the mechanism of growth and the law of growth allows for defining this force, i.e. for defining the expected or postulated mechanism, and to compare it with data as described by growth trajectories. This simple law allows for a mathematical formulation of postulated mechanism and for translating this mechanism into growth trajectories, which can be readily tested by data. Thus, this law allows for testing various mechanisms of growth by data.

The growth rate G is defined as

$$G \equiv \frac{1}{W}\frac{dW}{dt} \quad , \tag{4}$$

where t is time.

If we now insert the postulated driving force of economic growth defined by the eqn (2) into the eqn (3), we shall get the following equation describing economic growth.

$$\frac{1}{W}\frac{dW}{dt} = kW \, , \tag{5}$$

where $k \equiv c / r$.

We have now linked the driving force with economic growth trajectory. The parameter k is inversely proportional to the resistance to growth r and could be called the compliance factor or simply the compliance. In the formulation of the general law of growth (Nielsen, 2016d) we have defined $1/r$ as compliance. However, k differs only by a constant c so it plays the same role as $1/r$. The larger is the parameter k, the more efficient is the generation of wealth and the faster is the growth of W. We could easily extend this model by considering that c or r or both of them depend on time, but at this stage it is preferable to use the simplest possible assumption.

The eqn (5) does not describe the growth of an individual economic unit but the average economic growth of a country, region or globally. Economic growth of a single unit might be affected by many random forces but for a large assembly of such units, random forces might be averaging out. If they are not or if there is some other strong force not included in our simple assumption, then our predictions of growth will be contradicted by data and we shall have to modify our assumed mechanism of growth. We can check whether our assumption is correct by comparing the calculated trajectory with data.

The eqn (5) can be solved using the substitution $W = Z^{-1}$. Its solution is

$$W = \frac{1}{C - kt}. \tag{6}$$

This is hyperbolic growth. Data describing historical economic growth (Maddison, 2010) and their analysis (Nielsen, 2016b) show that our choice of the driving force was correct and that there is no need to assume the presence of any other type of forces. An example of comparing calculations with data is presented in Figure 1.

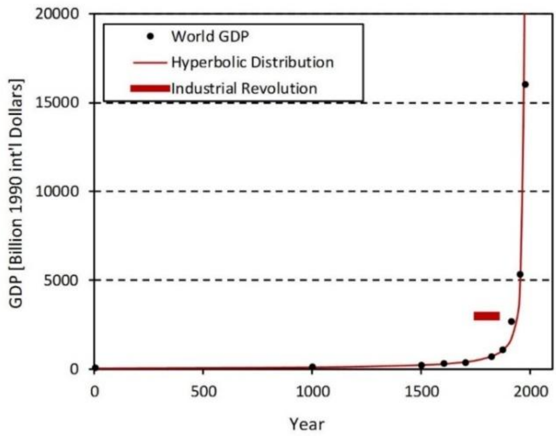

Figure 1. *World economic growth as described by the Gross Domestic Product (Maddison, 2010) compared with hyperbolic distribution. Single and simple driving force explains the mechanism of growth. This force was so strong that even the Industrial Revolution had no impact on changing the growth trajectory.*

Data and their analysis show that the historical economic growth was indeed hyperbolic and now we can understand why. *Historical economic growth was prompted by a single dominant force directly proportional to the existing volume of wealth, expressed usually as the GDP.* Hyperbolic economic growth describes the net historical growth of a large number of economic units. The larger was the existing wealth of a country or a region or globally, the larger was the driving force of economic growth. Currently, economic growth is no longer hyperbolic. It is no longer controlled by the simple force given by the eqn (2). Driving forces appear now to be now more complicated.

It should be noted that the growth described by compound interest is of a different kind. It is not a spontaneous and unconstrained growth controlled by the net driving force proportional to the size of the existing wealth. The force controlling the growth described by compound interest is constrained. It is dictated by human-imposed regulations. No bank in the world would pay interest increasing in the direct proportion to the balance of our deposits. For the money deposited in the bank, interest varies within a small range of values and consequently it is approximately constant. This type of growth is described by a constant or approximately constant force of growth, which generates exponential growth, the growth described by compound interest. Likewise, no bank in the world would give a loan with interest decreasing with the decreasing balance. These two types of transactions are controlled by man-made regulations. They are not controlled by the assumed by us, and confirmed by data, force describing the spontaneous and unconstrained historical economic growth. However, it does not mean that the current economic growth cannot be exponential. It can and it often is because, as indicated by data, the current economic growth is no longer prompted by the historically prevailing single force.

Mechanism of the historical growth of population

The most obvious and essential force, which has to be considered to explain the mechanism of the growth of population is obviously the biologically-controlled or prompted force of procreation, which is defined here as the difference between biologically controlled birth and death rates. Other forces might be included, if necessary, but this force is indispensable.

Let us assume that *on average*, the biologically controlled force of procreation is constant *per person*. Biologically controlled birth and death rates may vary over time but we assume that on average and per person the difference remains the same. This is a very simple assumption but again in science it is always advisable to use the simplest possible assumptions and make them more complicated only if necessary. Under this assumption,

$$\frac{F}{S} = c \,, \qquad (7)$$

where F is the biologically controlled force of procreation, S is the size of the population and c is certain average constant. It describes how, on average, each person contributes to the growth of population.

If we use this force in the general law of growth given by the eqn (3) we shall get

$$cS = rG , \tag{8}$$

where G is now given by

$$G = \frac{1}{S}\frac{dS}{dt} , \tag{9}$$

which leads to the following differential equation describing the growth of population

$$\frac{1}{S}\frac{dS}{dt} = kS . \tag{10}$$

Solution to this equation is

$$S = \frac{1}{C - kt} . \tag{11}$$

It is also a hyperbolic distribution, which gives excellent description of data (Nielsen, 2016a, 2016c). Example is shown in Figure 2.

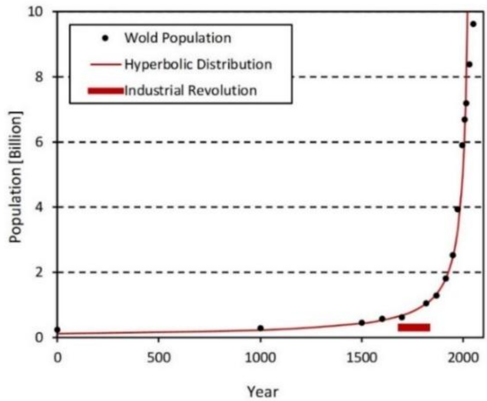

Figure 2. *Growth of the world population (Maddison, 2010) compared with hyperbolic distribution. Single and simple driving force explains the mechanism of growth. This force was so strong that even the Industrial Revolution had no impact on changing the growth trajectory.*

The mechanism of historical hyperbolic growth of population is explained as an unconstrained growth prompted solely by the biologically controlled force of procreation. This force is given by the average difference between biologically controlled birth and death rates and is assumed to be constant per person. This simple mechanism explains global and regional historical growth of population (Nielsen, 2016a, 2016c).

Mechanism of demographic transitions

If we include in our analysis a wider range of data describing the growth of the world population (Biraben, 1980; Clark, 1968; Cook, 1960; Durand, 1974; Gallant, 1990; Haub, 1995; Livi-Bacci, 1997; Maddison, 2010; McEvedy & Jones, 1978; Taeuber & Taeuber, 1949; Thomlinson, 1975; Trager, 1994, United Nations, 1973, 1999, 2013) we shall soon discover certain interesting details showing two demographic transitions in the past and the current ongoing transition (Nielsen, 2016a).

As we have shown earlier (Nielsen, 2016a), growth of the world population was hyperbolic between 10,000 BC and 500 BC, between AD 500 and 1200, and between AD 1400 and 1950. During these large sections of time, taking approximately 90% of the past 12,000 years, the mechanism of growth of population can be explained as being prompted by the simple, biologically controlled, force of procreation, which was on average constant per person. All other forces, even if present, had no influence on the growth of global population. They were either too weak or they were averaged out.

The time when the prevailing hyperbolic growth was significantly disturbed in the past 12,000 years was only between 500 BC and AD 500, between and AD 1200 and 1400 and now after around 1950. These are the only recorded demographic transitions in the past 12,000 years. The first transition was from a fast to a slow hyperbolic trajectory. The second transition was from a slow to a slightly faster trajectory and the current transition is to a yet unknown trajectory.

The first transition appears to coincide with the massive and widespread changes in the *style of living* associated with the intensified changes in the political landscape in various parts of the world, graphically and comprehensively explained by Teeple (2002). It is also probably not without significance that this transition coincides with the rise and fall of Roman Empire, the longest lasting political system in history, which by the first century BC ruled already over vast areas of land surrounding Mare Nostrum (the Mediterranean). After its fast expansion and after subjugating many independently-living societies under its rule, this powerful and seemingly unconquerable political structure disintegrated into many fragments of independent countries. However, during that long time, significant changes in the political landscape were also occurring outside the realm of the Roman Empire.

Between 10,000 and 500 BC, growth of population is described by a fast-increasing hyperbolic trajectory, as defined by the parameter *k*. After the BC/AD transition, the growth was directed to a significantly slower trajectory characterised now by the parameter *k*, which was about 6.4 times smaller. (The resistance to growth was now significantly larger.) Thus, the proposed explanation of the BC/AD transition is that it was caused by strong exogenous forces of political nature, forces causing the wide-spread and profound changes in the style of living. During that time, the resistance to growth was changing and eventually settled along a significantly larger value.

Demographic transition between AD 1200 and 1400 is much easier to explain. During that time, there was a temporary delay in the growth of human population. When closely inspected, it can be found that this delay coincides with *the most unusual convergence of demographic catastrophes*. It appears to have been caused by a *combined impact of five* large demographic catastrophes (Nielsen, 2016a): Mongolian Conquest (1260-1295) with the total estimated death toll of 40 million; Great European Famine (1315-1318), 7.5

million; the 15-year Famine in China (1333-1348), 9 million; Black Death (1343-1352), 25 million; and the Fall of Yuan Dynasty (1351-1369), 7.5 million.

During this transition, hyperbolic growth changed to a slightly faster trajectory, characterised by k only about 30% higher. This is the only available evidence that the growth of human population might have been affected by demographic catastrophes. However, their combined impact was small. The transition to a faster trajectory quickly compensated for the loss of time in the growth of population. This quick process of recovery could be explained by the regenerating impacts of Malthusian positive checks (Malthus, 1798; Nielsen, 2016f).

Currently, after a minor boosting around 1950, the growth of human population is slowing down. The possible explanation of the current diversion to a slower trajectory appears to be of *endogenous nature associated with human choices and motivations, voluntary or enforced*. While in many countries there is an increasing tendency to opt for smaller families, in China, small families have been enforced by legislation. This additional force appears to be the force of preventative checks (Malthus, 1798). They may have been active in the past but they were too weak to shape the growth trajectories.

So, the three demographic transitions in the past 12,000 years, including the ongoing transition, can be probably explained by three different forces: political forces active during the first transition, which lasted for about 1000 years; forces of demographic catastrophes, which were active for about 200 years; and the endogenous forces of personal choices, either voluntary or enforced by law during the current transition.

It would be difficult to describe mathematically all these complex forces. However, as already mentioned, the first two transitions were between hyperbolic trajectories characterised by different k factors. During these transitions, the k factor was changing. During the first transition, k factor dramatically decreased, which means that the resistance to growth dramatically increased. It increased by a massive factor of about 6.4. During the second transition, k factor slightly increased. The resistance to growth decreased by about 30%. The description of the past and present demographic transitions can be reduced to the description of changes in the compliance factor or in the corresponding resistance to growth. Resistance to growth was changing and we can study how it was changing. Such a study will not give a complete mathematical explanation of the mechanism of demographic transitions but will reduce this explanation to a single parameter: to changes in the compliance factor k or in the corresponding resistance to growth.

We can study these changes using a slightly modified eqn (8). If we assume that the resistance to growth was dependent on time (or equivalently that k depended on time), then we shall have the following equation describing growth trajectories during demographic transitions:

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = k(t)S(t) . \qquad (12)$$

Now, for better clarity, we are showing explicitly the dependence on time.

Solution of this equation is

$$S(t) = -\left[\int k(t)dt\right]^{-1} . \qquad (13)$$

If we assume that $k(t)$ is represented by an n-order polynomial,

$$k(t) = \sum_{i=0}^{n} a_i t^i \ , \qquad (14)$$

then

$$S(t) = \left[\sum_{j=0}^{n+1} b_j t^j\right]^{-1} , \qquad (15)$$

where $b_j = -a_{j-1} / j$ for $j > 0$ and b_0 is the constant of integration.

Even though we cannot describe mathematically the mechanism of growth during the demographic transitions, we can understand them a little better by studying changes in the growth factor $k(t)$, whose reciprocal values represent resistance to growth. Results are shown in Figure 3. The corresponding parameters are listed in Table 1. These calculations do not explain *why* the resistance to growth was changing (they do not explain the mechanism of the demographic transitions) but at least they are describing *how* the resistance to growth (or the compliance factor) was changing.

In the lower section of Figure 3, we show the growth trajectory during the AD era. It is made of two hyperbolic trajectories, between AD 500 and 1200 and between AD 1400 and 1950. The remaining segments of time represent demographic transitions described by the reciprocal values of polynomials, as given by the eqn (15). This section shows also one of the projected trajectories.

In the middle section, we show the overall fit to the data, which is represented by hyperbolic trajectories between 10,000 BC and 500 BC, between AD 500 and 1200 and between AD 1400 and 1950. The remaining segments of time represent demographic transitions described by the reciprocal values of polynomials [see eqn (15)].

In the top section, we show time dependence of the compliance factor $k(t)$, which can be calculated using the fitted $S(t)$. As we can see from the eqn (13)

$$k(t) = -\frac{dZ(t)}{dt}, \tag{16}$$

where $Z(t) \equiv S^{-1}(t)$.

In Figure 3, we show the compliance factor $k(t)$ only down to 2000 BC. However, this factor was constant between 10,000 BC and 500 BC but then started to decrease. The compliance was decreasing, the resistance to growth was increasing and the growth of population was slowing down. Around 80 BC, the compliance factor decreased to zero, the resistance to growth increased to infinity and the growth of population reached its maximum. The compliance factor continued to decrease and the size of population was decreasing. When the compliance factor reached its minimum, around AD 200, there was a turning point in the growth of population. The compliance factor was still negative but now it was increasing. Slowly, the deceleration in the growth of population was decreasing. Around AD 450 the compliance factor reached its second value of zero. The size of the population reached a minimum value and started to increase. By around AD 500, this demographic transition was over and the growth of population settled again along an unconstrained hyperbolic trajectory, but now is was a significantly slower trajectory characterised by a significantly smaller compliance factor or equivalently by the significantly larger resistance to growth.

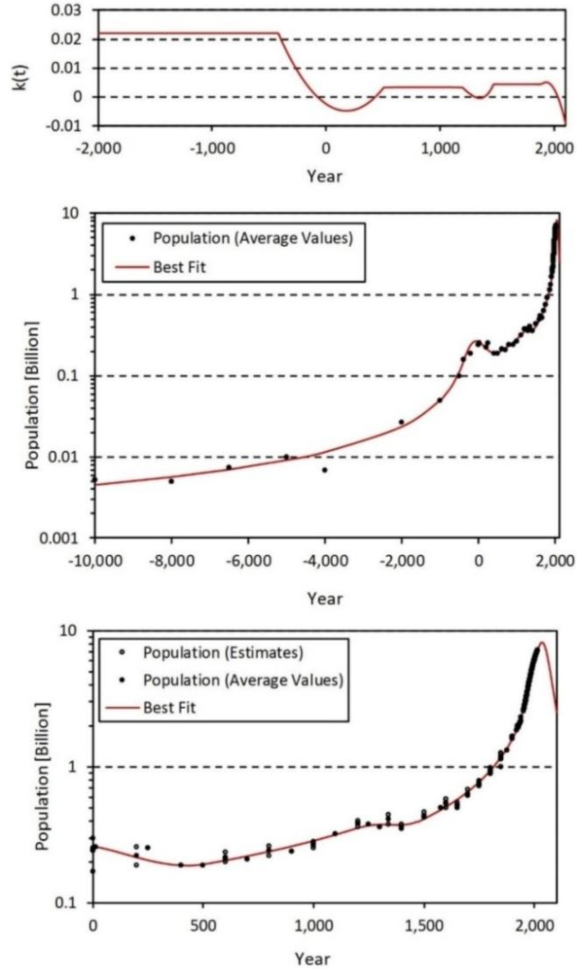

Figure 3. *Growth of the world population in the past 12,000 including mathematical description of the past two demographic transitions between hyperbolic trajectories and the ongoing transition to a yet unknown trajectory.*

Table 1. *Parameters describing the growth trajectory of the world population in the past 12,000 years.*

Unconstrained, hyperbolic growth (~89% of the total combined time) $k = const$	Demographic transitions (~11% of the total combined time) $k(t) = \sum_{i=0}^{n} a_i t^i$
10,000 BC – 500 BC $a = -2.282$; $k = 2.210\times10^{-2}$	500 BC – AD 500 $a_0 = -2.347\times10^{-3}$, $a_1 = -2.659\times10^{-5}$, $a_2 = 7.479\times10^{-8}$
AD 500 – 1200 $a = 6.940$; $k = 3.448\times10^{-3}$	AD 1200 – 1400 $a_0 = -1.022$, $a_1 = 2.618\times10^{-3}$, $a_2 = -2.198\times10^{-6}$, $a_3 = 6.068\times10^{-10}$
AD 1400 – 1950 $a = 9.123$; $k = 4.478\times10^{-3}$	1950 – present $a_0 = -1.820$, $a_1 = 1.891\times10^{-3}$, $a_2 = 4.899\times10^{-7}$

The onset of the second demographic transition occurred around AD 1200. Again, the compliance factor started to decrease and the growth of population started to slow down and even briefly decline. However, the growth quickly recovered and by around AD 1400 this short-lasting transition was over. Growth of population resumed its spontaneous and preferred hyperbolic trajectory, which was even a little faster than before, as indicated by the slightly larger k factor.

Around 1950, the compliance factor was boosted but only for a short time. The growth of population started to be a little faster than before, but very soon this temporary boosting was halted and the growth of population started to slow down as expressed in the continually decreasing compliance factor k.

Characteristic properties of hyperbolic growth

Hyperbolic growth might be more common than we think. In order to understand this type of growth it is useful to compare it with other processes and particularly with the more familiar exponential growth.

For the exponential growth, the size added per fixed unit of time is directly proportional to the total size of the growing entity, e.g. to the size of the population or the GDP (if they are assumed to increase exponentially). If the total size doubles, then the added size per unit of time also doubles. For the hyperbolic growth, the size added per fixed unit of time depends quadratically on the total size of the growing entity. If the size of the growing entity doubles, the added size per fixed unit of time quadruples. If the size triples, the added size per fixed unit of time increases nine-folds.

For the exponential growth, the doubling time is constant. For the hyperbolic growth, it decreases linearly with time. As the size of the growing entity increases, the doubling time decreases. Each consecutive doubling time is twice as small as the immediately preceding doubling time. So, for instance, if at a certain stage of growth, the doubling time is 24 years, then after 24 years it will be reduced to 14 years, after 14 years to 7 years, and so on. That is why, hyperbolic growth, or any other type of growth, but exponential, should never be characterised by the doubling time. Constant doubling time applies exclusively to the exponential growth.

For the exponential growth, the total driving force is constant. No matter how large is the growing entity, the force remains unchanged. Driving force per single unit decreases exponentially. If exponential growth were to describe economic growth then the driving force per unit of invested wealth, e.g. the driving force per invested dollar, would decrease exponentially with the size of the investment. The potential to generate economic growth per dollar would be decreasing exponentially with the size of the GDP. If exponential growth were to describe the growth of population, then the biologically driven force of procreation (the difference between the biologically generated or controlled birth and death rates) per person would be decreasing exponentially.

For the hyperbolic growth, the total driving force increases hyperbolically, i.e. in the direct proportion with the size of the growing entity, which means that the driving force per unit or per element of the whole assembly of hyperbolically growing entity, e.g. per person or per dollar, is constant. Each unit, on average and for a large assembly of growing units, contributes equally to support growth. For the hyperbolic growth, the potential of each invested unit of wealth, e.g. the potential of each dollar to create more wealth is constant. It does not depend on the size of invested wealth; it does not depend on the size of the GDP. For the hyperbolic growth of human population, the force of procreation per person remains constant; it does not decrease with the size of population.

For the hyperbolic growth, each element, each added component, makes on average, *a fixed contribution* to the overall driving force. Individual contributions may vary, but on average *the contribution of each component is constant over time*. The larger is the size of the growing entity the larger is the *combined force* pushing the growth forward. It is the growth that propels itself in a very specific way. In the unconstrained hyperbolic growth, the growth is propelled by the approximately equal contribution of *all* individual members of the growing entity. It is an interesting and distinct process where *growth generates growth* in a very specific way, i.e. where the driving force of growth per person or per unit of the growing entity is *constant*. In contrast, for the exponential growth, the *combined* driving force is constant but the driving force per unit of the growing entity *decreases* exponentially.

Now, we can see that there might be more examples of hyperbolic growth. Take, for instance, technology or knowledge. Knowledge generates knowledge by stimulating new ideas. Technology generates technology by stimulating new solutions to technological problems. This is the well-known process, which *even a single person can experience*. The more we learn, the easier it is to learn more. The more problems we solve, the easier it is to solve new problems. Ideas create new ideas, solutions create new solutions, and knowledge creates new knowledge. It is, therefore, not surprising that knowledge and technological innovations appear to have been increasing hyperbolically (Kurzweil, 2006; Vinge, 1993). There is a close correlation between the growth of population and technology (Kremer, 1993). The two processes are similar but they are prompted by different kind of forces.

Technology is certainly not prompted by the force of procreation (the biologically prompted sex drive and the biologically prompted process of aging and dying). The growth of population is obviously controlled by these processes. It could be also controlled by some additional forces but the historical growth of population shows that these other forces were either too weak or that they were averaging out.

Technology is prompted by concepts, solutions and by research activities. Growth of population is definitely not prompted by technological concepts, solutions and by research activities but by the force of procreation. Economic growth is similar to the growth of population but it is obvious that economic growth is not prompted by the biologically controlled force of procreation.

Another example of hyperbolic growth could be the growth of biodiversity. We could expect that biodiversity should generate greater biodiversity through competition, adaptation and biological solutions based on life-supporting mutations. We can also expect that the force driving the growth of biodiversity is proportional to the existing biodiversity. If it is directly proportional, then the growth of biodiversity is hyperbolic. Even if we consider minor or major extinctions of species one might expect that over a sufficiently long time the prevailing trend might be hyperbolic. If we think in terms of driving forces, we could probably identify other examples of hyperbolic growth. We can also understand easier the distinctions between various types of growth.

For processes described by hyperbolic trajectories, each system will be prompted *by its own mechanism reflected in a specific driving force*, but each system will be prompted by the same *type* of force. In each case, the force per unit of the growing entity will be constant. Hyperbolic similarities and close correlations between hyperbolic systems should never be interpreted as necessarily reflecting precisely the same mechanism of growth represented by precisely the same driving force. In general, each hyperbolic process will be expected to be propelled by a distinctly different force reflecting a distinctly different mechanism, but all these forces will be of the same *type*: their intensity will increase in the direct proportion to the size of the growing entity; their intensity per person, per biological object, per unit of measurement (such a dollar, for instance) will be always constant during the entire time of the unconstrained growth.

Hyperbolic growth is characterised by singularity where the growth escapes to infinity at a fixed time. Such a growth might be deemed impossible. However, historical economic growth and historical growth of populations were hyperbolic so obviously, they were possible. Growth trajectories can change and there is nothing unusual about that. A new force may be added to the existing force or the previously active force might be replaced by a new force. In the growth of global population there were only two instances in the past 12,000 years when a new force of growth was added temporarily to the force of procreation. First time, this additional force appears to be of political nature changing radically and on a large scale the style of living. Second time, it was in the form of demographic catastrophes, the only known case when demographic catastrophes were reflected in the trajectory describing the growth of population. Currently, there is also a diversion to a new trajectory. The force of procreation continues to be active but the new and significant force added to the force of procreation appears to be the force of preventative checks (Malthus, 1798).

It is also absolutely not necessary to imagine that in order to avoid the problem of singularity we have to find some mathematically-described force, which over a certain time would mimic hyperbolic growth but at around a certain time would gradually become non-hyperbolic, and that this unusual and yet unknown mathematical distributions would also reproduce the growth of human population. It is absolutely not necessary "to eliminate the unrealistic 'demographic explosion' from the model" (Karev & Kareva, 2014, p. 76), because it is not at all unusual for a trajectory to remain undisturbed over a certain time but then to be diverted to a new trajectory. The mechanisms of growth can change or can be modified by adding new type of force to the already existing force. We do not have to imagine that we should have a single force, which over a long time would describe hyperbolic growth and then would also describe a diversion to a new, non-hyperbolic growth. Karev attempted to find such a force but failed (Karev, 2005). He tried two such forces but they did not explain the mechanism of growth because they were incomprehensibly complicated (Nielsen, 2016g). They were also unsuccessful in describing the growth of population. A single and easy to understand force of procreation results in a far better description of data.

Current growth of population and economic growth is no longer described so consistently by a single type of force. For instance, economic growth in Greece was logistic over a certain time but then it changed to a pseudo-hyperbolic growth with singularity in 2017 (Nielsen, 2016h). This fast growth could not have been supported in any way and it collapsed. The current global economic growth is exponential (Nielsen, 2016i). Such a growth is insecure because it does not lead to a maximum or to a safe and sustainable level of the GDP. It continues to grow until it can be no longer supported.

The current global growth of population is less clearly defined and its projections are less certain. Analysis of the growth rate shows that growth of population may reach a certain maximum but it may also continue to increase for as long as it can be supported by the availability of natural resources (Nielsen, 2006).

Summary and conclusions

Historical economic growth and historical growth of population were hyperbolic (Nielsen, 2016a, 2016b, 2016c). We have explained their mechanism by postulating simple forces of growth. Hyperbolic growth is mathematically simple and its mechanism of growth is also simple.

For the economic growth, the mechanism of the historical hyperbolic growth is explained by *the net market force*, which on average was *directly proportional to the invested wealth* usually expressed as the Gross Domestic Product. For the growth of population, the mechanism of the historical hyperbolic growth is explained by *the biologically prompted force of procreation* defined as the difference between the biologically prompted birth rate and the biologically controlled process of aging and dying. It is assumed that this force was on average *constant per person.*

We do not explain the net market force and neither do we explain the biological force of procreation. We do not dissect these processes, isolate their components, study minute interactions between the mand then put them together to derive the net driving force. We only describe these forces in the simplest possible way using simple mathematical expressions based on simple and readily acceptable assumptions. We then use these simplified forces to explain the mechanisms of growth.

This type of approach is common in scientific investigations. For instance, we do not understand the force of gravity. We do not really know what it is. However, we can represent this force using a simple mathematical expression (Newton, 1687) and then use it to explain the mechanism of the movement of celestial by, we can land a man on the Moon and bring him back to Earth, explore our solar system, land our probes on Mars, detect the presence of the invisible matter and in general explain the dynamics of the Universe.

We do not understand nuclear forces but we can describe them mathematically and use this description to study, for instance, the mechanism of nuclear reactions and nuclear structure (Nielsen, 2016). Nobody understands quantum mechanics (Feynman, 1967) but this does not stop us from describing mathematically various quantum phenomena, explain them and even use the acquired knowledge to apply it, for instance, in quantum computing or cryptography. We do not understand the weak force and yet we can explain the process of radioactive decay and use radioactive isotopes in many applications, primarily in medicine but also industry and agriculture.

We do not understand why matter reveals itself as mass or energy. We do not understand the intricate details of this peculiar phenomenon but we can describe it by a simple and well-known equation (Einstein, 1905a). We can then use this simple equation to calculate how much energy will be released if a certain amount of mass manifests itself as energy. We can use this knowledge, combined with our fundamental knowledge of nuclear processes, to explain the mechanism of fusion and fission reactions. We can then go a step further and construct (unfortunately) a nuclear bomb and (maybe similarly unfortunately) to construct a controversial nuclear reactor to produce energy. However, we can also explore how this huge amount of energy locked in the mass could be used in a controlled fusion reaction and maybe at last to construct a clean and practically inexhaustible source of energy. We can also use this simple mass-energy relation to explain the mechanism of the production of energy in our Sun and in the distant stars. We do not know everything but what we already know can be useful.

We do not understand why electromagnetic radiation reveals itself as waves or particles, the property, which turns out to apply not only to electromagnetic radiation but also to all matter, but we can describe this relationship by simple mathematical expressions (Einstein, 1905b; de Broglie, 1924) and explain not only why the rainbow looks so nice but also the strange phenomenon of photoelectricity (Einstein, 1905b). Einstein is well known for his theory of relatively and for his mass-energy equation but he received his Nobel Prize for explaining photoelectricity, which demonstrates that light can manifest itself as being made of tiny particle.

We may not know all the details how nature works but we can still explain many phenomena we observe and even represent our explanations by useful and often simple mathematical expressions. We might not be able to explain everything. We might not answer every single question but we can still explain many phenomena in a satisfactory manner and answer many questions. A deeper understanding might come much later but only if we make sure that our current knowledge is not based on illusions and impressions but on the methodically checked interpretations of observed phenomena.

The fundamental principle in scientific research is to look for the simplest explanations of observed phenomena. These few examples from physics show that even complicated processes can be often represented by simple mathematical descriptions and that the interpretation of their mechanism can be significantly simplified.

Distributions describing historical growth of population and the historical economic growth look complicated, so complicated that they are routinely interpreted as being made of two distinctly different components, slow and fast, stagnant and explosive, each component governed by distinctly different and complicated mechanisms. The illusion is so persuasive that even most prominent researchers are easily misguided, particularly if the data are not properly analysed or if they are presented in a grossly distorted way (Ashraf, 2009; Galor, 2005a, 2005b, 2007, 2008a, 2008b, 2008c, 2010, 2011, 2012a, 2012b, 2012c; Galor & Moav, 2002; Snowdon & Galor, 2008).

The first indication that these distributions are not complicated is demonstrated when they are mathematically analysed. The analysis is trivially simple (Nielsen, 2014) and it shows that these distributions are hyperbolic (Nielsen, 2016a, 2016b, 2016c). Hyperbolic distributions look complicated but they are described by an exceptionally simple mathematical formula: a reciprocal of a linear function containing just two adjustable parameters.

This remarkable simplicity of hyperbolic distributions representing the historical growth of population and the historical economic growth suggests a simple mechanism of growth. We have now demonstrated that the mechanism of these two processes is indeed remarkably simple. They were prompted by the well-known and simple forces.

Data describing the growth of global population allow for a study of growth over an exceptionally long time. They show that for the most part of the past 12,000 years, growth of global population was hyperbolic: between 10,000 BC and around 500 BC, between around AD 500 and 1200 and between around AD 1400 and 1950. The remaining time of the past 12,000 years was taken by demographic transitions: between around 500 BC and AD 500, between around AD 1200 and 1400, and from around 1950.

We have proposed the explanation of the mechanism of these transitions. The first transition is explained by the dramatic and wide-spread changes in the style of living associated with significant changes in the political landscape. The second transition is explained as being caused by the combined impact of five major demographic catastrophes. This is the only example when demographic catastrophes appear to have had impact on shaping the population growth trajectory. However, this impact was insignificant. The slight delay in the growth of population was soon compensated because the growth of population was diverted to a slightly faster trajectory. We can explain the mechanism of this quick recovery by the regenerating effects of the Malthusian positive checks (Malthus, 1798; Nielsen, 2016f). The mechanism of the ongoing transition is explained by the Malthusian preventative checks.

A partial mathematical explanation of these transitions is by assuming that the growth of human population was still prompted by the biologically controlled force of procreation but that the resistance to growth (or equivalently the compliance factor) was changing. This simple assumption does not allow us to predict growth trajectories during demographic transitions but only to determine how the resistance to growth (or compliance factor) was changing during each transition.

Currently, neither the growth of population nor the economic growth can be described by the historically simple driving force. Generally, we have to use different descriptions for each specific case. For instance, current global economic growth can be described by a relatively simple but non-hyperbolic trajectory, which is now converging into the exponential growth (Nielsen, 2016i). Economic growth in Greece was logistic but then it was converted to a fast-increasing pseudo-hyperbolic growth, which inevitably resulted in the economic collapse because it came too close to the point of singularity (Nielsen, 2016h). Global growth of population can be described using different trajectories, each trajectory giving different prediction of growth (Nielsen, 2006).

The general law of growth (Nielsen, 2016d) helps to understand mechanisms of growth because it links growth trajectories with driving forces, which are usually easier to visualise and to understand. We have used this general law of growth and the simplest driving forces to explain the mechanism of the historical growth of population and the historical economic growth.

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8. New direction for the economic and demographic research

1. There is no need to explain the mechanism of the so-called Malthusian stagnation, or the Epoch of Malthusian Stagnation, as Galor called it, because there was no stagnation. Growth of the world population and economic growth in the past 2,000,000 years were steadily increasing along the predominantly hyperbolic trajectories. There were only two major transitions and one minor disturbance in the past 2,000,000 years. There is also now a slow transition to a slower pattern of growth, but there was never a prolonged epoch of stagnation that needs to be explained. This supposed stagnation is based on pure imagination stimulated by the misinterpretation of hyperbolic growth, which is slow over a long time but never stagnant.
2. There is no need to explain the so-called Malthusian trap. Malthus never proposed any form of a trap in the growth of population and in economic growth. This concept was invented later and for no justified reason the name of Malthus was attached to it. For no justified reason his name was also attached to the erroneous concept of stagnation. Malthus was cautious with his observations and it is not certain whether he would be pleased by this dubious distinction. There was no trap in the growth of population and in economic growth.
3. There is no need to explain the mechanism of Galor's three regimes of growth because they did not exist. This concept is based on fantasy reinforced by suitably distorted representations of data. His three regimes are consistently contradicted by data.

4. There is no need to explain a sudden increase in the growth of population or in economic growth labelled as takeoffs or in any other similar way, which is supposed to describe a sudden change in the pattern of growth, because there were no such sudden accelerations. Hyperbolic growth is slow over a long time and fast over a short time but there is no sudden transition from a slow to fast growth. There is, therefore, no need to explain also the so-called differential takeoffs, which were supposed to describe different time of takeoffs in the growth in more developed and less developed regions, as claimed by Galor, because there were no takeoffs. The concept of takeoffs is based on the incorrect interpretation of hyperbolic growth but the concept of differential takeoffs is harder to explain. It adds a new flavour to the incorrect interpretation of growth. Takeoffs or differential takeoffs never happened. There is, therefore, no need to explain their mechanism. There is also no need to explain the transition from stagnation to growth, claimed by Galor, because there was no such transition at any given time. If this claimed transition is supposed to describe the transition from a slow to fast growth, then it did not happen at any given time but monotonically over a long time. The time of the transition from slow to fast hyperbolic growth takes place over the whole range of hyperbolic growth. It is impossible to determine the time of this transition.
5. There is no need to explain the boosting effects of the Industrial Revolution because Industrial Revolution had no impact on shaping the economic growth trajectories and trajectories describing the growth of population. There was no boosting in the growth trajectories even in Western Europe and even the United Kingdom, the epicentre of the Industrial Revolution. This event introduced many changes in socio-economic conditions but it did not change trajectories describing the growth of population or economic growth.
6. There is no need to explain Galor's mysteries of growth because there are no mysteries. His mysteries of growth were created by his habitual distorted representations of data.
7. There is no need to explain the sudden spike in the growth rate describing growth of income per capita, as claimed by Galor, because there was no such sudden spike. Growth rate of income per capita was increasing monotonically without a sudden boosting at any time.
8. There is no need to explain the "mind boggling" and "perplexing phenomenon of the Great Divergence in the income per capita" claimed by Galor, because there was no Great

Divergence. Galor created the Great Divergence by his habitual distorted representations of data.

9. There is no need to explain the puzzling features of income per capita distributions and no need for proposing different mechanisms of growth to different perceived parts of these distributions because their puzzling features are no longer puzzling. They reflect the general mathematical property of dividing two hyperbolic distributions and they have nothing to do with an imagined mechanism attributable only to the growth of income per capita. Any two hyperbolic distributions, when divided, display the same pattern of growth. The only requirement is for the numerator to be characterised by an earlier singularity than the denominator.
10. Malthus noticed the dichotomous effects of his positive checks: they increase the rate of mortality but they also stimulate growth. He also suggested that these stimulating effects should be further investigated. Unfortunately, his seminal observation was ignored. The investigation of Malthusian positive checks demonstrated that he was right. They do not cause stagnation but stimulate growth by increasing the growth rate.
11. There is no need to include impacts of demographic catastrophes to explain the mechanism of growth of human population, because with only one notable exception they did not shape growth trajectories. The absence of their impacts can be explained by the fundamental properties of Malthusian positive checks and by the relatively small intensity of demographic catastrophes. They might have had significant local impacts but they did not shape the growth of global population and there is also no convincing evidence that they were shaping the growth of regional populations. The notable exception was between AD 1195 and 1470 when a minor distortion in the growth trajectory of the world population coincides with an unusual convergence of five major demographic catastrophes. The resulting short delay in the growth of population was quickly compensated by the renewed hyperbolic growth from AD 1470, which was even faster than before AD 1195.
12. Demographic Transition Theory and the Unified Growth Theory are contradicted by data, even by the same data, which were used in unsuccessful attempts to support these two theories because data were never rigorously analysed. These two theories have no useful place in the demographic and economic research. They are misleading, incorrect and scientifically unacceptable.

13. There has been too much fantasy in the demographic and economic research, which is in a way understandable because good sets of data were not available. Now they are. Significant changes have to be made if these two fields of research are meant to be recognised as science. Demographic and economic research has to be based on accepting that the historical growth of population and economic growth were following hyperbolic trajectories. Correct understanding of hyperbolic distributions is the fundamental requirement for the correct understanding of the past growth of population and of economic growth.
14. Mechanism of the hyperbolic economic growth and of the hyperbolic growth of population has been now explained. Hyperbolic growth is the simplest, unconstrained, growth prompted by the simplest fundamental forces. In the case of the growth of population, it is the indispensable, biologically-controlled, force of procreation, which, includes birth and death rates and which is on average constant per person. In the case of the economic growth, it is the simplest market force, which is directly proportional to the generated wealth. These were the dominating forces of growth in the past and they were shaping the growth of population and economic growth. This simple dominating mechanism was disrupted only three times in the past 2,000,000 years (1) between 46,000 BC and 27,000 BC when there was a transition from a slow to a fast hyperbolic growth, (2) between 425 BC and AD 510 when there was a transition from a fast to a significantly slower hyperbolic growth, and (3) between AD 1195 and 1470, when there was a minor disturbance coinciding with the unusual convergence of five exceptionally strong demographic catastrophes. From around 1950, other forces also started to contribute strongly to the growth of population and to the economic growth. Growth is no longer hyperbolic, as it was in the past, but it was diverted to slower trajectories. Future growth trajectories are unknown. However, there appears to be now no more room for a continuation of hyperbolic growth.
15. The currently accepted interpretations of the mechanism of economic growth and of the growth of population are not only in conflict with data but they are also dangerously misleading because they create the sense of security by claiming that after a long time of stagnation, extending over thousands or even millions of years, we have finally escaped the alleged, but non-existent, Malthusian trap and entered into the so-called sustained growth regime. Scientific evidence shows that the opposite is true. The past growth was stable, secure and

sustainable, as demonstrated by the largely stable hyperbolic distributions. In contrast, it is the current growth, which is potentially unsustainable. For the first time in human existence, human ecological footprint is larger than ecological capacity and it continues to increase. For the first time in human history we support our existence on the continually increasing ecological deficit.

16. Conjectures are acceptable in scientific research but they have to be controlled and moderated by a rigorous analysis of data. When conjectures are supported by conjectures, the created system of doctrines and explanations becomes quickly scientifically untenable.
17. Growth of the world population is now following a gradually decelerating trajectory. This slowing down growth commenced around 1963, after an initial minor boosting around 1950. From 1950, global economic growth also started to follow a slower trajectory. It is unfortunate that after the initial decline, growth rate describing global economic growth settled along a constant value, which describes exponential growth. There is a strong possibility that growth of population might also settle along an exponential trajectory because the growth rate is now decreasing so slowly that it might easily become constant. There is now an urgent need in the demographic and economic research to look for ways of making the growth of population and of economic growth sustainable.

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Ron W. Nielsen

Ron William Nielsen (aka Jan Nurzynski) was born in Poland. He studied physics and mathematics at the Jagiellonian University, Krakow, Poland. He is a nuclear scientist. He has carried out his research work in the Department of Nuclear Physics in the Institute of Advanced Studies at the Australian National University, Canberra, ACT, Australia but initially he worked briefly in the Institute of Nuclear Physics, Krakow, Poland. He also carried out his research work as a visiting professor in nuclear research centres in Switzerland and Germany. The objective of his research work was the investigation of the mechanism of nuclear reactions, application of nuclear reactions to nuclear spectroscopy and a study of nuclear polarization phenomena. He supported his work by using particle accelerators (cyclotrons and a electrostatic accelerators), a wide range of particle detection techniques and a wide range of mainframe computers for the theoretical interpretations of his experimental results. His work in nuclear physics is summarised in "Nuclear Reactions: Mechanism and Spectroscopy". After his retirement, he became interested in environmental issues and published a comprehensive "Green Handbook", where he discusses all critical events shaping our future. This book was endorsed by academics, including Nobel Laureate Prof. Dr. Paul Crutzen from Germany, and by other readers. More recently, he focused his attention on the issues associated with the growth of population and economic growth. The primary aim was to understand the Anthropocene, the recent strong anthropogenic activities and impacts, which have been proposed as a transition to a new geological epoch. In connection with this study, he formulated two analytical methods: the general law of growth, which can be used to study mechanism of any type of growth, and the related method based on the application of differential equations in the analysis of data and in predicting growth.

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