Ana Paula Martins

WAGES, WEALTH AND GROWTH

with Portugal case

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Wages, Wealth and Growth: with Portugal case

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Preface

Ch.1) This chapter explores the dynamic potential of point-wise utility functions optimization of representative agent economies. Such functions were generically considered to depend upon current consumption and wealth to be made available for next period usage or income generation, implying an endogenous (pseudo-)rate of time preference. At first inspection, the framework reproduced closely the dynamics and steady-state properties of the traditional Solow-Swan and Ramsey models – with population growth, exogenous technical progress, land, or increasing returns to scale - as well as, when human capital/knowledge was introduced, the Lucas-Uzawa endogenous growth set-up. General uncertainty – simulated at different decision stages - resulted in intuitively appealing solutions. Overlapping optimization of the capital stock generated forward-looking recursive dynamics. Homothetic preferences (CES or generalized Cobb-Douglas) - implying a constant consumption-(lead)wealth ratio along an optimal path and resulting in steady-state saving rates independent of CRS technologies features in simple structures -, were assumed for illustration, and also generic separable forms in the arguments. The latter were useful under uncertainty, allowing the inspection of the role of risk-aversion and diminishing marginal returns to capital in equilibrium and steady-state determination.

(Ch.2) This chapter extends the standard closed shop union model of wage determination by introducing endogeneity of union membership. The labor market outcome with endogenous membership may differ when unions behave monopsonisticaly relative to the case where they are "membership-takers", resulting in higher or lower wages (more or less favorable contract curve in efficient bargaining) according to the form union´s utility function and/or implicit decision process value union size. Some notes are added highlighting the role of membership fees in the membership function determination of a union that works as a nonprofit organization.

(Ch.3) This chapter discusses the relation between centralization in union bargaining and the wage-(un)employment mix. Empirical findings point to a positive relation between the degree of coordination in union bargaining and wages till a certain point, and a negative one afterwards. A theoretical argument fits such evidence, relying on the mechanism behind the free-rider problem in union bargaining. If earnings taxes were introduced to finance the unemployment insurance fund, that relation could change. The impact on the equilibrium wages and multipliers in the several scenarios is briefly explored. Indirectly, an explanation for the shape of the empirical "wage curve" is also derived.

(Ch.4) It is the purpose of this chapter to present some estimates of human capital earnings functions for Portugal, using published data on mean earnings by age, education and sex. We provide estimates of the implicit rates of return to human capital - schooling and general O.J.T. Differential effects by sex are discussed. An application of the methodology is used to analyze returns differentials between different schooling categories. Research on the specification of the earnings-experience profiles is also performed.

A.P. Martins
November 10, 2019

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1 Wealth-in-utility and time-consistent growth: Real excursions with an "overlapping" welfare function

Introduction

Maximization of an inter temporal utility function, usually with the form of accumulated discounted felicity (per period utility), has become a common objective of the representative agent in most macroeconomic and growth models. The formulation, after Ramsey (1928)[1], has proved successful in generating long-run and specially short-run and cyclical insights in the most varied economic subjects; however, it has the disadvantage of generating time inconsistent results. It is the purpose of this research to propose an alternative modelling framework capable of circumvent such shortcoming: assume that individuals proceed to the point-wise (or per period, in discrete time) maximization of an - eventually time indexed – utility function, with two types of arguments: perishable items (consumption, leisure), and assets. It is through the latter – a self as general bequest carried over to the next period - that concerns over future consumption are internalized.

On a superficial appraisal, the formulation would remind Sidrauski's (1967) money-in-utility functions. Or spirit-of capitalism-ones[2]. In fact, it is quite different on both the rationale as on the dynamic implications: in MIU models, money is imbedded in felicity functions – themselves measures of per period flows of individuals' well-being -, and its inclusion meant to represent its favorable role in transactions. In spirit of capitalism models, wealth is included in the felicity function of intertemporal utility and its inclusion has been able to account for the savings puzzle[3] and for the high volatility of stock prices[4]. Our purpose is different: wealth (real wealth…) captures all intertemporal welfare substitution pertaining to a decision period (horizon) – felicity discounting is no longer required or justified. One can say that the traditional growth models as Solow (1956) and Swan (1956), assuming a constant savings function, are, to some extent, an inspiration of such a device; of course, asset demand becomes clearer, more immediate, with the proposed objective function. Moreover, first-order conditions confer a permanent income or life cycle flavor to the optimal consumption path.

In fact, intertemporal dynamic effects are quite present: even if individuals maximize static utility functions, they are (still…) conditioned by existing past wealth – they prepare today the wealth stock that will be available next period. On the other hand, they must form expectations to correctly assess today's real value of their possessions – or in a general equilibrium framework, those expectations will concur to generate their actual value. If we introduce leisure, future work-consumption-wealth decisions will also affect today's value of what we can call "full-time wealth". Moreover, a wealth plan for two periods may be evaluated today (a conditional decision over capital made in the past "overlapping" with the one made in the present) – and forward dynamics arise, with the same potential as future expectations models.

Overlapping generations[5] can also be simulated through the aggregation of coexisting cohorts supplies and demands – with younger generations exhibiting a stronger preference for capital relative to older ones. Technically, with the proposed function it becomes a matter of heterogeneity of contemporaneous agents.

In this article, we concentrate on the study of the potential of the modelling device – and work with discrete variables, even if continuous-time generalizations are straight-forward. Hence, we set out to replicate the dynamics of some of the widely recognized neoclassical benchmark environments, but under the new representative agent's behavior: firstly, the basic Solow-Swan and Ramsey-Cass-Koopmans one-sector models with their multiple extensions. Secondly, the endogenous growth Lucas-Uzawa experiment. Finally, we digress over the mechanisms behind exogenous shocks and their volatility transmission – or heterogeneity… - to economic aggregates under the current framework.

The exposition proceeds as follows: section 1 introduces the utility function and the representative agent dynamic problem; short-run dynamics and steady-state properties are explored in section 2, with the supportive market equilibrium briefly justified in section 3. Exogenous technical progress and the Lucas-Uzawa hypothesis concerning human capital formation are analyzed in section 4, implications of increasing returns to scale and fixed resources (land) studied in section 5. Section 6 deals with the effects of exogenous uncertainty, experimenting with both additive and multiplicative shocks. In section 7, an enlarged utility function, including lead capital as well, originates a recursive solution. A final appraisal and possible extensions produce a concluding section.

The wealth-in-utility welfare function

We will assume a generic utility function:

$$U_t(c_t, w_t) \tag{1}$$

c_t denotes (per capita) consumption in period t, w_t is the stock of wealth the individual gathered in period t – made available in period t+1[6]. Theoretically, such type of "reduced" form arguments are suggested by Bellman's equation formulations of standard accumulated discounted felicity functions[7] – yet, these imply a special recursive structure of today's wealth evaluation which on the one hand, we leave free, and, on the other, we do not make correspondence to. Rather, a weight of future consumption is embedded in preferences over w_t.

Assume a simple economy: only capital, k_t, can constitute wealth. At each point in time, the representative consumer-producer must decide whether to produce investment goods, i_t, adding to his pre-existing capital stock, or consumption goods, c_t, exhausted in the period, which are homogeneously generated by a CRS production function, implying an average labor product one denoted by $f(k_t)$, with $f(0) = 0$ and$f_k(k_t) > 0$ around the relevant range of k_t:

$$c_t + i_t = f(k_{t-1}) \tag{2}$$

Each unit of capital depreciates at rate d per period. Wealth will evolve according to:

$$k_t = k_{t-1} + i_t - d\, k_{t-1} \tag{3}$$

Hence, at each point in time, *given* a level k_{t-1}, the representative agent's problem – assuming that the utility function is immutable, so that $U_t(c_t, k_t) = U(c_t, k_t)$ for all t - is:

$$\underset{c_t, k_t}{Max}\, U(c_t, k_t) \tag{4}$$

s.t: $$k_t = (1 - d)\, k_{t-1} + f(k_{t-1}) - c_t \tag{5}$$

Given k_{t-1}

or in lagrangean form:

$$\underset{c_t, k_t, \lambda_t}{Max}\, L(c_t, k_t, \lambda_t) = U(c_t, k_t) + \lambda_t\, [k_t - (1 - d)\, k_{t-1} - f(k_{t-1}) + c_t] \tag{6}$$

F.O.C., along with the restriction, require:

$$\frac{\partial L}{\partial c_t} = U_c(c_t, k_t) + \lambda_t = 0 \tag{7}$$

$$\frac{\partial L}{\partial k_t} = U_k(c_t, k_t) + \lambda_t = 0 \tag{8}$$

from where

$$U_c(c_t, k_t) \; = \; U_k(c_t, k_t) \qquad (9)$$

A first implication is therefore that in each period the consumer is going to equate marginal utility from consumption to that he derives from wealth available for next period income-product generation. At the empirical level, (9) suggests a relation – an "income-expansion" path - between consumption in a period and the lead stock of wealth: past consumption choices condition more closely the current level of attained wealth – and not the other way around...

Another is that if U(c, k) is homothetic, condition c_t and k_t will move at the same proportional change rate along any optimal path – i.e., c_t / k_t is kept constant. For instance:

1) A Cobb-Douglas utility function, $U(c_t, k_t) = A\, c_t^{◎} k_t^{◎}$, would generate: $c_t = \frac{\alpha}{\beta} k_t$. c_t / k_t increases with ◎ and decreases with ◎ .

2) A CES utility function, $U(c_t, k_t) = A \left[a_1 c_t^{\rho} + a_2 k_t^{\rho} \right]^{\frac{\mu}{\rho}}$ with $a_1, a_2 > 0$, $a_1 + a_2 = 1$, ◎≤ 1, would imply: $c_t = \left(\frac{a_1}{a_2} \right)^{\frac{1}{1-\rho}} k_t$ – where, as is well-known, $\frac{1}{1-\rho}$ = ◎ corresponds to the elasticity of substitution between the two arguments. c_t / k_t increases with a_1 and decreases with a_2; it increases (decreases) with ◎ provided $\frac{a_1}{a_2}$ > (<) 1.

For S.O.C. of the problem to hold, $U[c_t, (1 - d)\, k_{t-1} + f(k_{t-1}) - c_t]$ should be concave [8] in c_t, or $U_{cc} + U_{kk} - 2\, U_{ck} < 0$.

Short-run dynamics and steady-state properties

Along (9):

$$\frac{\partial c_t}{\partial k_t} = \frac{U_{kk} - U_{ck}}{U_{cc} - U_{ck}} = \frac{\dfrac{U_{kk} - U_{ck}}{U_k}}{\dfrac{U_{cc} - U_{ck}}{U_c}} \tag{10}$$

$\frac{U_{kk} - U_{ck}}{U_{cc} - U_{ck}}$ is expected to be larger than zero. It reflects how consumption is exchanged for capital along any optimal path, i.e., maintaining equality between the marginal utility with respect to capital and that to consumption – keeping the marginal rate of substitution between consumption and capital fixed and equal to 1. It has a similar status to a discount rate – the rate of time preference - in standard discounted utility models: a unit of capital available at the end of the period would be exchangeable or equivalently evaluated to perpetual future consumption flows at that rate, so that rate would be the trade-off with today's consumption, c_t we should be measuring by $\frac{\partial c_t}{\partial k_t}$ over an optimal path.

Under homothetic utility functions, $\frac{\partial c_t}{\partial k_t}$ is expected to be constant – once it is evaluated at (9); using the previous examples: for the Cobb-Douglas, $\frac{\partial c_t}{\partial k_t} = \frac{\alpha}{\beta}$; for the CES, $\frac{\partial c_t}{\partial k_t} = \left(\frac{a_1}{a_2}\right)^{\sigma}$.

Additively separable utility functions will imply $U_{ck} = 0$, and $\frac{\partial c_t}{\partial k_t}$ is just the ratio between the concavity (or absolute risk-aversion) of U in k to that in c.

An alternative definition of the rate of time preference would be

$$\frac{U_c(c_{t+1}, k_{t+1})[1 - d + f_k(k_{t+1})]}{U_c(c_t, k_t)} - 1 = \frac{U_k(c_{t+1}, k_{t+1})[1 - d + f_k(k_{t+1})]}{U_k(c_t, k_t)}$$

- 1. Such ratio would be suggested by the traditional F.O.C. of the Ramsey problem. With a minor adjustment:

$$\frac{U_c(c_{t+1},k_{t+1})[1-d+f_k(k_t)]}{U_c(c_t,k_t)} - 1 = \frac{U_c(c_{t+1},k_{t+1})[1-d+f_k(k_t)]}{U_k(c_t,k_t)} - 1 \tag{11}$$

would measure the relation between the marginal contribution of today's unit of capital (of the potentially consumable input) for tomorrow's utility – internalizing that $U(c_{t+1}, k_{t+1}) = U[(1 - d)\, k_t + f(k_t) - k_{t+1}, k_{t+1}]$ - relative to the immediate one, minus 1. And, then, in steady-states coinciding with $f_k - d$. One could say that if this definition measures how utility is implicitly evaluated, the other affers it in terms of the consumption capital trade-off. We shall prefer the former definition.

. The equation driving capital dynamics is (5) obeying (9),

$$\frac{\partial k_t}{\partial k_{t-1}} = (1 - d) + f_k(k_{t-1}) - \frac{\partial c_t}{\partial k_t}\frac{\partial k_t}{\partial k_{t-1}} = [1 - d + f_k(k_{t-1})] / (1 + \frac{\partial c_t}{\partial k_t})$$

Replacing (10) in the previous expression and solving for

$$\frac{\partial k_t}{\partial k_{t-1}} = [1 - d + f_k(k_{t-1})]\,\frac{U_{cc} - U_{ck}}{U_{cc} + U_{kk} - 2U_{ck}} \tag{12}$$

The dynamics of the system can now be studied with reference to the properties of (12). $\frac{\partial k_t}{\partial k_{t-1}}$ is expected to be positive provided U is concave in both arguments. $\frac{\partial k_t}{\partial k_{t-1}} < 1$ and the solution will be stable iff (around the steady-state)

$$f_k(k_{t-1}) - d < \frac{U_{kk} - U_{ck}}{U_{cc} - U_{ck}} = \frac{\partial c_t}{\partial k_t} \tag{13}$$

Otherwise, it will be unstable. Stability requires that the marginal product of capital, deducted of the depreciation rate (coinciding, in the steady-state, with our second alternative for the definition of the rate of time preference), be smaller than the pseudo-discount rate.

. Being stable, the system will converge to the solution for which $k_t = k_{t-1}$, that is, using (5):

$$c_t = f(k_t) - d\, k_t = f(k_{t-1}) - d\, k_{t-1} \tag{14}$$

positively sloped while $f_k(k_t) > d$ – certainly for low levels of k under diminishing marginal returns -, and obey (9): k* will be such that

$$U_c[f(k^*) - d\, k^*, k^*] = U_k[f(k^*) - d\, k^*, k^*] \tag{15}$$

(9) establishes an immediate "saddle-path" trajectory for contemporaneous consumption and capital to follow. The c_t on such path is reached from, for given k_{t-1}:

$$U_c[c_t, (1-d)\, k_{t-1} + f(k_{t-1}) - c_t] = U_k[c_t, (1-d)\, k_{t-1} + f(k_{t-1}) - c_t] \tag{16}$$

originating a slope

$$\frac{\partial c_t}{\partial k_{t-1}} = [1 - d + f_k(k_{t-1})]\ \frac{U_{kk} - U_{ck}}{U_{cc} + U_{kk} - 2U_{ck}} = [1 - d + f_k(k_{t-1}]\ \frac{1}{1 + \frac{1}{\frac{\partial c_t}{\partial k_t}}} \tag{17}$$

We can plot the implicit function (16) in space (k_{t-1}, c_t) – below on Fig. 1. Under stability, it will have a smaller slope than the saddle-path (9) evaluated at the lag – i.e., in co-ordinates (k_{t-1}, c_{t-1}), which is also plotted. We plot phaseline (14) – for the lag [9] - as well – above it k_{t-1} is decreasing: $k_t - k_{t-1} = f(k_{t-1}) - d\, k_{t-1} - c_t < 0$. Below (14), k_{t-1} is rising. (14) has slope, $\frac{\partial c_t}{\partial k_{t-1}} = f_k(k_{t-1}) - d$; it will be smaller than that of (16) provided that (13) holds, i.e., that the system is stable, - and then also smaller than the slope of $\frac{\partial c_{t-1}}{\partial k_{t-1}}$ from (9).

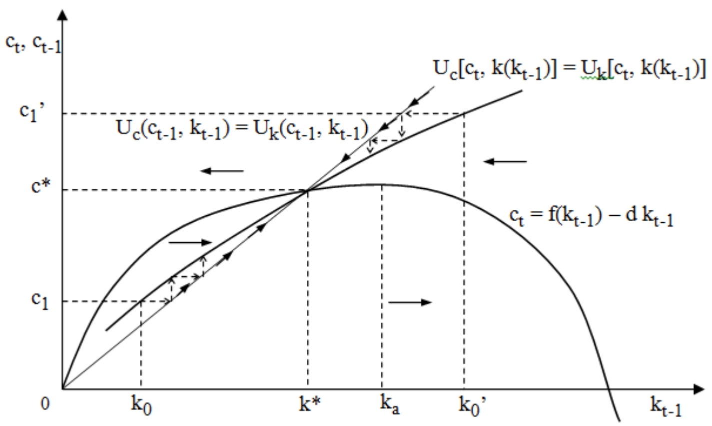

Fig. 1

If we start at a point like k_0 (k_0') consumption will be c_1 (c_1') on line given by (16), and k_1 (k_1') then is read over line (9): we follow the ascending (descendent) steps signaled in the Figure.

Under instability, (16) should have a smaller slope than (14) around the steady-state, and then also (9) would have a smaller slope than (16). The path would be divergent from the steady-state – where both lines meet -, but nevertheless fluctuate (within the space) between (new) lines (16) and (9).

Apparently, the saddle-path properties would resemble those of the Ramsey's problem – see Azariadis, p. 74, for example.

We did not find – unlike in the neoclassical framework – any reason why k_a, the point for which function (14) exhibits a maximum and, therefore, $f_k(k_a) = d$, should be larger than k*. Apparently, then, it is possible that $f_k(k^*) < d$. That might not be the case if we had postulated, instead of (5), a state equation $k_t = (1 - d) k_{t-1} + f(k_t) - c_t$, allowing k_t to be immediately available as a production input; not only that would not seem so realistic, as it would render manipulations somewhat more tedious.

. One can show, using (15), that

$$\frac{\partial k^*}{\partial d} = \frac{(U_{cc} - U_{ck})k^*}{(U_{cc} - U_{ck})(f_k - d) + U_{ck} - U_{kk}} = \frac{k^*}{f_k - d + \frac{U_{ck} - U_{kk}}{U_{cc} - U_{ck}}} = \frac{k^*}{f_k - d - \frac{\partial c_t}{\partial k_t}} \quad (18)$$

If the system is stable, $\frac{\partial k^*}{\partial d} < 0$.

. Let us assess the likelihood of stability by an example. Assume an homothetic utility function and that $c_t = a\, k_t$, where a is a constant; then, (14) implies $f(k^*) = (a + d)\, k^*$. From (13), stability requires that $f_k(k^*) < a + d$; with the previous, that $k^* f_k(k^*) < f(k^*)$. With CRS, $f(k) - k f_k(k)$ equals the marginal product of labor which will be positive. Hence the steady-state will necessarily be stable. In such case, capital dynamics are completely described by $k_t = \frac{(1-d)k_{t-1} + f(k_{t-1})}{1+a}$, positively sloped - once $\frac{\partial k_t}{\partial k_{t-1}} = \frac{1 - d + f_k(k_{t-1})}{1+a} > 0$ and $1 - d > 0$ – and concave (in k_t) iff $f_{kk}(k) < 0$ – crossing the 45º line at k* in space (k_{t-1}, k_t):

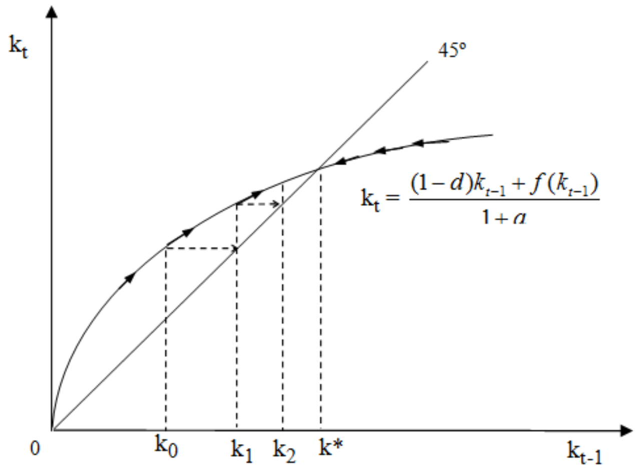

Fig. 2

With those preferences:

$$\frac{\partial k^*}{\partial a} = \frac{k^*}{f_k(k^*) - d - a} \qquad (19)$$

Given that stability holds, $\frac{\partial k^*}{\partial a} < 0$: the optimal k* will decrease with the pseudo-rate of time preference.

. Population growth - at an exogenous constant rate n, i.e., $L_t = (1 + n) L_{t-1}$- would not imply any qualitative change to the previous model, provided we keep considering k as the capital-labor ratio, or capital stock per capita: $k_{t-1} = \frac{K_{t-1}}{L_t}$; as in the Solow-Swan model [10], the left hand-side of the capital equation (5) becomes:

$$(1 + n)\, k_t = (1 - d)\, k_{t-1} + f(k_{t-1}) - c_t \qquad (20)$$

Then (9) becomes

$U_c(c_t, k_t) = U_k(c_t, k_t)/(1+n)$ (21)

The marginal rate of substitution between consumption and capital is now going to be kept at level (1 + n) - $\frac{\partial c_t}{\partial k_t}$ is now $\frac{U_{kk}-(1+n)U_{ck}}{(1+n)U_{cc}-U_{ck}}$. At any level of k_{t-1}, line (9) - the saddle-path, depicted in Fig. 1 - will most likely go up (to (21)): over (21), $\frac{\partial c_t}{\partial n}$ = $\frac{U_c}{U_{ck}-(1+n)U_{cc}}$ and (by SOC) probably positive. For $k_t = k_{t-1}$, $c_t = f(k_{t-1}) - (n+d)\, k_{t-1}$ – line (14) also in Fig. 1 - lowers with n at any level of k_{t-1} -, and in the steady-state:

$$(1+n)\, U_c[f(k^*) - (d+n)\, k^*, k^*] = U_k[f(k^*) - (d+n)\, k^*, k^*] \qquad (22)$$

Now:

$$\frac{\partial k^*}{\partial n} = \frac{[U_{cc}(1+n)-U_{ck}]k^*-U_c}{[U_{cc}(1+n)-U_{ck}](f_k-d-n)+(1+n)U_{ck}-U_{kk}} = \frac{k^*-\dfrac{U_c}{U_{cc}(1+n)-U_{ck}}}{f_k-d-n+\dfrac{(1+n)U_{ck}-U_{kk}}{U_{cc}(1+n)-U_{ck}}} \qquad (23)$$

For stability – the denominator will be negative -, and for S.O.C. to hold – which suggests that the second term of the numerator will likely be positive -, the optimal capital-labor ratio will decline with n.

It is easily deducted that for homothetic preferences of the CES form the steady-state savings rate 1 – c*/f(k*), because c* = f(k*) – (d + n) k* = a k* where a = $\left(\frac{a_1}{a_2}\right)^{\sigma}(1+n)^{\sigma}$ is a constant, is equal to:

$$s^* = \frac{d+n}{a+d+n} \tag{24}$$

It decreases with a, the "rate of time preference", with $\left(\frac{a_1}{a_2}\right)^{\sigma}$,

it increases with d and, provided (1 + n) >◎ (d + n), with n- as one encounters in specific cases of the neoclassical model, using say Cobb-Douglas technology and constant elasticity felicity function [11]. Unlike in these, it will be independent of technology features.

Free market equilibrium

A decentralized market equilibrium can easily support the previous problem, provided the firms' production function F(K, L) is CRS. Being wages w_t and the interest rate r_t (net of depreciation), firms will maximize profits so as to equate the first to the marginal product of labor, $f(k_{t-1}) - k_{t-1} f_k(k_{t-1})$, the second plus depreciation to that of capital, $f_k(k_{t-1})$, and the economy will follow that same described path with no need for intervention. If agents care for their off-springs – sharing the capital stock -, population growth will not alter the conclusion.

Even decreasing returns to scale – with each individual owning his own production plant and using his own equipment according to $f(k_{t-1})$ – would revert to the previous solution and be efficient, provided there is no population growth.

Technical progress and human capital

Technical progress, may generate explosive paths. We shall analyze under which circumstances it may generate stable growth rates. We consider two scenarios: one, in which technical progress is exogenous. Another – in the Uzawa (1965) - Lucas (1988) tradition -, in which it is the product of applied resources to a second sector, that requires but qualified labor to accumulate knowledge or human capital stock, also used in the production of the other goods.

. Admit, then, that the production function is CRS of the type:

$$F(K_{t-1}, A_{t-1} L_t) = A_{t-1} L_t F(\frac{k_{t-1}}{A_{t-1}}, 1) = A_{t-1} L_t f(\frac{k_{t-1}}{A_{t-1}}) \quad (25)$$

A_{t-1} is an efficiency factor affecting labor – labor-augmenting, Harrod-neutral technical progress -, exogenously growing at proportional rate x:

$$A_t = (1 + x) A_{t-1} \quad (26)$$

Then one can convert (20) to:

$$(1 + n) \frac{k_t}{A_{t-1}} = (1 – d) \frac{k_{t-1}}{A_{t-1}} + f(\frac{k_{t-1}}{A_{t-1}}) - \frac{c_t}{A_{t-1}} \quad (27)$$

or

$$(1 + n) (1 + x) \frac{k_t}{A_t} = (1 – d) \frac{k_{t-1}}{A_{t-1}} + f(\frac{k_{t-1}}{A_{t-1}}) - \frac{c_t}{A_t} (1 + x) \quad (28)$$

Provided that U(c, k) is homothetic, condition (21) allows for c_t and k_t to move at the same proportional rate along any optimal path, and it will also be true that:

$$(1 + n) U_c(\frac{c_t}{A_t}, \frac{k_t}{A_t}) = U_k(\frac{c_t}{A_t}, \frac{k_t}{A_t}) \quad (29)$$

Then the problem is stated in such a way that $\frac{k_t}{A_t} = \hat{k}_t$ and $\frac{c_t}{A_t} = \hat{c}_t$ enjoy the same properties as k_t and c_t in the previous model: there will be a steady state *level* $\hat{k}*$ and $\hat{c}*$ that will be stable under similar requirements as before. It involves – as it does for the intertemporal utility function, neoclassical, case - a balanced-growth path for c_t and k_t, moving at the proportional rate x per period, at which A_t grows as well. The steady-state adjusted capital-labor ratio will be such that $\hat{k}_t = \hat{k}_{t-1}$ and we can re-arrange (28) to:

$$\hat{c}_t = [f(\hat{k}_t) - (d + n + x + n\,x)\ \hat{k}_t\,] / (1 + x) \tag{30}$$

In Fig. 1, the line – equivalent to (14) before - will descend with x, and therefore, with stability, the steady-state $\hat{c}*$ and $\hat{k}*$ will decrease with it, once (29) is invariant to x (as long as the latter is positively sloped).

Under homothetic preferences, the steady-state savings rate 1 – $c_t/f(k_t)$ = 1 – (1+x) $\hat{c}*/f(\hat{k}*)$, because $\hat{c}* = [f(\hat{k}*) - (d + n + x + n\,x)\hat{k}*]/(1 + x) = a\,\hat{k}*$ where a is a constant but dependent on n, is equal to:

$$s* = \frac{d+n+x(1+n)}{a+d+n+x(1+a+n)} = \frac{1}{1+\dfrac{a(1+x)}{d+n+x(1+n)}} \tag{31}$$

It will increase with x (provided d < 1, which is expected).

. We are going to allow human capital h, to be included in the individual's utility function: on the one hand, it accrues to the individual's productivity potential. On the other, it carries over earnings ability to future periods. The individual can split his time between studying – creating h according to per period function g(.) using only his time endowment, normalized at 1 per period – or producing, l_t.

At each point in time, the planner solves the representative agent's problem:

$$\underset{c_t, k_t, h_t, l_t}{Max}\ U(c_t, k_t, h_t) \tag{32}$$

s.t: $$k_t = (1 - d)\,k_{t-1} + f(k_{t-1}, h_{t-1}\,l_t) - c_t \tag{33}$$

$$h_t = (1 - e)\,h_{t-1} + g[h_{t-1}\,(1 - l_t)] \tag{34}$$

Given k_{t-1} and h_{t-1}

or in lagrangean form:

$$\underset{c_t, l_t, k_t, \lambda_t, \nu_t}{Max} \; L(c_t, l_t, k_t, h_t, \lambda_t, \nu_t) = U(c_t, k_t, h_t) + \lambda_t [k_t - (1-d) k_{t-1} - f(k_{t-1}, h_{t-1} l_t) + c_t] + \tag{35}$$

$$+ \nu_t \{h_t - (1-e) h_{t-1} - g[h_{t-1} (1 - l_t)]\}$$

F.O.C. require:

$$\frac{\partial L}{\partial c_t} = U_c(c_t, k_t, h_t) + \lambda_t = 0 \tag{36}$$

$$\frac{\partial L}{\partial l_t} = - \lambda_t h_{t-1} f_2(k_{t-1}, h_{t-1} l_t) + \nu_t h_{t-1} g'[h_{t-1} (1 - l_t)] = 0 \tag{37}$$

$$\frac{\partial L}{\partial k_t} = U_k(c_t, k_t, h_t) + \lambda_t = 0 \tag{38}$$

$$\frac{\partial L}{\partial h_t} = U_h(c_t, k_t, h_t) + \nu_t = 0 \tag{39}$$

from where we derive that:

$$U_c(c_t, k_t, h_t) = U_k(c_t, k_t, h_t) \tag{40}$$

$$U_c(c_t, k_t, h_t) f_2(k_{t-1}, h_{t-1} l_t) / U_h(c_t, k_t, h_t) = g'[h_{t-1} (1 - l_t)] \tag{41}$$

The two equations generate a saddle path system for c_t and l_t – as a function of (contemporaneous) k_t and h_t -, if we replace k_{t-1} and h_{t-1} from the two state equations in (41).

A stable steady-state for k and h may exist, but for some functional forms, a steady growth rate may be compatible with the optimal solution:

. Assume g(.) is linear in the argument: g(z) = b z and therefore g'(z) = b. Then h_t grows at rate b (1 – l_t) – e. Take also $U(c_t, k_t, h_t)$ to be of the CES type (or similar), so that (40) insures that c_t and k_t will grow at the same proportional rate, i.e., $c_t = a\, k_t$ where a is a constant. That will determine a saddle-path requirement for c_t and k_t.

Then one can re-write condition (33) as:

$$\frac{k_t}{h_t} = [(1-d)\frac{k_{t-1}}{h_{t-1}} + f(\frac{k_{t-1}}{h_{t-1}}, l_t)] / \{[1 + b(1-l_t) - e](1+a)\} \qquad (42)$$

As $f(k_{t-1}, h_{t-1}\ l_t)$ is CRS in the two arguments its partial derivatives are homogeneous of degree 0 and condition (41) becomes

$$bU_h(c_t,k_t,h_t)/U_c(c_t,k_t,h_t) = f_2(k_{t-1},h_{t-1}\ l_t) = f[k_{t-1}/(l_t h_{t-1}),1] - \frac{k_{t-1}}{h_{t-1}}\frac{1}{l_t} f_k[k_{t-1}/(l_t h_{t-1}),1] = \qquad (43)$$

$$= \frac{1}{l_t}[f(k_{t-1}/h_{t-1}, l_t) - \frac{k_{t-1}}{h_{t-1}} f_k(k_{t-1}/h_{t-1}, l_t)] = f_2(k_{t-1}/h_{t-1}, l_t)$$

As U is CES, $U_h(c_t, k_t, h_t) / U_c(c_t, k_t, h_t) = m(c_t/h_t)$ where m(.) is a function independent of the arguments of U but c_t/h_t, in which it is increasing. Then, because $c_t = a\ k_t$, (43) can be written as:

$$b\ m(a\frac{k_t}{h_t}) = f_2(k_{t-1}/h_{t-1}, l_t) \qquad (44)$$

$$b\ m(a[(1-d)\frac{k_{t-1}}{h_{t-1}} + f(\frac{k_{t-1}}{h_{t-1}}, l_t)] / \{[1 + b(1-l_t) - e](1+a)\}) = f_2(\frac{k_{t-1}}{h_{t-1}}, l_t) \qquad (45)$$

(42) and (44) allow us to determine, at each point in time, $\frac{k_t}{h_t} = \widehat{k}_t$ and l_t as a function of, solely, $\frac{k_{t-1}}{h_{t-1}} = \widehat{k}_{t-1}$ and describe the whole system dynamics. From (45), and as l_t is not a state variable, its path is determined by it,

$$\frac{\partial l_t}{\partial \widehat{k}_{t-1}} = (f_{k2}(\widehat{k}_{t-1}, l_t) - b\ a\ m'(.)\ [1 - d + f_k(\widehat{k}_{t-1}, l_t)] /\{[1 + b (1 - l_t) - e] (1 + a)\}) /$$
$$/ \{ b\ a\ m'(.)\ (f_2(\widehat{k}_{t-1}, l_t) / \{[1 + b (1 - l_t) - e] (1 + a)\} +$$
$$+ b (1 + a) [(1 - d)\ \widehat{k}_{t-1} + f(\widehat{k}_{t-1}, l_t)] / \{[1 + b (1 - l_t) - e] (1 + a)\}^2) - f_{22}(\widehat{k}_{t-1}, l_t)\} \qquad (46)$$

A (contemporaneous) saddle-path could be generated replacing instead $\widehat{k}_t$ implicit in the state equation (42) in (44). *Given* $\widehat{k}_{t-1}$, l_t is on (45); then, $\widehat{k}_t$ would be on that "saddle-path".

From (42),

$$\frac{\partial \widehat{k}_t}{\partial \widehat{k}_{t-1}} = [1 - d + f_k(\widehat{k}_{t-1}, l_t)] / \{[1 + b (1 - l_t) - e] (1 + a)\}+$$
$$+ (f_2(\widehat{k}_{t-1}, l_t) / \{[1 + b (1 - l_t) - e] (1 + a)\} +$$
$$+ b (1 + a) [(1 - d)\ \widehat{k}_{t-1} + f(\widehat{k}_{t-1}, l_t)] / \{[1 + b (1 - l_t) - e] (1 + a)\}^2)\ \frac{\partial l_t}{\partial \widehat{k}_{t-1}} \qquad (47)$$

It will be positive if (but not only if) $\frac{\partial l_t}{\partial \widehat{k}_{t-1}} > 0$. The system will be stable provided (47) is smaller than 1 (in absolute value). It will be smaller than 1 iff ((46) is smaller than):

$$\frac{\partial l_t}{\partial \widehat{k}_{t-1}} < [\{[b (1 - l_t) - e] (1 + a) + a + d\} - f_k(\widehat{k}_{t-1}, l_t)] [1 + b (1 - l_t) - e] /$$
$$/ \{f_2(\widehat{k}_{t-1}, l_t) [1 + b (1 - l_t) - e] + b [(1 - d)\ \widehat{k}_{t-1} + f(\widehat{k}_{t-1}, l_t)] \} \qquad (48)$$

For $\widehat{k}_t = \widehat{k}_{t-1}$, the dynamic equation (42) becomes:

$$\widehat{k}_{t-1} = f(\widehat{k}_{t-1}, l_t) / \{[b (1 - l_t) - e] (1 + a) + a + d\} \qquad (49)$$

which has slope:

$$\frac{\partial l_t}{\partial \widehat{k}_{t-1}} = [\{[b\,(1 - l_t) - e]\,(1 + a) \; + a + d\} - f_k(\widehat{k}_{t-1}, l_t)\,] \,/$$

$$/\,(\,f_2(\widehat{k}_{t-1}, l_t) + b\,(1 + a)\;\widehat{k}_{t-1}) \qquad (50)$$

With CRS, it will be positive, once $f(\widehat{k}_{t-1}, l_t) > \widehat{k}_{t-1}\, f_k(\widehat{k}_{t-1}, l_t)$ insuring positive numerator.

We can plot line (49) on space (k_{t-1}, l_t), along with (45), function $l_t = j(\widehat{k}_{t-1})$. Above (49) – because the right hand-side of (42) rises with l_t -, $\widehat{k}_{t-1} < \widehat{k}_t$ and $\widehat{k}_t$ is rising: let $g(\widehat{k}_{t-1}, l_t)$ denote the right-hand-side of (42), implying $\widehat{k}_t - \widehat{k}_{t-1} = g(\widehat{k}_{t-1}, l_t) - \widehat{k}_{t-1} = 0$ over (49); above it, $\widehat{k}_t - \widehat{k}_{t-1} > 0$ ($\widehat{k}_t$ is rising) iff $\widehat{k}_{t-1} < g(\widehat{k}_{t-1}, l_t)$. Below (49), the opposite occurs.

If the system is stable – (47) has a slope smaller than 1, being positive, and (48) holds -, (49) will have a higher slope, (50), than that of (46) (because the right hand-side of (48) evaluated at (49) is equal to (50)). If (46) > 0, the saddle-path should have a slope between the two around the steady-state; if negative, it should be more negative that (46). Graphically:

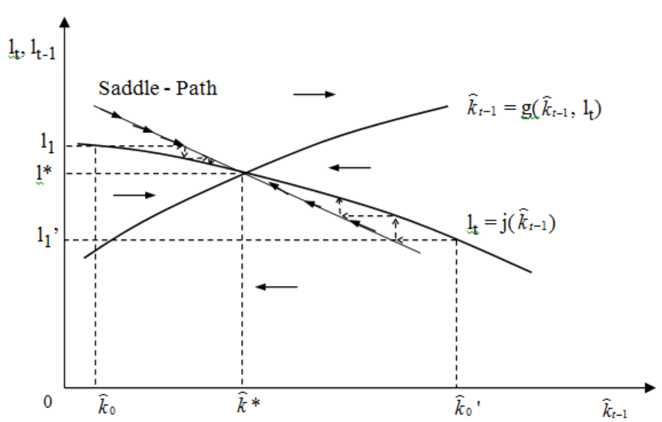

Fig. 3

Increasing returns to scale and land

Assume that the aggregate production function is homogeneous of degree $\circledcirc$ in the two arguments, aggregate stock of capital, K_t and labor force L_t. IRS (increasing returns to scale) occur for $\circledcirc > 1$. As L_t is exogenous, if we can view the production function as CRS in the two arguments, K_{t-1} and $(A_{t-1} L_t)$ of:

$$F(K_{t-1}, A_{t-1} L_t) \quad \text{where} \quad A_{t-1} = L_t^{(\circledcirc-1)/(1-g)} \qquad (51)$$

where g is the degree of homogeneity of $F(K_t, L_t)$ in K_t only – this occurs for a Cobb-Douglas technology, for example -, then we fall under the conditions of exogenous technical progress – and along the stable balanced growth path, k_t and c_t will grow at the same proportional rate as $L_t^{(\circledcirc-1)/(1-g)}$: $(1 + n)^{(\circledcirc-1)/(1-g)} - 1$ [12].

Note, however, that a constant population will allow – unlike in Romer (1986) - for a stable steady-state even with increasing marginal returns to capital, once these are not required – recall (13) – for stability. But of course, they may contend indirectly with the requirement, at least after some level of k_t.

. Admit, on the other extreme that there is also a fixed resource, asset, land, denoted by D, that enters the production function and cannot be changed. Its property is evenly distributed among the population and the representative agent's utility function also depends on it. The aggregate production function is of the type $F(K_{t-1}, L_t, D)$, homogeneous in the two arguments K_t and L_t in such a way that we can write $F(K_{t-1}, L_t, D) = L_t A_{t-1} f(k_{t-1}/A_{t-1}, 1, D)$ where A_{t-1} is a power of L_t. An individual solves:

$$\underset{c_t, k_t, h_t, l_t}{Max}\; U(c_t, k_t, D/L_t)$$

$$\text{s.t:}(1 + n)\, k_t = (1 - d)\, k_{t-1} + A_{t-1} f(k_{t-1}/A_{t-1}, D) - c_t \qquad (52)$$

Given k_{t-1}

Then, with population growth, there will be a steady-state balanced growth path in the economy where c_t and k_t grow (or decrease…) at the same proportional rate that A_{t-1} – conditioned by the degree of homogeneity of $F(K_{t-1}, L_t, D)$ in K_{t-1} and L_t only – , as long as along the optimal path, $U_c(c_t, k_t, D/L_t) = U_k(c_t, k_t, D/L_t)$ allows it – say, $U(., ., .)$ is of the CES type in the three arguments. [13]

If $F(K_{t-1}, A_{t-1} L_t, D)$ is homogeneous of degree 1 in the three arguments, there will be, in equilibrium with a (gross…) payment of $F_D(K_{t-1}, A_{t-1}L_t, D)$ in real – consumption and/or final product - terms to owners of land per unit of the resource, adjusting $F(K_{t-1}, A_{t-1}L_t, D) = F_k(K_{t-1}, A_{t-1}L_t, D)\ K_{t-1} + F_L(K_{t-1}, A_{t-1}L_t, D)\ L_{t-1} + F_D(K_{t-1}, A_{t-1}L_t, D)\ D$; or, denoting $\widehat{k}_{t-1} = K_{t-1} / A_{t-1}L_t$, $F(K_{t-1}, L_tA_{t-1}, D) / L_tA_{t-1} = F_k[\widehat{k}_{t-1}, D/ (L_tA_{t-1})]\ \widehat{k}_{t-1} + F_L[\widehat{k}_{t-1}, 1, D/ (L_tA_{t-1})] + F_D[\widehat{k}_{t-1}, 1, D/ (L_tA_{t-1})]\ D/ (A_{t-1} L_t)$. In an economy of instantaneous firms, a relative price $p^D{}_t$ – the price of land in units of either capital, product and, consumption may have to emerge – to account for the fact that they contribute differently to production and consumption and that land does not depreciate – even in the absence of technical progress… Then: $U_D(c_t, k_t, D/L_t) / U_c(c_t, k_t, D/L_t) = U_D(c_t, k_t, D/L_t) / U_k(c_t, k_t, D/L_t) = p^D{}_t$.

Uncertain wealth

Additive uncertainty in stationary models

One can hypothesize that the value of the capital stock is a random variable, say added of a noise e_t, translating expectations of future gains from savings applications or expected appreciation or other. Decisions must be made ex-ante, that is, before e_t is observed, and therefore they exhibit no recurrent consequences, or these being independent as long as the external shocks also are. Nevertheless, the disturbance must cause (general) precautionary reaction: now the consumer maximizes expected welfare.

$$\underset{c_t, k_t}{Max} \; E_e U(c_t, k_t + e_t) \quad (53)$$

$$\text{s.t:} k_t = (1 - d) k_{t-1} + f(k_{t-1}) - c_t \quad (54)$$

Given k_{t-1}

or in lagrangean form:

$$\underset{c_t, k_t, \lambda_t}{Max} L(c_t, \lambda_t) = E_e U(c_t, k_t + e_t) + \lambda_t [k_t - (1 - d) k_{t-1} - f(k_{t-1}) + c_t]$$

F.O.C., along with the restriction, require:

$$\frac{\partial L}{\partial c_t} = E_e U_c(c_t, k_t + e_t) + \lambda_t = 0 \quad (55)$$

$$\frac{\partial L}{\partial k_t} = E_e U_k(c_t, k_t + e_t) + \lambda_t = 0 \quad (56)$$

from where

$$E_e U_c(c_t, k_t + e_t) = E_x U_k(c_t, k_t + e_t) \quad (57)$$

We can now use Taylor's approximation to expand the marginal utilities around k_t (or we could have expand $U(c_t, k_t + e_t)$ before optimization). Taking the corresponding expected value - assuming the noise has null mean - and denoting the variance of e_t multiplied by 2 (for simplification) by s2: $Var(e_t) = E[e_t^2] = 2$ s2,

$$U_c(c_t, k_t) + U_{ckk}(c_t, k_t) \text{ s2} = U_k(c_t, k_t) + U_{kkk}(c_t, k_t) \text{ s2} \quad (58)$$

or

$$U_c(c_t, k_t) - U_k(c_t, k_t) = [U_{kkk}(c_t, k_t) - U_{ckk}(c_t, k_t)] \text{ s2}$$

For s2 larger than 0, $U_c(c_t, k_t) - U_k(c_t, k_t) > 0$, suggesting a more favorable capital relative to consumption transformation path than the s2=0 case iff $U_{kkk}(c_t, k_t) > U_{ckk}(c_t, k_t)$, that is, if U_{kk} raises more – or – U_{kk}, measuring the concavity of U in k, related to the aversion to a risk added to k, decreases more - per unit increase of capital than per unit rise in consumption.

Now,

$$\frac{\partial c_t}{\partial k_t} = \frac{U_{kk} + U_{kkkk} s2 - U_{ck} - U_{ckkk} s2}{U_{cc} + U_{cckk} s2 - U_{ck} - U_{ckkk} s2} \tag{59}$$

Stability still requires $f_k(k_{t-1}) - d < \frac{\partial c_t}{\partial k_t}$ around the steady-state.

The new consumption path (k_{t-1}, c_t) will satisfy (57) and (54), i.e.:

$$U_c[c_t, (1-d) k_{t-1} + f(k_{t-1}) - c_t] + U_{ckk}[c_t, (1-d) k_{t-1} + f(k_{t-1}) - c_t]\, s2 =$$
$$= U_k[c_t, (1-d) k_{t-1} + f(k_{t-1}) - c_t] + U_{kkk}[c_t, (1-d) k_{t-1} + f(k_{t-1}) - c_t]\, s2 \tag{60}$$

$$\frac{\partial c_t}{\partial s2} = \frac{U_{kkk} - U_{ckk}}{U_{cc} + U_{kk} - 2U_{ck} + U_{cckk} s2 - 2U_{ckkk} s2 + U_{kkkk} s2} \tag{61}$$

Second order conditions require the denominator to be negative – they will be satisfied if not only $U[c_t, (1-d) k_{t-1} + f(k_{t-1}) - c_t]$ but also $U_{kk}[c_t, (1-d) k_{t-1} + f(k_{t-1}) - c_t]$ is concave in c_t. The consumption path will lower with s2 at given k_{t-1} iff $U_{kkk} > U_{ckk}$, i.e., if the marginal utility of consumption is more concave in k than the marginal utility of capital is. Then, as long as that path is positively sloped (which is expected by S.O.C.), – recall Fig. 1 -, k* will rise with uncertainty. Such steady-state value, implying $c^* = f(k^*) - d\, k^*$, requires:

$$U_c[f(k^*) - d\, k^*, k^*] + U_{ckk}[f(k^*) - d\, k^*, k^*]\, s2 = \tag{62}$$
$$= U_k[f(k^*) - d\, k^*, k^*] + U_{kkk}[f(k^*) - d\, k^*, k^*]\, s2$$

Or $\quad U_c[f(k^*) - d\, k^*, k^*] - U_k[f(k^*) - d\, k^*, k^*] =$

$$= \{U_{kkk}[f(k^*) - d\, k^*, k^*] - U_{ckk}[f(k^*) - d\, k^*, k^*]\}\, s2$$

The steady-state savings rate, $s^* = 1 - c^*/f(k^*) = d\, k^* / f(k^*)$, will respond to uncertainty according to:

$$\frac{\partial s^*}{\partial s2} = \frac{d[f(k^*) - k^* f_k(k^*)]}{f(k^*)^2} \frac{\partial k^*}{\partial s2} \tag{63}$$

As with CRS $f(k^*) - k^* f_k(k^*)$ equals the, positive, marginal product of labor, s^* will respond to $s2$ in the same direction as k^* does.

Admit separability between c and k in the utility function such that cross-derivatives are null and we can write

$$U(c_t, k_t) = u^c(c_t) + u^k(k_t) \tag{64}$$

Then (58) becomes:

$$u^c{}_c(c_t) = u^k{}_k(k_t) + u^k{}_{kkk}(k_t)\, s2 \tag{65}$$

and along the saddle-path

$$\frac{\partial c_t}{\partial k_t} = \frac{u^k_{kk} + u^k_{kkkk} s2}{u^c_{cc}} \tag{66}$$

In the steady-state:

$$u^c{}_c[f(k^*) - d\, k^*] = u^k{}_k(k^*) + u^k{}_{kkk}(k^*)\, s2$$

$$\frac{\partial k^*}{\partial s2} = \frac{u^k_{kkk}(k^*)}{u^c_{cc}[f_k(k^*) - d] - u^k_{kk}(k^*) - u^k_{kkkk}(k^*) s2} =$$

$$\frac{\dfrac{u^k_{kkk}(k^*)}{u^c_{cc}(k^*)}}{f_k(k^*) - d - \dfrac{u^k_{kk}(k^*) + u^k_{kkkk}(k^*) s2}{u^c_{cc}(k^*)}} \tag{67}$$

If the system is stable, the denominator is negative. Then, $\frac{\partial k^*}{\partial s2}$ will be positive and k* increases with uncertainty iff $\frac{u^k_{kkk}(k^*)}{u^c_{cc}(k^*)} = \frac{-u^k_{kkk}(k^*)}{-u^c_{cc}(k^*)} < 0$: it will if $u^c(c)$ – and U(c,k) - is concave in c and $u^k_k(k)$ – as $U_k(c, k)$ - is convex in k. The condition establishes that if - $\frac{u^k_{kkk}(k^*)}{u^c_{cc}(k^*)}$ - that resembles Kimball's (1990) [14] measure of *absolute prudence*, determining how a control variable reacts to added uncertainty in the static context - is positive, k* will rise with uncertainty.

. Let us consider the reasonable alternative: that optimization behavior is made ex-post and works "deterministically" and that uncertainty only affects production [15]:

$$\underset{c_t, k_t}{Max}\, U(c_t, k_t)$$

s.t. $$k_t = (1 - d)(k_{t-1} + e_{t-1}) + f(k_{t-1} + e_{t-1}) - c_t \qquad (68)$$

Given k_{t-1} and e_{t-1}

e_{t-1} is known at time t. Then, obviously, there will not be a "steady-state" for k_t: it will fluctuate according to e_{t-1}, obeying (68) and (9).

Interestingly, (9) and (68) – in general, the current setup - provide a rationale for a co-integrating relation with a structural error-correction mechanism.

Using Taylor expansion on the state equation:

$$k_t = (1 - d)(k_{t-1} + e_{t-1}) + f(k_{t-1}) + f_k(k_{t-1})\, e_{t-1} + \frac{1}{2} f_{kk}(k_{t-1})\, e_{t-1}^2 - c_t \qquad (69)$$

From F.O.C, equality between marginal utility of consumption and capital will be satisfied. We could inspect the effect on the "expected" phaseline of a rise in s2, but we would not account for the simultaneous determination of k_t and c_t. Rather, we must consider that the later responds to k_{t-1} according to:

$$U_c[c_t, (1-d)(k_{t-1}+e_{t-1}) + f(k_{t-1}+e_{t-1}) - c_t] = \quad (70)$$
$$= U_k[c_t, (1-d)(k_{t-1}+e_{t-1}) + f(k_{t-1}+e_{t-1}) - c_t]$$

(70) establishes a relation $c_t = c(k_{t-1} + e_{t-1})$ identical to that without uncertainty. Expanding around k_{t-1},

$$c_t = c(k_{t-1}+e_{t-1}) = c(k_{t-1}) + \frac{\partial c_t}{\partial k_{t-1}} e_{t-1} + \frac{d^2 c_t}{dk_{t-1}^2} e_{t-1}^2 / 2$$

We can write then that:

$$k_t = (1-d)(k_{t-1}+e_{t-1}) + f(k_{t-1}+e_{t-1}) - c(k_{t-1}+e_{t-1}) = \quad (71)$$
$$= (1-d)(k_{t-1}+e_{t-1}) + f(k_{t-1}) + f_k(k_{t-1})\, e_{t-1} + \frac{1}{2} f_{kk}(k_{t-1})\, e_{t-1}^2 - c(k_{t-1}) - \frac{\partial c_t}{\partial k_{t-1}} e_{t-1} - \frac{1}{2}\frac{d^2 c_t}{dk_{t-1}^2} e_{t-1}^2$$

Let again $Var(e_t) = E[e_t^2] = 2\,s2$. The expected value of k_t at time t is, therefore:

$$E[k_t] = (1-d)\, k_{t-1} + f(k_{t-1}) + f_{kk}(k_{t-1})\, s2 - c(k_{t-1}) - \frac{d^2 c_t}{dk_{t-1}^2}\, s2 \quad (72)$$

Stability requires $\frac{\partial k_t}{\partial k_{t-1}}$ to be between –1 and 1. Assume that $\frac{d^2 c_t}{dk_{t-1}^2}$ is negligible. Then:

$$\frac{\partial k_t}{\partial k_{t-1}} = (1-d) + f_k(k_{t-1}) + f_{kkk}(k_{t-1})\, s2\ ] / (1 + \frac{\partial c_t}{\partial k_t})$$

where $\frac{\partial c_t}{\partial k_t}$ comes from the equality between marginal utility of capital and consumption. Admit, for example, homothetic preferences such that $c_t = a\, k_t$. Then:

$$k_t = [(1 - d)\, k_{t-1} + f(k_{t-1}) + f_k(k_{t-1})\, e_{t-1} + \frac{1}{2} f_{kk}(k_{t-1})\, e_{t-1}^2] / (1 + a) \quad (73)$$

$$\frac{\partial k_t}{\partial k_{t-1}} = [(1 - d) + f_k(k_{t-1}) + f_{kk}(k_{t-1})\, e_{t-1} + \frac{1}{2} f_{kkk}(k_{t-1})\, e_{t-1}^2] / (1 + a) \quad (74)$$

On average, we can expect stability iff

$$s2 f_{kkk}(k_{t-1}) < a + d - f_k(k_{t-1})$$

If $f_{kkk}(k_{t-1}) > 0$ – a plausible assumption -, that requires a low volatility of capital value or productive potential – a low s2. In other words, even if stability were guaranteed under deterministic conditions, if $f_{kkk}(k_{t-1}) > 0$, it is no longer so.

Consider expectations of (73). For a steady-state level of capital, k*:

$$f(k^*) + f_{kk}(k^*)\, s2 = (a + d)\, k^* \quad (75)$$

$$\frac{\partial k^*}{\partial s2} = \frac{f_{kk}(k^*)}{a + d - f_k(k^*) - f_{kkk}(k^*)s2} \quad (76)$$

With stability, diminishing marginal returns to capital - $f_{kk}(k^*) < 0$ – imply that k* decreases with uncertainty (and also c* if the saddle-path is positively sloped, which is expected by SOC).

The steady-state savings rate, $s^* = 1 - c^*/f(k^*) = d\, k^* / [f(k^*) + f_{kk}(k^*)\, s2]$, will be:

$$s^* = \frac{d}{a + d} \quad (77)$$

It will be invariant to uncertainty.

. A juxtaposition of the two effects would be realistic: that the agent solves:

$$\underset{c_t, k_t}{Max} \; E_e U(c_t, k_t + e_t)$$

s.t: $k_t = (1 - d)(k_{t-1} + e_{t-1}) + f(k_{t-1} + e_{t-1}) - c_t$

Given k_{t-1} and e_{t-1}

The previous decomposition allows us to distinguish the utility and technology channels through which uncertainty – dispersion of tastes - affects the equilibrium.

Multiplicative uncertainty

.Suppose:

$$\underset{c_t, k_t}{Max} \; E_e U[c_t, k_t (1 + e_t)] \quad (78)$$

s.t: $$k_t = (1 - d) k_{t-1} + f(k_{t-1}) - c_t \quad (79)$$

Given k_{t-1}

Admit further separability of the utility function so that we can write: $U[c_t, k_t (1 + e_t)] = u^c(c_t) + u^k[k_t (1 + e_t)]$ so that $E_e U[c_t, k_t (1 + e_t)] ◎ u^c(c_t) + u^k(k_t) + u^k_{kk}(k_t)\, k^2$ s2. Then:

$$u^c_c(c_t) = u^k_k(k_t) + [u^k_{kkk}(k_t)\, k_t^2 + 2\, u^k_{kk}(k_t)\, k_t]\, s2 \quad (80)$$

and along the saddle-path:

$$\frac{\partial c_t}{\partial k_t} = \frac{u^k_{kk} + (k_t^2 u^k_{kkkk} + 4 k_t u^k_{kkk} + 2 u^k_{kk}) s2}{u^c_{cc}} \quad (81)$$

In the steady-state:

$$u^c_c[f(k^*) - d\, k^*] = u^k_k(k^*) + [u^k_{kkk}(k^*)\, k^{*2} + 2\, u^k_{kk}(k^*)\, k^*]\, s2$$

$$\frac{\partial k^*}{\partial s2} = \frac{u^k_{kkk}(k^*) k^{*2} + 2 u^k_{kk}(k^*) k^*}{u^c_{cc}[f_k(k^*) - d] - u^k_{kk}(k^*) - [u^k_{kkkk}(k^*) + 2k^* u^k_{kkk}(k^*)] s2} =$$

$$= \frac{\dfrac{u^k_{kkk}(k^*)k^{*2} + 2u^k_{kk}(k^*)k^*}{u^c_{cc}}}{[f_k(k^*) - d] - \dfrac{u^k_{kk}(k^*) + [u^k_{kkkk}(k^*) + 2k^* u^k_{kkk}(k^*)]s2}{u^c_{cc}}} \qquad (82)$$

If the system is stable, it will be positive iff $\frac{u^k_{kkk}(k^*)k^{*2} + 2u^k_{kk}(k^*)k^*}{u^c_{cc}} < 0$: it will if $u^c(c)$ – and U(c,k) - is concave in c and $[u^k_{kk}(k)\, k^2]$ rises with k.

. Finally, let preferences be homothetic and admit

$$k_t = (1 - d)\, k_{t-1}\, (1 + e_{t-1}) + f[k_{t-1}\, (1 + e_{t-1})] - c_t \qquad (83)$$

Now

$$k_t = [(1 - d)\, (k_{t-1} + e_{t-1}) + f(k_{t-1}) + f_k(k_{t-1})\, k_{t-1} e_{t-1} + \frac{1}{2} f_{kk}(k_{t-1})\, k_{t-1}^2\, e_{t-1}^2] / (1+a) \qquad (84)$$

In the expected steady-state

$$f(k^*) + k^{*2} f_{kk}(k^*)\, s2 = (a + d)\, k^* \qquad (85)$$

$$\frac{\partial k^*}{\partial s2} = \frac{k^{*2}\, f_{kk}(k^*)}{a + d - f_k(k^*) - [k^{*2}\, f_{kkk}(k^*) + 2k^* f_{kk}(k^*)]s2} \qquad (86)$$

As before, with stability, diminishing marginal returns to capital - $f_{kk}(k^*) < 0$ – insure that k* (and c*) decreases with uncertainty.

. If we are in the presence of exogenous labor-augmenting technical progress, balanced growth would be recovered (at least on average), with system dynamics towards k_t/A_t approaching that of k_t in the current framework. Given that uncertainty factors capital, we do not expect effects of uncertainty on balanced growth rates even if effects remain in steady-sate ratios.

Overlapping optimization: Recursive structures

One could argue that the previous problem fails to capture forward-looking intertemporal effects. That may not be so, but we could assume then that k_{t+1} also enters the individual's utility function at time t and that the following period's capital constraint is (therefore) also considered in the current problem. Then the finite-horizon problem – problems - would be [16]:

$$\max_{c_t, k_t, k_{t+1}} U(c_t, k_t, k_{t+1}) \quad , \quad t = 1,2,\ldots, T \tag{87}$$

s.t: $k_t = (1 - d)\, k_{t-1} + f(k_{t-1}) - c_t \quad , \; t = 1,2,\ldots, T$

$$k_{t+1} = (1 - d)\, k_t + f(k_t) - c_{t+1} \quad , \; t = 1,2,\ldots, T\text{-}1 \tag{88}$$

Given k_0, k_{T+1}

or in lagrangean form:

$$\max_{c_t, k_t, k_{t+1}, \lambda_t, v_t} L(c_t, \lambda_t) = U(c_t, k_t, k_{t+1}) + \lambda_t \,[k_t - (1 - d)\, k_{t-1} - f(k_{t-1}) + c_t] +$$

$$+ \nu_t \,[k_{t+1} - (1 - d)\, k_t - f(k_t) + c_{t+1}] \tag{89}$$

F.O.C., along with the restrictions, require for t = 1,2,…,T-1 that

$$\frac{\partial L}{\partial c_t} = U_c(c_t, k_t, k_{t+1}) + \lambda_t = 0$$

$$\frac{\partial L}{\partial k_t} = U_{k1}(c_t, k_t, k_{t+1}) + \lambda_t - \nu_t \,[(1 - d) + f_k(k_t)] = 0$$

$$\frac{\partial L}{\partial k_{t+1}} = U_{k2}(c_t, k_t, k_{t+1}) + \nu_t = 0$$

from where

$$U_c(c_t, k_t, k_{t+1}) = U_{k1}(c_t, k_t, k_{t+1}) + U_{k2}(c_t, k_t, k_{t+1})\,[(1 - d) + f_k(k_t)] \,,$$
$$t = 1,2,\ldots,T\text{-}1 \tag{90}$$

$$U_c(c_T, k_T, k_{T+1}) = U_{k1}(c_T, k_T, k_{T+1}) \tag{91}$$

There are (T + T) unknowns – (c_t, k_t), t = 1,2,…,T -, and T + T equations – (90) and (91) and the T generic state equations, (88) [17]. Therefore, the problem should have a well-defined solution, obeying

$$c_t = c(k_{t-1}, c_{t+1}),\ k_t = k1(k_{t-1}, c_{t+1}),\ k_{t+1} = k2(k_{t-1}, c_{t+1}),\ t = 1,2,\ldots T-1 \tag{91}$$

$$c_T = cT(k_{T-1}, k_{T+1}),\ \ k_T = kT(k_{T-1}, k_{T+1})$$

The rate of time preference would become:

$$\frac{\partial c_t}{\partial k_t} = \frac{U_{k1k1} + U_{k1k2}(1-d+f_k) + U_{k2} f_{kk} - U_{ck1}}{U_{cc} - U_{ck1} - U_{ck2}(1-d+f_k)} \tag{92}$$

Again, we could propose as an alternative definition: $\frac{U_c(c_{t+1}, k_{t+1}, k_{t+2})[1-d+f_k(k_{t+1})]}{U_c(c_t, k_t, k_{t+1})}$ -1 or $\frac{U_c(c_{t+1}, k_{t+1}, k_{t+2})[1-d+f_k(k_t)]}{U_c(c_t, k_t, k_{t+1})}$ - 1.

S.O.C. would require U{c_t, (1 – d) k_{t-1} + f(k_{t-1}) - c_t, (1 – d) [(1 – d) k_{t-1} + f(k_{t-1}) - c_t] + f[(1 – d) k_{t-1} + f(k_{t-1}) - c_t] – c_{t+1}} concave in c_t, i.e., that U_{cc} - U_{k1c} - U_{k2c} (1 – d + f_k) - U_{ck1} + U_{k1k1} + U_{k2k1} (1 – d + f_k) –[U_{ck2} - U_{k1k2} - U_{k2k2} (1 – d + f_k)](1 – d + f_k) + U_{k2} f_{kk}< 0. They are therefore satisfied with decreasing marginal utility with respect to each argument (U_{jj}< 0, all j), positive U_{cj}'s, j = k1, k2, and negative U_{k1k2}.

In infinite horizons, (90) and the state equations define the properties of the optimal path; a boundary – or limiting transversality-like - condition could replace the establishment of k_{T+1}. A steady-state would satisfy:

$$U_c[f(k^*) - d\,k^*, k^*, k^*] = U_{k1}[f(k^*) - d\,k^*, k^*, k^*] + U_{k2}[f(k^*) - d\,k^*, k^*, k^*]\,[1 - d + f_k(k^*)] \tag{94}$$

One can study the optimal solution dynamics by analyzing the system (around the steady-state, at least) [18]:

$U_c(c_t, k_t, k_{t+1}) = U_{k1}(c_t, k_t, k_{t+1}) + U_{k2}(c_t, k_t, k_{t+1})\,[1 - d + f_k(k_t)]$ (95)

$k_{t+1} = (1 - d)\, k_t + f(k_t) - c_{t+1}$ (96)

From the two, we can generate:

$k_{t+1} = g1(k_t, c_t)$ (This, immediately from (95)) (97)

$c_{t+1} = g2(k_t, c_t)$ (In our system, $g2(k_t, c_t) = (1 - d)\, k_t + f(k_t) - g1(k_t, c_t)$) (98)

The (2x2) Jacobian matrix $A = [\, a_{ij} \,]$, would contain:

$$a_{11} = \frac{\partial k_{t+1}}{\partial k_t} = \frac{U_{k1k1} + U_{k1k2}(1-d+f_k) + U_{k2} f_{kk} - U_{ck1}}{U_{ck2} - U_{k1k2} - U_{k2k2}(1-d+f_k)}$$

$$a_{12} = \frac{\partial k_{t+1}}{\partial c_t} = \frac{U_{k1c} + U_{k2c}(1-d+f_k) - U_{cc}}{U_{ck2} - U_{k1k2} - U_{k2k2}(1-d+f_k)}$$

$$a_{21} = \frac{\partial c_{t+1}}{\partial k_t} = [1 - d + f_k(k_t)] - a_{11}$$

$$a_{22} = \frac{\partial c_{t+1}}{\partial c_t} = - a_{12}$$

It has trace T and determinant D, having correspondence with the eigenvalues of A, r_1 and r_2, in such a way that:

$$T = a_{11} + a_{22} =$$

$$\frac{U_{k1k1} + U_{cc} + (U_{k1k2} - U_{ck2})(1-d+f_k) + U_{k2} f_{kk} - 2U_{ck1}}{U_{ck2} - U_{k1k2} - U_{k2k2}(1-d+f_k)} = r_1 +$$

$$r_2 < 0$$

$$D = a_{11}\, a_{22} - a_{12}\, a_{21} = - [1 - d + f_k(k_t)]\, a_{12} =$$

$$= - [1 - d + f_k(k_t)] \frac{U_{k1c} + U_{k2c}(1-d+f_k) - U_{cc}}{U_{ck2} - U_{k1k2} - U_{k2k2}(1-d+f_k)} = r_1 r_2 < 0$$

If in moduli one eigenvalue is larger than one and the other is smaller than one, the system is unstable and possesses a saddle-path that converges to the steady-state. The determinant and the trace most likely are negative (if we assume decreasing marginal utility with respect to each argument, positive U_{cj}'s, j = k1, k2, and negative U_{k1k2}); $T^2 - 4D$ is then positive and the two roots are real. As D and T are negative, one eigenvalue is positive and the other is negative. As their sum - T - is negative, numbering the regions in space (T, D) according to Azariadis (1998), p.65-66:

If $D = -[1 - d + f_k(k_t)]\, a_{12} < -1$, i.e. $f_k(k_t) > d$ for a_{12} larger or equal to 1, the steady-state is:

Case A: a source (unstable) if D < T – 1: both have moduli larger than 1 - Region (2).

Case B: a saddle if D > T – 1, D < - (T + 1) - for D < -1 (T < 0), only the first bound is relevant: both eigenvalues are on the same side of –1, different sides of 1: one is in (-1, 1), the other in (1, ∞)- Region (3).

If - 1 < D < 0; as T < 0, the steady-state is either

Case C: a saddle if D > T – 1, D < - (T + 1) – for –1 < D < 0 (T < 0), only the second bound is relevant: both eigenvalues are on the same side of –1, different sides of 1: one is in (-1, 1), the other in (1, ∞)- Region (3).

Case D: a sink (stable) if D > - (T + 1): both eigenvalues fall in (-1,1) – Region (7b).

Case E: a flip (period-doubling) bifurcation (Azariadis, p. 93) if D = - (T + 1).

We can compute:

$$T - 1 = a_{11} - a_{12} - 1$$
$$-(T + 1) = -(a_{11} - a_{12} + 1)$$

As D = - [1 – d + $f_k(k_t)$] a_{12}, D > T – 1 implies a_{11} – 1 < - [$f_k(k_t)$ - d] a_{12}. If $f_k(k_t)$ > d, this will necessarily occur (because a_{11}< 0 and a_{12}> 0 are most likely). Then, we rule out case A.

D < - (T + 1) translates to - [1 – d + $f_k(k_t)$] a_{12}< - (a_{11} - a_{12} + 1). We could not prove that the opposite cannot occur, which would discard Case D (and E). But at least D < - (T + 1) would cover – once T < 0 – a larger range of possibilities.

Alternatively, we can rely on the simpler analysis of the variables' trends around the functions k_{t+1} =g1(k_t, c_t) and c_{t+1} = g2(k_t, c_t) evaluated at the steady-state, i.e., for k_{t+1} = k_t and c_{t+1} = c_t – the phaselines. We plot the resulting conclusions – valid for linear approximations around the steady-state - in the phase diagrams, Fig. 4 and 5, below:

Taking (97) for steady-state k_t, k_t = g1(k_t, c_t) (or (95)) and evaluating its slope at k_{t+1} = k_t and c_{t+1} = c_t, i.e. on:

$$U_c(c_t, k_t, k_t) = U_{k1}(c_t, k_t, k_t) + U_{k2}(c_t, k_t, k_t)\,[1 - d + f_k(k_t)] \qquad (99)$$

We derive:

$$\frac{\partial c_t}{\partial k_t} ◎ ◎ -1a_{11}) / a_{12} > 0 \qquad (100)$$

It is (if T < 0, a justified assumption) positively sloped: above (99), k_t is rising. k_{t+1} – k_t = g1(k_t, c_t) – k_t; it will be larger than 0 and k_t is rising iff g1(k_t, c_t) > k_t – once at a given k_t, as $g1_c$ = a_{12}> 0, for values of c_t to the right of the line g1 shows larger values (and then, larger than k_t).

Repeating the same exercise for (98), c_t = g2(c_t, k_t) that we can solve for c_t = g3(k_t) - that differs from (14). We have that over it:

$$\frac{\partial c_t}{\partial k_t} = a_{21} / (a_{12} + 1) > 0. \qquad (101)$$

It is also most likely positively sloped: to the right of $c_t = g2(k_t, c_t)$, c_t is increasing. $c_{t+1} - c_t = g2(k_t, c_t) - c_t$; it will be larger than 0 and c_t is rising iff $g2(k_t, c_t) > c_t$ – once at a given c_t, as $g2_k = a_{21} > 0$ (most likely), g2 is larger for larger values of k_t than over the line.

Then either:

$c_t = g2(k_t, c_t)$ has a higher slope than $k_t = g1(k_t, c_t)$ – Fig. 4 – and we have a saddle-path (Case C):

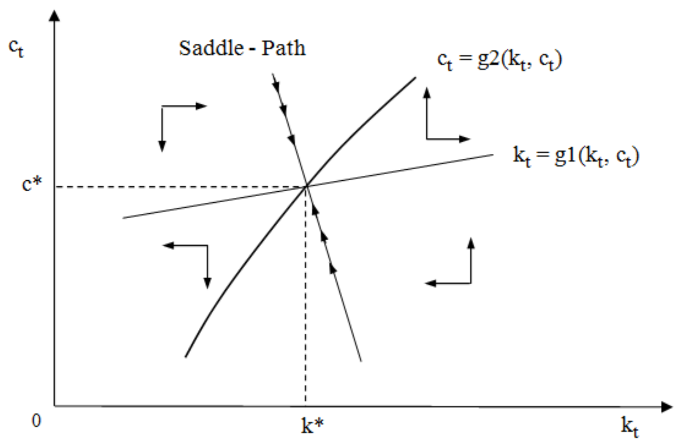

Fig. 4

The arrows point to the steady-state, where the two functions meet, in only two quadrants (the system is not stable) where the saddle-path must lie, exhibiting the pattern shown in the figure. Interestingly, the saddle-path appears negatively sloped – a different pattern from that of Fig.1.

Or $c_t = g2(k_t, c_t)$ has a smaller slope than $k_t = g1(k_t, c_t)$ – depicted in Fig. 5 – and we have a (stable) "sink" steady-state – Case D:

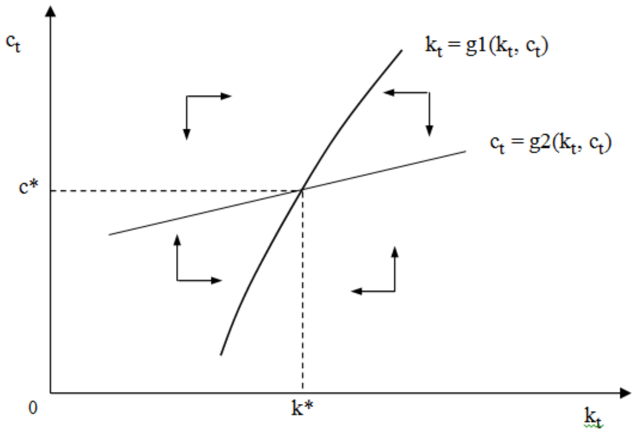

Fig. 5

A final comment on the optimization structure should be added. One could forward an optimization procedure where intertemporal efficiency was also required:

$$\underset{c_t, k_t, M_t}{Max}\ U(c_t, k_t, k_{t+1})$$

$$k_{t+j} = (1-d)\, k_{t+j-1} + f(k_{t+j-1}) - c_{t+j} \quad , \quad j = 0, 1 \text{ (or } 0, 1, 2, 3) \quad (102)$$

$$U(c_{t+j}, k_{t+j}, k_{t+j-1}) \circledcirc \overline{U}_{t+j,t} \quad , \quad j = 1 \text{ (or } 1, 2, 3) \qquad (103)$$

Given k_{t-1}

In lagrangean form:

$$\underset{k_t, k_{t+1}, \xi_t}{Max}\ L(c_t, k_t, k_{t+1}, \circledcirc_t) = U(c_t, k_t, k_{t+1}) + \lambda_t [k_t - (1-d)\, k_{t-1} - f(k_{t-1}) + c_t] +$$
$$+ \circledcirc_t [k_{t+1} - (1-d)\, k_t - f(k_t) + c_{t+1}] +$$
$$+ \circledcirc_t \{\overline{U}_{t+1,t} - U(c_{t+1}, k_{t+1}, k_{t+2})\} \qquad (104)$$

However, at time t, $\overline{U}_{t+j,t}$ is still forthcoming – $\overline{U}_{t+j,t}$ would not even have to equal $\overline{U}_{t+j,t+s}$: the corresponding multiplier should in fact be 0. In fact we are assuming that different people may proceed to subsequent optimization and freeing the analogous constraint that implicitly structures accumulated discounted utility maximizers.

Conclusion

We explored the potential of point-wise optimization of individuals' utility functions dependent on consumption and wealth to reproduce the dynamic behavior of the macroeconomy. The framework proved to be capable of generating similar dynamics as traditional and neoclassical real growth models – based on intertemporal utility maximization. Exogenous population growth, technical progress and increasing returns showed similar consequences and stability requirements as "conventional" models do.

With the introduction of human capital, more sophisticated scenarios could be explored. Under the current framework, the inclusion of human capital along with material wealth in the utility function becomes natural. Again, Uzawa's technological setup implied the same dynamic patterns.

One can ask then why would the proposed function should be of use? Firstly, because it does not require an hypothesis of accumulated discounted utility maximand. Rather, an implicit or pseudo-rate of time preference was mathematically deducted, relating the rate of change of consumption with that of capital along an optimal path - homothetic preferences rendering it constant. Eventually, time inconsistency [19] based on discounting patterns therefore disappears.

Secondly, because it shifts the attention towards and stresses wealth formation – resulting from, caused by, the accumulation of past saving-investment – consumption abstinence – flows. Maybe we should not be looking for a long-run consumption function, but for a wealth or asset (demand) function – dependent on past consumption... Or functions of the two – wealth and lagged consumption - may just be more closely co-integrated.

Thirdly, for its mathematical tractability – even if we foresee that the inclusion of money, bonds, taxes, public goods, multiple assets or market goods – debt, a potential negatively valued argument of the WIU function -, or yet life-cycle labor-leisure choices, natural extensions or applications that we did not pursue here, may complicate it again. That allowed us to make reasonable deductions of matters like the effect of exogenous uncertainty – shocks – on tastes or technology over intertemporal dynamics of economic stocks and flows – reviewing the role of risk-aversion and pace of diminishing marginal returns to capital in growth determination.

Finally, forward-looking dynamics were found to be compatible with capital-in-utility (wealth-in-utility) modeling – sufficing to include the capital stock of two (an extra) future periods in the welfare function, optimization being thus made conditional on future decisions.

Obviously, applications of the same principles to the intertemporal decisions of the firm are in the agenda.

Notes

[1] Cass (1965) and Koopmans (1965). Kurz (1968) includes wealth effects in the felicity function.
[2] Or status effect models – see Bakshi & Chen (1996) for an example. See also Zou (1998).
[3] Zou (1995).
[4] Bakshi & Chen (1996).
[5] Diamond (1965).
[6] One could postulate as well $U_t(c_t, w_{t+1})$. Provided that in the fundamental dynamic wealth equation(s) below w_t (k_t) is replaced by w_{t+1} (k_{t+1}) for all t and w_{t+1} is determined with c_t, the conclusions would remain.
[7] See, for example, Romer (1996), Ex. 4.11., p. 192-193. Or Blanchard & Fischer (1989), p.284.
[8] With quasi-concavity of $U(c_t, k_t)$ in the arguments being sufficient to generate convex indifference contours in space (k_t, c_t) - where for given k_{t-1}, the (budget) constraint (5) is linear with slope –1 - and that F.O.C. imply a maximum.
[9] Theoretically, if c_{t-1} entered the system as well, the curve to be plotted in a phase diagram, the phaseline, would not be this one – see section 7 below. Given the simple structure of the problem, the argument holds. Also, notice we are plotting c_t against k_{t-1}, which is not usual, but here sensible.
[10] See Azariadis (1998), p. 4, for example.
[11] See Martins (1989), for example.
[12] Arrow's (1962) IRS technology generates similar consequences.
[13] Under increasing returns, a competitive solution will hardly guarantee a pareto optimal result. Market solutions with externalities are found in Martins (1987) and (1989), for example.
[14] See also Martins (2004).
[15] A generalization of Ramsey's problem with ex-ante uncertainty in production was studied by Brock & Mirman (1972). Of course, anticipation would have more complex effects than ours when intertemporal optimization is considered.
[16] We could think that including the second restriction in the former, simpler, problem would render a recursive structure. It does not, once, as we have two lagrange multipliers, with only two controls, we are left with only the two state equations per periodic problem. Hence, k_{t+1} is "conditionally" decided today, and we have to confirm – or specify - our decision on k_{t+1} next period. Also, only k_t and k_{t+1} appear in the current function, for current decision – hence, only two constraints (one for each of them) are relevant.
[17] As one restriction always overlaps in two subsequent period problems, decisions over capital are forced to be consistent.
[18] Technically, the F.O.C generate now similar dynamic traits as Ramsey's structure – see Azariadis, p. 210, for example.
[19] See an early reference in Strotz (1955), and Frederick Loewenstein & O'Donoghue (2002) for a recent survey.

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2 Unions and wage determination: Can monopsonist unions reduce unemployment?

Introduction

We present some of the analytical consequences of introducing endogenous membership in the standard union model. Endogeneity of union status in bargaining models has been previously addressed in the literature [1], and this includes very early references [2]. However, a simple and clear methodological distinction of the issues underlying the final labor market outcome has not, to our knowledge, been advanced.

Hence, the analysis of labor market outcomes in the presence of endogenous membership involves three levels of considerations: one is how the union's objective function is affected by the number of "insiders" [3]. The other is how applicants - potential insiders - react to conditions offered by union membership, which calls for the definition of a membership demand function. Finally, unions may or may not be able to decide membership size - which determines whether the union must behave competitively or not towards membership demand.

Usually, it is not considered explicitly - or it is irrelevant and taken as given - how union size affects the union's utility function.

On the one hand, for given wages and total employment, an increase in membership decreases the probability of (union) employment of union members. But also, union size may affect positively the union´s ability to behave as a monopoly. Moreover, a larger number of members at given wages (say, in an utilitarian environment) would probably be seen as a positive fact. On the other hand, as it is the argument behind the median voter structure, the decision process ruling union behavior is also a factor affecting union's goals. In sum, the union´s objective function may be considered to depend not only on employment and wage but also on membership - its effect on utility may be positive or negative.

The membership function measures how the labor force gets unionized and is therefore assumed to be positively related to the wages the union jobs offer [4]. Some theoretical models of union behavior have treated demand for union services as coming from median voter models [5]. We note that, specially with corporate bargaining, the membership function is ultimately related to labor supply - it is as if unions, that may behave as a "monopsonist" [6] in the hiring market, make the "interface" between the labor supply and firms' labor demand. If closed-shop agreements are ruled out, and if the union has no ability to avoid membership - or agreements with respect to wages must be extended to any employed worker [7] - it will probably have to take into account such fact in optimization - i.e., make "conjectures" about membership behavior (demand) or internalise membership response to union wages.

In section I, we discuss the role of endogenous membership and compare the situation where a monopoly union behaves as a monopsonist with respect to membership demand to the one where it behaves "competitively". Section II reproduces the exercise for an environment with efficient bargaining.

Some special cases are presented in section III, with less usual unions' utility functions: directly using absolute unemployment (and wages) as arguments; average utility of members; money value of aggregate surplus or "economic rent" obtained by members.

In section IV, we present an analytical and graphical derivation of the "microfoundations" of the membership function [8]. We diverge from the issues that have been previously raised and emphasize the behavior of the union as a nonprofit organization facing bargaining costs.

The modelling is kept as simple as possible in order to focus on the special mechanism in study.

The exposition ends with a brief summary in section V.

The role of labor supply - Endogenous union membership and the monopoly union

1. The demand for union membership will probably increase with the wage set by the union, i.e., we consider that membership M, M = M(W) and is increasing in W.

Assume the union maximizes total utility, and the unions' utility depends, as usual on total employment, L; wage, W, and also on M, i.e., [9]

$$\max_{L,\,W,\,M} U(L, W, M) \quad (1)$$

$$\text{s.t.:} \quad L = L(W) \;; \qquad M = M(W)$$

L(W) denotes a negatively sloped demand for labor [10]. Alternatively, we can write (I.1) as:

$$\max_W U[L(W), W, M(W)] \quad (2)$$

The optimal solution will be W^M such that [11]:

$$U_L L_W + U_W + U_M M_W = 0 \quad (3)$$

2. Assume U = U(L, W, M) is increasing and quasi-concave in its arguments. Then, given that U_M and M_W are positive, the utility function will be increasing in the point W* where union ignores union membership demand, i.e., in the solution of:

$$U_L[L(W), W, M(W)]\, L_W + U_W[L(W), W, M(W)] = 0 \qquad (4)$$

which would correspond to the usual monopoly union solution, in the Dunlop (1944) tradition.

Therefore:

$$W^M > W^* \quad ; \quad M^M > M^* \quad ; \quad L^M < L^* \qquad (5)$$

Unemployed members, $u(W) = M(W) - L(W)$, will be more than if union membership effect was taken as exogenous, i.e., $u^M > u^*$.

Consider the graphic representation of the problem in the following way: Take the problem written as:

$$\max_{W, M} U[L(W), W, M] = \omega(W, M) \qquad (6)$$

s.t.: $M = M(W)$

$\omega(W, M)$ arises from the substitution of L by labor demand in the general utility function $U(L, W, M)$. F.O.C. will yield:

$$-\frac{\omega_W}{\omega_M} = M_W \quad ; \qquad (7)$$

$\omega_W = U_L[L(W), W, M]\, L_W + U_W[L(W), W, M]$; $\omega_M = U_M[L(W), W, M]$

We are assuming $\omega_M > 0$; a typical "reduced" union indifference curve defined over W and M, $\omega(W, M) = \bar{\omega}$, will be positively sloped near the optimal solution - where $\omega_W < 0$ – and, for an internal solution, concave. Utility increases to southeast. The indifference curves will have a point for which $\omega_W = 0$, representing a membership-taker first-order condition: for low wage levels, an indifference curve will be negatively sloped.

The solution of W* and W^M are depicted in Fig. 1. W* is the wage at which an indifference curve achieves $\omega_W = 0$ on the membership demand curve, M = M(W). Plotting also the underlying demand function L(W), we can visualize not only membership, but also demand, and corresponding unemployment level in the two situations.

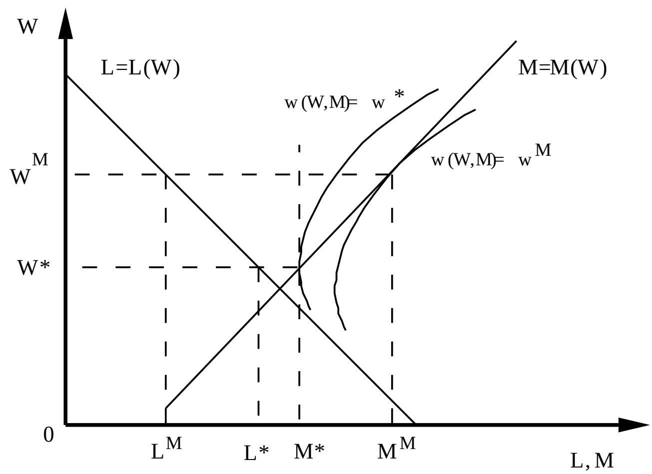

Fig. 3

Figure 1.

3. Suppose that $U_M < 0$. Then, indifference curves - $\omega(W, M) = \bar{\omega}$ - in (M,W) space would increase to northwest, we would want them to be convex, and W* would be higher than W^M.

Proposition 1. If membership demand increases with the wage set in the negotiations, and the union´s utility increases with the number of members:

1. The monopoly union that behaves as a monopsonist in the membership market will choose a higher wage, higher membership and lower employment than the one that behaves competitively in the membership market.

2. The opposite occurs if either the membership function is negatively sloped or union´s utility function decreases with membership.

4. Alternatively to formulation (6), the monopoly union problem can be written in terms of L and W:

$$\max_{L,\,W} U[L, W, M(W)] = \circledcirc(L, W) \quad \text{s.t.: } L = L(W) \tag{8}$$

F.O.C. will yield:

$$\frac{\nu_W}{\nu_L} = -L_W \quad ; \quad \circledcirc_W = U_W + U_M M_W \tag{9}$$

The graphical representation of this problem is identical to the one in which M is taken as exogenous, but with respect to the modified utility function $\circledcirc(L, W)$. At the tangency of the optimal indifference curve with labor demand, provided $U_M M_W > 0$, $\frac{U_W}{U_L} < \frac{\nu_W}{\nu_L}$; the tangency with a membership-taker´s indifference curve will be to the southeast of the solution (9).

(9) defines a relation between the wage and employment, W = g(L); its intersection with labor demand yields the optimal solution, (W^M, L^M) We can see it depicted in Fig. 2. The slope of g(L), $\frac{dg}{dL} = -\frac{\nu_{LL} L_W + \nu_{WL}}{\nu_{LW} L_W + \nu_L L_{WW} + \nu_{WW}}$ (and probably positive). Being $U_M M_W > 0$, it lies to the left of the function W = h(L), solving $\frac{U_W}{U_L}\Big|_{M=M(W)} = -L_W$, which would intersect labor demand at the membership-taker solution W*.

Endogenous union membership with efficient bargaining

1. The efficient bargaining solution comes from [12]:

$$\max_{L,\,W}\; U[L,W,M(W)] + B \circledcirc (L,W) \qquad (10)$$

where B is directly related to the relative power of the employer in negotiations, and would yield:

$$\frac{\nu_W}{\nu_L} = \frac{\Pi_W}{\Pi_L} \qquad (11)$$

If the firm('s) is a profit maximizer:

$$\frac{\nu_W}{\nu_L} = \frac{U_W + U_M M_W}{U_L} = \frac{L}{W - P F_L} \qquad (12)$$

If $U_M M_W > 0$, the locus (L,W) such that [13]

$$\frac{U_W}{U_L}\Big|_{M=M(W)} = \frac{L}{W - P F_L} \qquad (13)$$

will be to the right of the efficient locus given by the tangency points of (14), because at any point (L,W):

$$\frac{U_W}{U_L}\Big|_{M=M(W)} < \frac{U_W + U_M M_W}{U_L} \qquad (14)$$

So, once $\dfrac{\partial \frac{U_W}{U_L}}{\partial L} > 0$, for each level W, a lower L will be chosen in an efficient contract agreement in the case where membership is taken as endogenous. Alternatively, at a tangency between an indifference curve and an isoprofit curve that satisfies (12), $\frac{U_W}{U_L}\Big|_{M=M(W)}$ - the slope of $U(L,W,M) = \bar{U}$ evaluated at $M = M(W)$ - is smaller than $\frac{L}{W - P F_L}$; to achieve tangency of an indifference curve, i.e., (14), with the same isoprofit curve, the solution must lie to the southeast of (13). CC, defining tangency of indifference curves with isoprofit curves when membership is endogenously considered by the union, will be to the left (in space (L,W)) of the locus defined by (14) – the contract curve of the traditional membership-taker union, so to speak -, C′C′. We can see both curves in Fig. 2.

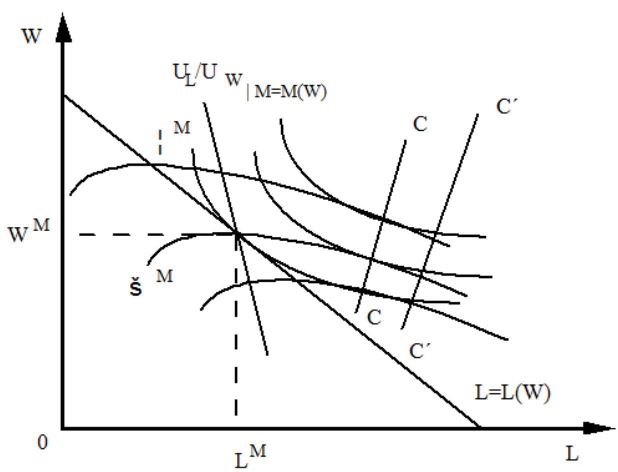

Figure 2.

3. If $U_M M_W < 0$, the conclusions would be reversed and the "monopsonist" contract curve lies to the right of the "membership-taker" union - implying that for the same employment level, lower wages will be achieved.

Proposition 2. If membership demand increases with the wage set in the negotiations, and the union′s utility increases with the number of members:

1. The efficient bargaining locus of the "monopsonist" union will lie to the left (less L for given W; higher W for given L) of that of the "competitive" union.

2. The opposite occurs if either the membership function is negatively sloped or union´s utility function decreases with membership.

4. In an efficient contract solution, an increase in membership may decrease B, i.e., B = B(M) and $B_M < 0$ -, or decrease the cost of rising wages, and additional effect could be in place, this favoring a shift to the right of the contract curve.

Analytical examples

Case A. Unions Utility Depending on u and W

1. A special case of $U_M < 0$ would be the utility function of the problem:

$$\max_{L, W, M, u} \ U(u, W) \tag{15}$$
$$\text{s.t.:} \quad u = M - L \ ; \quad L = L(W) \quad ; \quad M = M(W)$$

where $U_u < 0$ and $U_W > 0$. We see that $U_M = U_u$ and $U_L = - U_u$; so the union values employed and total members utility symmetrically.

Consider the excess supply of members

$$u(W) = M(W) - L(W) \tag{16}$$

We have that

$$u_W = M_W - L_W > - L_W > 0 \tag{17}$$

Problem (15) can be written as:

$$\max_{u\ W} \ U(u,W) \tag{18}$$
$$\text{s.t.:} \quad u = u(W)$$

The problem can be represented in the (u,W) space - see Fig. 3. A typical indifference curve slopes upward - as well as u(W) - and the utility level increases to the northwest.

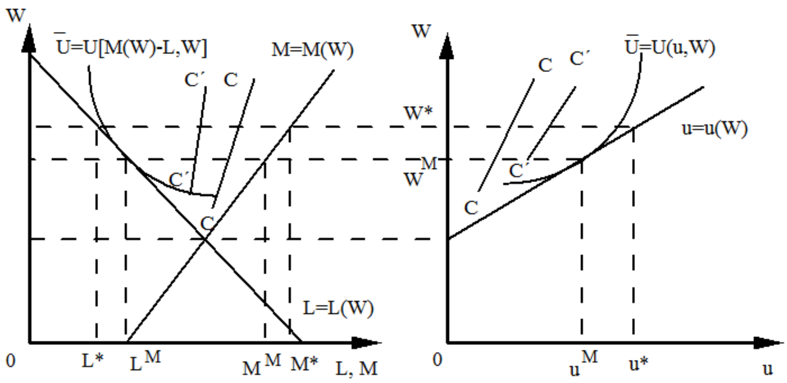

Figure 3.

The optimal solution will be such that:

$$-\frac{U_W}{U_u} = u_W = M_W - L_W \tag{19}$$

Efficient bargaining satisfies:

$$\max_{L,\, u,\, W} \; U(u,W) + B \circledcirc (L,W) \tag{20}$$

s.t.: u = M(W) - L

or

$$\max_{L,\, W} \; U[M(W) - L, W] + B \circledcirc (L, W) \tag{21}$$

The optimal solution obeys:

$$-\frac{U_u M_W + U_W}{U_u} = \frac{L}{W - P F_L} \quad \text{or} \tag{22}$$

$$- \frac{U_W}{U_u} - M_W = \frac{L}{W - P\,F_L} = \frac{\Pi_W}{\Pi_L} \qquad (23)$$

Consider then the problem in space (L,W). We will have that for any tangency,

$$\frac{U_W}{U_L}\Big|_{M=M(W)} = - \frac{U_W}{U_u} > - \frac{U_W}{U_u} - M_W \qquad (24)$$

Therefore the efficiency locus CC (monopsonist) in the (L,W) space will lie to the right of the curve C′C′ (membership -taker) - the opposite occurring in the (u, W) space - given by

$$\frac{U_W}{U_L}\Big|_{M=M(W)} = \frac{\Pi_W}{\Pi_L} \qquad (25)$$

2. Take a particular example where the union′s utility function is of the form:

$$U(u, W) = u^{-\circledcirc} (W - W_a)^{\circledcirc \; \circledcirc} \qquad (26)$$

2.1. The monopoly union solution will yield:

$$\frac{\theta}{\gamma} \frac{u}{W - W_a} = M_W - L_W = u_W \qquad (27)$$

Assume that $W_a = 0$. Then, we can manipulate the expression to yield:

$$\frac{\theta}{\gamma} = u_W \frac{W}{u} = \circledcirc_{u,W} = (M_W \frac{W}{M}) \frac{M}{u} - (L_W \frac{W}{L}) \frac{L}{u} = \qquad (28)$$

$$= \circledcirc^M \frac{M}{u} + \circledcirc^D \frac{L}{u}$$

Denote the unemployment rate $u_r = \frac{u}{M}$. Then, we can solve (28):

$$u_r = \frac{\eta^M + \eta^D}{\frac{\theta}{\gamma} + \eta^D} \quad (29)$$

where η^M denotes the (positive) elasticity of membership demand (labor supply) and η^D the labor demand elasticity in absolute value. The unemployment rate will be between 0 and 1 if $\eta^M < \frac{\theta}{\gamma}$; this condition guarantees that the unemployment rare increases with the elasticity of demand. (29) also suggests that the unemployment rate will be higher the larger is the elasticity of membership demand (labor supply), η^M - yet, an interior solution may be impossible for constant wage-elasticity labor demands.

The competitive solution is given by (29) with $\eta^M = 0$, a lower unemployment rate.

2.2. The efficient bargaining locus will be (assuming $W_a = 0$).

$$\frac{\theta}{\gamma}\frac{u}{W} - M_W = \frac{L}{W - P F_L} \quad (30)$$

In terms of the unemployment rate:

$$u_r = \frac{\eta^M (1 - \frac{P F_L}{W}) + 1}{\frac{\theta}{\gamma}(1 - \frac{P F_L}{W}) + 1} \quad (31)$$

Proposition 3. If membership demand increases with the wage set in the negotiations, and the union´s utility is of the form (26):

1. The monopoly union solution of the "monopsonist" union will lead to lower wages and lower unemployment than the "membership-taker" union.

2. The efficient bargaining locus of the "monopsonist" union will lie to the right (higher L for given W; lower W for given L) of that of the "competitive" union.

3. The unemployment rate of the "monopsonist" union will respond positively to both the elasticity of demand (in absolute value) and the elasticity of the membership function with respect to the wage rate.

Case B. Union Maximizes Average Utility

Consider that the union maximizes the average utility, i.e.,

$$\max_{L,\,W,\,M} \frac{U(L, W, M)}{M} \tag{32}$$

s.t.: $L = L(W)$; $M = M(W)$

or, alternatively:

$$\max_{W} \frac{U[L(W), W, M(W)]}{M(W)} \tag{33}$$

This utility function may be justified in analogous terms as the labor managed firm [14] objective function (revenue per worker): union members, in their decision processes concerning letting "outsiders" come in, maximize the "amount of utility" that accrues to each member.

The F.O.C. for an interior solution will give an optimal W^{2M} such that

$$(U_L L_W + U_W + U_M M_W)\, M - U M_W = 0 \tag{34}$$

Recall that at W^M, $U_L L_W + U_W + U_M M_W = 0$. Therefore, U is (already) decreasing at W^M: as expected (because as W increases M increases, decreasing, for fixed U, $\frac{U}{M}$), $W^{2M} < W^M$. The wage is now smaller than in the case where it maximizes total utility - demand will be higher, membership lower and the unemployment of members lower.

Let us compare the solution with the one where membership is exogenously considered, W*. For this solution, $U_L L_W + U_W = 0$. So we have two possibilities; at W*, either:

a) $U_M M_W \; M - U M_W > 0$, or $U_M \frac{M}{U} > 1$ (elasticity of U respect to M is larger than 1).

In this case, at W*, U is increasing and so $W^{2M} > W^*$. We will have, therefore:

$$W^* < W^{2M} < W^M \tag{35}$$

Graphically, this problem will yield the same conclusions as the one of Case A.

b) $U_M M_W \; M - U M_W < 0$, or $U_M \frac{M}{U} < 1$ (elasticity of U respect to M is smaller than 1).

In this case, at W*, U is decreasing and so $W^{2M} < W^*$. We will have, therefore:

$$W^{2M} < W^* < W^M \tag{36}$$

It is easy to show that the solution of this case will have similar properties as the one in the example. Typically, it corresponds to a similar graph as the one of Fig. 3.

Consider the utilitarian union: $U(W, L, M) = L\,u(W) + (M - L)\,u(W_a) = L\,[u(W) - u(W_a)] + M\,u(W_a)$, where $u(W)$ - increasing and concave in its argument - is the typical member utility function. Then the union maximizes the expected utility of the representative worker [15] - an objective function well known in the literature - $\frac{L\,[u(W) - u(W_a)]}{M} + u(W_a)$, which is equivalent to maximize $\frac{L\,[u(W) - u(W_a)]}{M}$; then (III.23) holds as long as $W > W_a$.

Proposition 4. If membership demand increases with the wage set in the negotiations, the union´s utility increases with the number of members, the monopoly union maximizes average (over all members) utility and behaves as a monopsonist in the membership market:

1. the wage, membership and unemployment will be lower (and employment higher) than if the "monopsonist" union maximizes total utility.

2. the wage, membership and unemployment will be:

- lower than if the union behaved "competitively" in the membership market if the elasticity of the union´s utility function with respect to M is smaller than one (in this case the union´s objective function decreases with M).
- higher than if the union behaves "competitively" in the membership market if the elasticity of the union´s utility function with respect to M is larger than one.

Case C: Union Maximizes Money Value of Surplus

Consider the utility function that corresponds to the collective rent [16]. It is sometimes assumed that the alternative wage is the one corresponding to the equilibrium solution without the union. If we have a membership "demand" $M = M(W)$, we can interpret it (as any labor supply curve) as valuing the alternative use of time (leisure) by workers. Consider the inverse demand and membership functions:

$$W = W^{D}(L) \quad \text{and} \quad W = W^{M}(M) \tag{37}$$

Denote by W_a the wage that equalizes membership and demand, i.e.:

$$W_a = W^{D}(L_a) = W^{M}(L_a) \tag{38}$$

We can postulate an utility function where what is maximized is the monetary surplus of employed members, i.e.:

$$U(W, L) = W\,L - \int_0^L W^{M}(u)\,du \tag{39}$$

The monopoly union problem will be:

$$\max_{L,\,W} \; W\,L - \int_0^L W^{M}(u)\,du \tag{40}$$

$$\text{s.t.:} \qquad W = W^{D}(L)$$

or

$$\max_{L} \; W^{D}(L)\,L - \int_0^L W^{M}(u)\,du \tag{41}$$

F.O.C. originate:

$$W^{D}(L) + L\,\frac{dW^{D}(L)}{dL} = W^{M}(L) \tag{42}$$

that is, marginal revenue of the union - $\frac{d(WL)}{dL}$ - equals membership demand (labor supply) wage in the employment level (implicitly) chosen.

Denote the above solution (L_1, W_1). Comparing with the solution (L_0, W_0), corresponding to the rent maximizer union with fixed W_a (i.e., that looks at membership supply as perfectly elastic at the wage that equates labor demand and membership supply) - see Fig. 4 -, we conclude that - as long as membership supply is not perfectly elastic - we achieve a lower wage and higher employment in the case where the surplus - the area below W until L between W line and the membership function - is maximized.

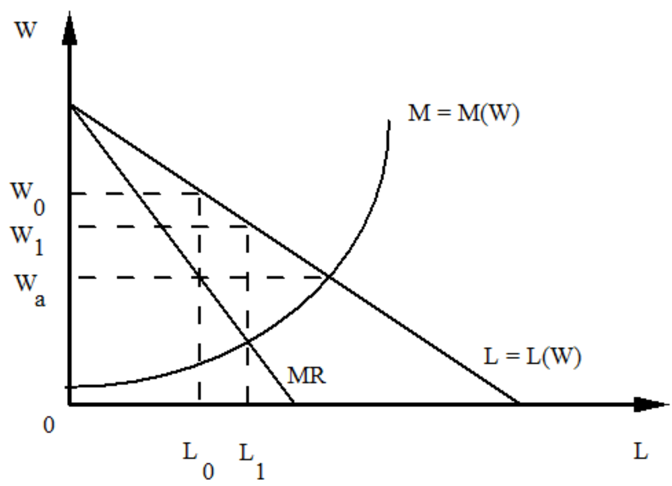

Figure 4.

Summarizing:

Proposition 5. If membership demand increases with the wage set in the negotiations, the monopoly union maximizes the members aggregate rent and behaves as a monopsonist in the membership market, the wage, membership and unemployment will be higher than if the union behaves "competitively" in the membership market. It will be lower than if the union considers supply of members as perfectly elastic at the "competitive wage".

We should notice that even if the union acts with benevolent intentions, say, members are altruistic and so is the union, it is still the case that the role of the union is very different from the one of a social planner. In some cases, if unions behave as monopsonists towards the labor market, the outcome may be worse in terms of unemployment than if they did not; in others, it may be better. Notwithstanding that it is (still...) the case, behind these models, that unions are considered a means of achieving redistribution purposes but not efficiency.

Membership fees and Bargaining costs: Wage determination and membership demand

We have considered a membership demand function M(W) without referring to its formation [17]. On the one hand, one can - specially if corporate bargaining is considered - interpret it as labor supply. Alternatively, we could see it as arising from a more general problem and identify membership response to membership fees in general form [18].

1. Denote membership fees by a. Membership demand will likely be

$$M = M(W, a) \quad ; \qquad M_W > 0 \, ; M_a < 0 \tag{43}$$

Let C denote union bargaining costs. They will be increasing in M and, eventually, W.

$$C = C(M, W) \tag{44}$$

The union behaves as a nonprofit organization, i.e., works under a budget constraint (which gives rise to membership supply $M = M^S(W, a)$):

$$C(M, W) = M\,a \tag{45}$$

The union´s utility function will depend negatively on a, once members income decreases as a increases:

$$U = U(L, W, M, a) \tag{46}$$

The monopoly union problem can be written as:

$$\max_{L, W, M, a} U(L, W, M, a) \tag{47}$$

s.t.: $L = L(W)$; $M = M(W, a)$; $C(M, W) = M\,a$

Let us consider the restrictions (44) and (45). We can derive:

$$a = \frac{C(M,W)}{M} \tag{48}$$

Replacing in the membership demand function (43), $M = M(W, a)$, we get:

$$M = M[W, \frac{C(M,W)}{M}] \tag{49}$$

From an explicit form (49), we can solve for $M = M(W)$. Graphically - see Fig. 7 -, we can see how this function is formed. In the space (M,W), we have the union average cost curves (for different levels of W); say curve C_0 corresponds to the average cost of attaining wage W_0, i.e., has the form $a = \frac{C(M,W_0)}{M}$ and C_1, for a given $W_1 > W_0$, $a = \frac{C(M,W_1)}{M}$; The intersection of these curves with $M = M(W_0, a)$ and $M = M(W_1, a)$ respectively, yields a membership/membership fees relation $M = M(a)$. To each intersection corresponds, therefore a given level of M - the relation $M = M(W)$ is represented in quadrant II. From here we conclude that M(W) may not be positively sloped - it will not be if average costs rise sufficiently fast with W relative to the shift of M(a, W); it will have the same slope as M(a). The relation between a and W is in quadrant IV and will always be positive.

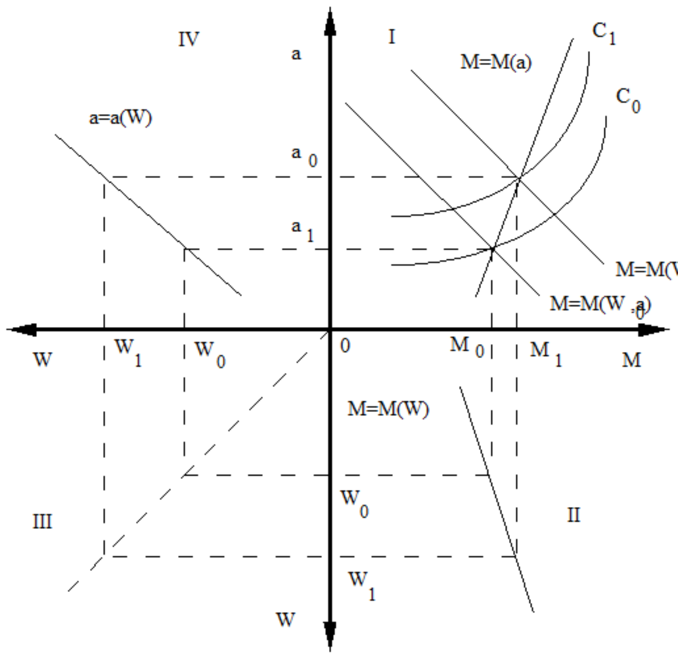

Figure 5.

If we replace restrictions (48) and (49) on the utility function we will obtain a problem of the general form of (I.1) which arguments are L, M and W:

$$\max_{L,\,W,\,M} \; U[L,\, W,\, M,\, \frac{C(M,W)}{M}] \qquad (50)$$

$$\text{s.t.:} \qquad L = L(W) \;\; ; \qquad M = M(W) = M[W,\, \frac{C(M,W)}{M}]$$

Analogously, an efficient solution will answer:

$$\max_{L,\,W,\,M} \; U[L,\, W,\, M,\, \frac{C(M,W)}{M}] + B\,[P\,F(L) - W\,L] \qquad (51)$$

$$\text{s.t.:} \qquad M = M(W) = M[W,\, \frac{C(M,W)}{M}]$$

2. The previous problem assumes - as noted before - the union behaves as a monopsonist in the "membership market" - the labor market. That is, presumably, a reasonable assumption in "corporate systems". But assume, instead, that the union behaves competitively. This would correspond to the following:

The union, given membership M, decides

$$\max_{L,\,W,\,a} \; U(L, W, M, a) \qquad (52)$$

$$\text{s.t.:} \quad L = L(W) \;; \quad C(M, W) = M\,a$$

That is:

$$\max_{L,\,W} \; U\left[L, W, M, \frac{C(M,W)}{M}\right] \qquad (53)$$

$$\text{s.t.:} \quad L = L(W)$$

The solution of the problem will yield W and L as a function of M. Then, we can write:

$$M = M^S(W) \qquad (54)$$

Using the budget constraint, we can also obtain

$$a = \frac{C[W, M^S(W)]}{M^S(W)} = a^S(W) \qquad (55)$$

Competitive equilibrium in the "membership market" can be derived from:

$$M = M^S(W) \;; \quad a = a^S(W) \;; \quad M = M(W, a) \qquad (56)$$

Ultimately, in this market we observe wage determination. Notice that this setting represents the problem

$$\max_{L, W} U(L, W, M) \quad (57)$$

s.t.: $L = L(W)$

and exogenous "membership demand" M = M(W) to which endogenous M solution was compared to.

Summary and conclusions

This paper gathers some notes and enlargements to the standard collective bargaining problem in which unions maximize utility and firms maximize profits.

We extended the simple standard model in order to include membership considerations, introducing a union membership demand positively related to wages - eventually arising in a setting where unions behave as nonprofit organizations.

In some cases, wages and unemployment will be higher (the contract curve will shift to the right in efficient bargaining) when the monopoly union can behave as a monopsonist towards the labor supply or membership demand than when it acts competitively, i.e., take membership as exogenously given. This will occur if union's utility function depends positively on number of members. The opposite is expected if the unions' decision process values negatively the number of members. Some examples of both cases are presented as an illustration.

This study contains an additional point to the explanation of the hump-shaped relation between wages and centralization in wage bargaining [19], here working through awareness of labor supply response, rather than union (or firm-union) rivalry. If membership is seen as more elastic to the wage rate when bargaining is coordinated economy-wide, provided that the decision process imply that unions value negatively the number of "insiders", a monopsonist union - representing the corporate bargaining result - will choose a lower wage and lower unemployment than the "competitive" or "membership-taker" union - this being associated to a smaller degree level of centralization, as in industry-wide bargaining.

Notes

[1] See Booth (1995), section 4.6, pp. 108-116, for a thorough recent survey.
[2] Dunlop (1944), cited in Farber (1986), considers a membership function increasing in the wage rate net of membership fees.
[3] Using Lindbeck & Snower (1990) term. Notice that we will never measure "outsiders", so we do not have a really insider-outsider scenario.
[4] There are empirical arguments justifying the use of such function. Duncan & Leigh (1985), for example, provide an empirical analysis involving the treatment of endogeneity of union status, not rejected by statistical tests in their study. See Booth (1995), section 6, pp. 157-182 and references therein.
[5] Grossman (1983), for example, analyses endogenous membership in a system with seniority rules and prohibited closed shops. Booth (1984) links membership demand to individual decision of potential members which compare union's expected payoff with available alternatives.
[6] We do not follow the interpretation of Lewis (1959), cited in Farber (1986), that the union wants to ´extract from the members all the rents (...)´ through membership fees, but instead we think of an union that may eventually behave as a nonprofit organization.
[7] This is a feature of the Portuguese bargaining results, for example.
[8] Relative demand for union status has been modelled as a function of wages by, for example, Booth & Chatterji (1993) in an open shop scenario. They arrive at a union membership demand function where union density is a function of the wages net of membership fees, convex in gross wages. See also Naylor & Raaum (1993) and Naylor & Cripps (1993). More recently, membership demand has received attention in the open shop union literature - see Corneo (1997), Holmlund & Lundborg (1999) and Moreton (2001).
[9] We consider that the union cannot avoid membership. If it could, the second restriction would be replaced by $M \leq M(W)$ and, if $M(W)$ is positively sloped, would only become active at very low levels of W.
[10] If the firm behaves competitively maximizing profits, the restriction $L = L(W)$ is equivalent to $P F_L(L) = W$.

[11] Second-order condition requiring

$$U_L L_{WW} + U_{LL} L_W^2 + 2 U_{LW} L_W + U_{WW} +$$

$$+2 U_{LM} L_W M_W + 2 U_{WM} L_W M_W + 2 U_{MM} M_W^2 + U_M M_{WW} \leq 0$$

This is consistent with both positive or negative U_M.

[12] See Earle & Pencavel (1990) - the "canonical bargaining form". The Nash maximand solution

$$\max_{L,\,W} \; [U(L,W) - \bar{U}]^{\circledcirc} \, [\circledcirc (L,W) - \bar{P}]$$

considered to arise in a bargaining where alternatives to agreement are $\bar{U}$ and $\bar{P}$ for the parties involved, would complicate some of the mathematics - and gives the same efficient combinations (L,W). $\circledcirc$ corresponds to the ratio of the firm discount rate to the union´s discount rate, and will, therefore, be higher the higher the relative bargaining power of the union. See Layard, Nickell & Jackman (1991), for example.

[13] As in McDonald & Solow 's (1981) traditional contract curve.

[14] Vanek (1970) is the mandatory reference for the analysis of the labor-managed economy, being the firm's objective function value-added per worker; we use the same argument to postulate the objective function of the union: total utility - whatever it may be, representing a measure of the aggregate social welfare of members - per member. This analysis is a form of modelling motivations behind the decision on the number of "insiders" and is a way to go around the well-known median voter problem of equilibrium determinacy with union-wage setting.

[15] Which may differ from the median voter's expected utility, as noted in Booth (1995), for example. Notice that the traditional median voter conclusions would not be applicable once voters also decide on the number of voters...

[16] See, for example, Kaufman (1991) for the analytical illustration of the monopoly union solution when the alternative wage is fixed or exogenous. de Menil (1971), cited in Blair & Crawford (1984), assumes that "unions maximize the surplus above the opportunity cost of the employed labour".

[17] We also ignore leadership problems or voting mechanisms - including seniority issues. These have been dealt with in the literature - see Farber (1986) for references. The considerations on membership in this paper would therefore apply with more accuracy to corporate bargaining settings.

[18] See the reference to and alternative derivations in Farber (1986). More recently, Booth & Chatterji (1995) that models a union as a nonprofit-seeking provider; Booth & Chatterji (1993), Naylor & Raaum (1993) and Naylor & Cripps (1993) model social custom, solidarity and reputation in membership demand.

[19] See, for example, Calmfors & Driffill (1988). Also, Tarantelli (1986). Flanagan (1999) contains a recent review of international evidence.

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3 Unemployment and wages and centralization in wage bargaining: Some analytical explanations

Introduction

Empirical findings point to a positive relation between the degree of coordination or centralization in union bargaining and wages (or, rather, the unemployment rate) till a certain point, and a negative one at high levels of centralization [1]. We depart from an argument previously stated in the literature which explains part of the observed relation (wage increases with centralization when bargaining is decentralized) - exposed in section 1 -, and suggest - in section 2 - an explanation for the other part (after some degree of centralization, wage decreases with it).

We do not invoke for the explanation any efficient bargaining considerations [2]: a closed shop monopoly union environment with respect to the target (firm, industry or total economy) is always assumed; unions maximize collective income. Also, monetary considerations are "sterilized" – unions and firms perform in a real environment. The difference in behavior results from the way unions perceive the alternative to employment of their members; with economy-wide bargaining, unemployment is certain and necessarily internalised in union's expectations; with decentralised union bargaining, eventually dismissed unemployed members are seen as competing for any union´s jobs.

If unemployed, members receive the unemployment benefit; employed members face earnings taxes. It is likely that unions, as workers, are responsive to after-tax wages. This motivates section 3, that briefly contrasts the two equilibrium outcomes under income taxation.

Going a step further, it can be that the unemployment benefit bill is passed on to employed members through the tax system. In section 4, we compare the two wage bargaining arrangements for the case where earnings taxes are levied to finance the unemployment benefit [3]. Interestingly, the relation between wages and coordination in union bargaining could be reversed, with economy-wide bargaining generating higher wages than industry bargaining.

The exposition ends with a brief summary of the main conclusions in section 5.

Firm and industry-wide Bargaining

1. Suppose unions maximize members´ earnings, and behave in a decentralized way. Each union solves

$$\max_{L,\,W} \; W L + W_a (M - L) \qquad (1)$$

$$\text{s.t.:} \quad L = L(W)$$

where W denotes the wage achieved by union members, L employment, W_a, the alternative received by unemployed members, and M the exogenous number of union members; L(W) is the demand for union members´ labor, a negatively sloped function, i.e., $L'(W) < 0$. The union will pick [4]:

$$W = \frac{W_a}{1 - \dfrac{1}{\eta_{L,W}}} \quad ; \quad W \geq W_a \qquad (2)$$

and $\frac{W - W_a}{W_a} = \frac{1}{\eta_{L,W} - 1}$; $\frac{W - W_a}{W} = \frac{1}{\eta_{L,W}}$

where $\eta_{L,W} = -\frac{L'(W)\,W}{L(W)}$ denotes the wage elasticity of labor demand in absolute value. Given that $L'(W) < 0$, in the internal solution, $W \geq W_a$. The percentage deviation of the negotiated wage over the alternative W_a - the "wage markup" - is equal to the inverse of the excess of the demand elasticity over unity. The wage set will be higher the lower is the elasticity of demand. In the optimal solution, for positive wages, we will necessarily have that $\eta_{L,W} > 1$. Also:

$$\frac{dW}{dW_a} = \frac{L'(W)}{2\,L'(W) + (W - W_a)\,L''(W)} \qquad (3)$$

For second-order conditions of problem (1) to hold, the denominator of (3) is negative [5]. Hence, being labor supply negatively sloped, (3) is positive. If labor demand is linear, $\frac{dW}{dW_a} = \frac{1}{2}$: a unit increase in the alternative increases the optimal wage - earnings of an employed member - by 0,5. If labor demand is convex, i.e., $L''(W) > 0$, $\frac{dW}{dW_a} > \frac{1}{2}$, if it is concave and $L''(W) < 0$, $\frac{dW}{dW_a} < \frac{1}{2}$. If $\frac{dW}{dW_a} < 1$, the difference between W and W_a shrinks when the alternative rises; hence, this happens necessarily with linear or concave demand, but may or may not be the case if demand is convex.

The union maximand increases with W_a. Denote by U* its value at the equilibrium solution; then, $\frac{dU^*}{dW_a} = M - L > 0$.

2. For a particular union, the alternative wage, W_a, is taken as exogenous. It may be seen as the weighted average of the unemployment benefit, b, received in case of unemployment, and the wage (also exogenous to the union control) received if the individuals get employment in other sectors. That is, unemployed members compete for other (any) unions′ jobs. If all sectors are unionised, this wage will equalize across sectors. Let u denote the unemployment rate in the economy:

$$u = \frac{M - L(W)}{M} \qquad (4)$$

Then:

$$W_a = u\,b + (1-u)\,W \qquad (5)$$

Replacing in (2) [6]:

$$W = \frac{b\,u}{u - \frac{1}{\eta_{L,W}}} = \frac{b}{1 - \frac{1}{u\,\eta_{L,W}}} \;;\quad W \geq b \qquad (6)$$

and $$\frac{W - b}{b} = \frac{1}{u\,\eta_{L,W} - 1} \;;\quad \frac{W - b}{W} = \frac{1}{u\,\eta_{L,W}}$$

In the optimal solution - see (6) -:

$$\eta_{L,W} > \frac{1}{u} > 1 \qquad (7)$$

3. With an explicit form for L(W), we may be able to derive an explicit solution for u and W as a function of b. Using (6) and (4) -

considering a inelastic labor supply M -, one can show that in internal solutions:

$$\frac{dW}{db} = \frac{u\,L'(W)}{2\,L'(W) + (W - W_a)\,L''(W) + L'(W)\,\frac{(1-u)^2}{u}} = \qquad (8)$$

$$= \frac{u^2}{u\,(W - W_a)\,\frac{L''(W)}{L'(W)} + 1 + u^2}$$

Hence, the wage and the unemployment rate will increase with the (exogenous) unemployment benefit b. If, but not only if, u < 0,5, (8) will be smaller than 1 – and equilibrium wages will rise by less than an exogenous increase in the unemployment benefit. Also, if demand is linear, $L''(W) = 0$, $\frac{dW}{db} = \frac{u^2}{1+u^2} \leq \frac{1}{2} < 1$. If demand is concave, $L''(W) < 0$, and also $\frac{dW}{db} < \frac{u^2}{1+u^2} \leq \frac{1}{2}$; if demand is convex, $L''(W) > 0$, $\frac{dW}{db} > \frac{u^2}{1+u^2}$.

$\frac{dU^*}{db} = (M - L)\,u - (M - L)\,\frac{(1-u)^2}{u}\,\frac{dW}{db}$, nonnegative and, around the same unemployment level, smaller than $\frac{dU^*}{dW_a} = M - L$ $= M\,u$ was. However, the union´s utility function is incorrectly perceived ex-post; with respect to the relevant maximand, $\frac{d[W\,L + b\,(M - L)]}{db} = M - L - L\,\frac{1-u}{u}\,\frac{dW}{db} = \frac{1}{u}\,\frac{dU^*}{db}$, the effect is larger than $\frac{dU^*}{db}$, but still expected to be smaller than $\frac{dU^*}{dW_a}$.

4. The "unsolved", so to speak, equation (6) is useful to interpret estimated (observed) relations between unemployment and wages. Assume the unemployment benefit level can be controlled for, or is constant over a particular sample – of time

series, or of cross section data; then, provided the wage elasticity of demand is the same for all observed units (an economy in different years; or in different sample regions; or in different industry sectors or professions) – and the same institutional arrangements, i.e., monopoly union wage-setting -, autonomous changes/differences in labor demand (for instance, business cycle induced movements; regional size effects; industry-profession labor demand specificities) will lead to different wage-(un)employment mixes across units, but over that stable relation between the two aggregates. Under those circumstances, (6) asserts that the higher is the wage level, the lower will be the unemployment rate [7] in the economy.

5. For any variable wage-elasticity demand curve:

$$\frac{d\eta}{dW} = -\frac{(1+\eta_{L,W})\,L'(W) + W\,L''(W)}{L(W)} \qquad (9)$$

In the optimal solution given by (3), $\eta_{L,W} > 1$; given second order conditions of problem (1), provided $W_a\ L''(W)$ is small or non positive (e.g., if demand is linear or concave, or around $W_a = 0$ if convex), $\frac{d\eta}{dW} > 0$. $\frac{d\eta}{du}$ will have the same sign. Whatever causes a rise in W (and u) will increase the equilibrium wage elasticity of demand.

Assume a constant elasticity labor demand function, $L(W, \eta_{L,W})$. Then, the analysis of the previous paragraph does not apply. Consider, in this case, a change in the elasticity of demand $\eta_{L,W}$; differentiating the last equation of (6) and using (4):

$$\frac{dW}{d\eta} = \frac{b\,(L_\eta \frac{\eta}{M} - u)}{(\eta\,u - 1)^2 - b\,L_W \frac{\eta}{M}} \qquad (10)$$

$$\frac{du}{d\eta} = \frac{b\,u\,L_W - L_\eta\,(\eta\,u-1)^2}{M\,(\eta\,u-1)^2 - b\,L_W\,\eta} \qquad (11)$$

If $L_\eta = \frac{\partial L}{\partial \eta} \leq 0$ [8], the wage rate decreases with η; if L_η is small in absolute value (or not too negative), the unemployment rate will also decrease with η[9].

Denote the industry level elasticity of demand by η_I. The lower is this elasticity, the higher will be the wage. If decentralizing bargaining – e.g., moving from industry bargaining to firm-level bargaining - has the same effects as increasing the elasticity of the implicit aggregate labor demand function, considerations pertaining to the interpretation of (10) and (11) apply. Hence, decentralized (firm) level bargaining will lead to a lower wage, once labor demand is seen as more elastic.

This result would seem to suggest that if bargaining was staged at the economy-wide level - because labor demand is even less elastic -, we would observe an even higher wage rate. This need not be the case, as we will see in the next section.

Economy-wide bargaining

1. Consider an economy-wide bargaining process. The alternative wage will be the unemployment benefit b with probability one - (2) will apply with $W_a = b$:

$$W = \frac{b}{1 - \frac{1}{\eta_{L,W}}} \quad ; \quad W \geq b \qquad (12)$$

and $$\frac{W-b}{b} = \frac{1}{\eta_{L,W} - 1} \quad ; \quad \frac{W-b}{W} = \frac{1}{\eta_{L,W}}$$

Let W_i denote wage with type i bargaining, i = I (industry), G (economy-wide). Then:

$$\frac{W_I}{W_G} = \frac{1-\dfrac{1}{\eta_G}}{1-\dfrac{1}{\eta_I u_I}} \tag{13}$$

The ratio will be higher:

- the higher η_G
- the lower η_I
- the lower the unemployment rate in the industry bargaining, u_I.

The ratio will be larger than one as long as:

$$\eta_G > \eta_I u_I \tag{14}$$

Given that (for interior solutions) $0 < u_I < 1$, even if $\eta_G < \eta_I$, we expect this condition to hold; if $\eta_G = \eta_I$, the wage with industry-level bargaining will be higher than with economy-wide bargaining. The reason lies on the fact that for a industry union, the alternative wage is perceived as higher than with economy-wide bargaining [10].

2. The previous comments apply regardless of whether aggregate demand shifts or not when we move from industry to economy-wide bargaining. Let us assume that it does not – the only difference comes from enhanced employment competition of dismissed employees in the former.

As $L'(W_I)(W_I - W_a) + L(W_I) = 0$ – first-order conditions of problem (1) - and $W_a \geq b$, $L'(W_I)(W_I - b) + L(W_I) = L'(W_I)(W_a - b) < 0$ (Or given (3) and because $W_a \geq b$), $W_G \leq W_I$. Then, economy-wide bargaining will lead to a lower wage – yet, to a higher wage bill, once wage-elasticity of demand is larger than one.

The unemployment benefit "wage multiplier" becomes, under economy-wide bargaining, (3), replacing W_a by b:

$$\frac{dW}{db} = \frac{L'(W)}{2\,L'(W) + (W - b)\,L''(W)} = \tag{15}$$

$$= \frac{1}{2 + (W - b)\,\frac{L''(W)}{L'(W)}}$$

(15) is positive and if labor demand is linear, $\frac{dW}{db} = \frac{1}{2}$. Comparing with (8), we conclude that if the term in L"(W) is negligible – e.g., demand is linear or b is close to the equilibrium wage -, the impact of a unit increase in b on W is much smaller under industry bargaining – and smaller than u times the economy-wide “multiplier”.

$\frac{dU^*}{db}$ = M – L > 0. As discussed previously, it is expected to be larger than under industry bargaining.

Assuming unions are utilitarian, members are risk neutral and a fixed and exogenous unemployment benefit level:

Proposition 1. 1.1. Economy-wide (corporate) bargaining will lead to a lower wage than industry-wide bargaining if (in equilibrium)

$$\eta_G > \eta_I\, u_I$$

The intuition behind the result lies on the fact that in decentralized bargaining the probability of employment outside a particular union is seen as positive - and, thus, higher than in corporate bargaining.

1.2. If the unemployment benefit is (exogenously) set around the wage rate (and for the same labor demand in the two cases), or demand is linear, the positive impact on the wage (negative impact on employment) of a unit increase in the unemployment benefit is magnified with economy-wide bargaining.

1.3. With industry-wide bargaining, the “unemployment benefit multiplier” will be smaller than one when the unemployment rate is smaller than 0,5. It will never be larger than $\frac{1}{2}$ if labor demand is linear or concave.

Introducing income taxes

1. Consider that we have taxes, s per unit of earned income, on employed workers and the union responds to after-tax wages. The utilitarian union with risk neutral individuals will solve

$$\underset{L,\,W}{\text{Max}} \quad (W - s)\,L + W_a\,(M - L) \tag{16}$$

s.t.: $L = L(W)$

The union will choose:

$$W = \frac{W_a + s}{1 - \dfrac{1}{\eta_{L,W}}} \quad ; \; W - s \geq W_a \;\text{ or }\; W \geq W_a + s \tag{17}$$

One can show, in line with (3) and (8), that:

$$\frac{dW}{dW_a} = \frac{dW}{ds} = \frac{L'(W)}{2\,L'(W) + (W - s - W_a)\,L''(W)} \tag{18}$$

The gross wage rate increases with the alternative W_a and unit tax s according to the same multiplier – second-order conditions require the denominator to be negative. If unions respond to after-tax wages, because $\frac{dW}{ds} > 0$, they will choose higher unemployment and equilibrium wages than if they respond to gross wages. However, the net wage rate may decrease with the tax rate (iff (18) is smaller than unity); this will occur if L"(W) is small.

$\frac{dU^*}{dW_a} = M - L$ and $\frac{dU^*}{ds} = -L$: a rise in the unemployment benefit will increase union´s welfare, an increase in the tax rate will decrease it.

Assume the unemployment benefit is not taxed – unemployed workers are recipients in the fiscal system. Then, replacing W_a for b in (17) and (18), we obtain, respectively, the wage curve and the benefit and tax rate multipliers with economy-wide bargaining.

2. If union(s)´ behavior is not fully centralized,

$$W_a = u\,b + (1-u)\,(W - s) \qquad (19)$$

Replacing in (17) and solving for W,

$$W = \frac{b+s}{1 - \frac{1}{u\,\eta_{L,W}}} \;;\; W - s \geq b \text{ or } W \geq b + s \qquad (20)$$

The union responds to an increase in b, the unemployment benefit, in the same way as to an increase in s, taxes.

$$\frac{dW}{db} = \frac{dW}{ds} = \frac{u\;L'(W)}{2\,L'(W) + (W - s - W_a)\,L''(W) + L'(W)\frac{(1-u)^2}{u}} \qquad (21)$$

(21) is still positive but smaller than (18) – with W_a replaced by b in (18) - under the same restrictions of the previous comparison of (15) and (8). If (but not only if) u < 0,5, or demand is linear, (21) will be smaller than 1 and net wages will decrease with s.

$\frac{dU^*}{db} = u\,(M - L) - (M - L)\frac{(1-u)^2}{u}\frac{dW}{db} > 0$, and $\frac{dU^*}{ds} = -L - u\,L\,(1 + \frac{1-u}{u}\frac{dW}{ds})$. Taxes seem to have a more negative effect on perceived utility than with economy-wide bargaining.

Proposition 2. If unions respond to net wages.

2.1. The tax and the unemployment benefit have similar treatment in the "bargained real wage curve".

2.2. The equilibrium wage and unemployment rate respond (positively and) equally to tax as to unemployment benefit changes (even if union´s welfare decreases with taxes and increases with the unemployment benefit level).

2.3. After-tax wages decrease with the unit tax rate if (but not only if) demand is linear or concave. Under industry bargaining, also if (but not only if) the unemployment rate is smaller than 0,5.

2.4. Proposition 1. still holds.

3. If unions recognize that taxes enter the alternative wage, they could behave as if

$$\max_{L,\,W} \; (W - s)\, L + W_a^* \,(M - L) - s^* \,(M - L)$$

$$\text{s.t.:} \quad L = L(W)$$

where $W_a^* = u\,b + (1 - u)\,W$ and $s^* = (1 - u)\,s$; under decentralized bargaining, s^* is outside the union control. Then, we can show that:

$$W = \frac{W_a^* + u\,s}{1 - \dfrac{1}{\eta_{L,W}}} \quad ; \quad W - u\,s \geq W_a^* \text{ or } W \geq W_a^* + u\,s$$

Replacing W_a^* and solving for W, we arrive at (20). We therefore do not complicate the analysis further with this refinement.

Transfers: A balanced budget constraint

1. Suppose taxes are levied to finance the unemployment benefit under a balanced budget constraint:

$$u\,b = (1-u)\,s \tag{22}$$

Yet, unions fail to recognize it in their decisions.

The comparison of this scenario with the one of the previous section depends on the tax level. If taxes, without intentional restriction, are set at the level that insures (22), equilibrium wages are the same. If, departing from (17) or (20), to insure the budget constraint, the government must increase the tax rate, equilibrium wages (and unemployment) increase, according to (18) and (21) respectively. However, it may not have to increase the tax rate when, in the previous equilibrium, u b > (1-u) s – that will depend on the size of the deficit, u b – (1 – u) s, and on the sign of

$$\frac{d[u\,b-(1-u)\,s]}{ds} = -(1-u)\left(1-\eta_{L,W}\frac{b+s}{W}\frac{dW}{ds}\right) \tag{23}$$

This can be, in either of the two bargaining systems, positive or negative. For economy-wide bargaining and linear demand, if but not only if $\eta_{L,W} < 3$; for industry-wide bargaining, if (but not only if) $\eta_{L,W} < \frac{(1-u)^2+u}{u^2} = \frac{(1-u)^2}{u^2} + \frac{1}{u}$, (23) is negative and an increase in the tax rate closes a positive deficit [11].

The previous statements can be explained as follows: when the tax rate increases, optimal gross wages increase - according to the previous section findings; hence, also unemployment. Then, two counteractive effects are in place with respect to the total impact on the deficit of the rise in s: on the one hand, the direct negative effect of the increase in the unit fiscal contribution. But, on the other, the increase in the unemployment insurance bill due to the larger number of unemployed, and decrease of (employed) taxpayers – a positive effect(s), expected large if labor demand reacts sizeably (i.e., labor demand elasticity is large) to the implicit rise in wages. If the second prevails, the unit tax rate should decrease to close a positive deficit.

(23) may (also) be positive or negative at zero taxes. If u b – (1 – u) s changes smoothly with s, and we depart from a positive deficit at s = 0, the deficit may be increasing or decreasing as we rise s - (23) may be positive or negative. But the deficit must start decreasing with s somewhere and be decreasing with s when we

achieve the balance. For a positive (23) around the balanced budget under such smoothness, either there are multiple solutions for the s that solves the balanced budget; or the internal conditions found for the problem with no taxes do not guarantee 0 < u < 1 (0 < L(W) < M) – that is, the interior solution for s=0 is not available.

2. Assume industry bargaining. Replacing (22) in (20):

$$W = \frac{b}{(1-\frac{1}{u\,\eta_{L,W}})(1-u)} \ ^{12} \ ; \ W \geq \frac{b}{1-u} \text{ or } W(1-u) \geq b \qquad (24)$$

In internal solutions, the wage multiplied by the employment probability in the economy must be larger than the unemployment benefit.

The relation between W and u in the bargained real wage curve is:

- negative if u is low: $\frac{1}{\eta_{L,W}} < u < \frac{1}{\eta_{L,W}^{1/2}}$

- positive if u is high: $u > \frac{1}{\eta_{L,W}^{1/2}} > \frac{1}{\eta_{L,W}}$

(Note that we must be working - see (17) - in a point where $\eta_{L,W} > \frac{1}{u} > 1$.)

This type of inversion seems to occur for some wage curves' estimates using local data surveyed in Blanchflower & Oswald (1994): when unemployment and unemployment squared are introduced in the wage regressions, the first coefficient is negative and the second-one is positive.

Condition (24) is, thus, appropriate for the comparison of wages set under different demand curves. Let us assume a particular economy and pursue the methodology of the previous sections.

Inserting (22) in (19),

$$W_a = W(1-u) \qquad (25)$$

Denote by W_I the solution of (1) $L'(W_I)(W_I - W_a) + L(W_I) = 0$ – first-order conditions of problem (1) – and W_a, the alternative wage function (5), $W_a \geq b$. Let W_I^s be the (new) solution of (16) combined with (19) and (22) that yield W_a^s: $L'(W_I^s)(W_I^s - s_I^s - W_a^s) + L(W_I^s) = L'(W_I^s)[W_I^s - s_I^s - W_I^s(1-u_I^s)] + L(W_I^s) = 0$. Then $L'(W_I^s)[W_I^s - W_a] + L(W_I^s) + L'(W_I^s)[W_a - s_I^s - W_I^s(1-u_I^s)] = 0$; at W_I^s, $W_a - s_I^s - W_I^s(1-u_I^s) = u_I^s b - s_I^s < 0$, $L'(W_I^s)(W_I^s - W_a) + L(W_I^s) < 0$ and the maximand of (1) is already decreasing at W_I^s. The denominator of (8) is negative: $L'(W_I^s)[W_I^s - W_a(W_I^s)] + L(W_I^s)$ decreases with W_I^s; hence, $W_I < W_I^s$ [13].

Using (25):

$$\frac{dW}{db} = \tag{26}$$

$$= \frac{\frac{u}{1-u}L'(W)}{2L'(W) + (W - s - W_a)L''(W) + L'(W)[(\eta_{L,W} - 1)(1-u) - (\eta_{L,W} - \frac{1}{u})]} =$$

$$= \frac{\frac{u}{1-u}}{2 + (W - s - W_a)\frac{L''(W)}{L'(W)} + [(\eta_{L,W} - 1)(1-u) - (\eta_{L,W} - \frac{1}{u})]}$$

(26) may be negative or positive [14]. It will be positive if (but not only if) equilibrium conditions yield $\eta_{L,W} < \frac{(1-u)^2 + u}{u^2}$. If

positive, around the same solution for (20), it will be larger than (21) [15].

We can show that:

$$\frac{ds}{db} = \frac{u}{1-u} + (\eta_{L,W} - \frac{1}{u}) \frac{dW}{db} \tag{27}$$

The required unit tax rate to insure (22) may increase or decrease with the unemployment benefit. (27) has the same sign as $\frac{dW}{db}$ (of the denominator of the second version in (26)). Also, or, rather alternatively, once s is seen as endogenously constrained by the balanced budget requirement,

$$\frac{dW}{ds} = \frac{L'(W)}{2\,L'(W) + (W - s - W_a)\,L''(W) + L'(W)\,(\eta_{L,W} - 1)\,(1-u)} \tag{28}$$

$\frac{dW}{ds}$ is always positive. If demand is linear or concave, it is smaller than $\frac{1}{2}$. Around the s that insures a balanced budget, (28) will be larger than (21) if (but not only if) $\eta_{L,W} < \frac{(1-u)^2 + u}{u^2}$.

$$\frac{dU^*}{db} = -\,M - L - [(1-u)(M-L)(\eta_{L,W} - 1) + L(\eta_{L,W} - \frac{1}{u})]$$

$\frac{dW}{db}$ and may be positive or negative. One can show that the sign will be symmetric to the one of $\frac{dW}{db}$. This is in contrast with the previous scenario: on the one hand, an increase in the unemployment benefit may actually decrease union´s welfare; secondly, when it does, we observe a rise in wages – and in the unit tax rate. Yet, $\frac{dU^*}{ds} < 0$ always.

3. Assume that global bargaining is in place. Then, in (17), W_a = b and, considering the budget constraint (22), we can derive:

$$W = \frac{b}{(1-\frac{1}{\eta_{L,W}})(1-u)} \quad ; \; W \geq \frac{b}{1-u} \text{ or } W(1-u) \geq b \qquad (29)$$

The wage set in the economy, for given demand elasticity $\eta_{L,W}$ and unemployment benefit b, will vary positively with the unemployment rate in the economy.

Comparing this situation, (29), with the one in section 2, in which unions ignore s, (12), we conclude that, provided labor demand elasticity is constant, wages, are now higher. That is, if we compare two economies with the same unemployment benefit and demand elasticity, even if demands differ in other respects, under global bargaining wages will be higher for the economy with tax funding. (We cannot say the same for the non-coordinated case because there, also u will vary).

Denote by W_G the solution of (12) $L'(W_G)(W_G - b) + L(W_G) = 0$ – first-order conditions of problem (1) - and $W_G \geq b$; and W_G^s, the (new) solution of (29) combined with (22), $L'(W_G^s)(W_G^s - s - b) + L(W_G^s) = L'(W_G^s)[W_G^s - b] + L(W_G^s) - L'(W_G^s)s = 0$; at W_G^s, $L'(W_G^s)(W_G^s - b) + L(W_G^s) < 0$ and the maximand of (12) is already decreasing: $W_G < W_G^s$ [16]. Gross wages and unemployment are higher under the tax.

Moreover:

$$W - s = \frac{b}{1 - \frac{1}{\eta}} \; \frac{1 - u\,(1 - \frac{1}{\eta})}{1 - u} \tag{30}$$

That is, if η is constant, after-tax wages will be higher than gross wages were in the case in which taxes were ignored or absent.

Simulating the impact of the unit increase in b as before:

$$\frac{dW}{db} = \frac{\frac{1}{1-u}\, L'(W)}{2\,L'(W) + (W - s - b)\,L''(W) + b\,L'(W)^2\, M / L^2} = \tag{31}$$

$$= \frac{\frac{1}{1-u}}{3 + (W - s - b)\,\frac{L''(W)}{L'(W)} - \eta}$$

If positive, the multiplier is larger than (18) around the same equilibrium and expected to be larger than (26) around the same solution and for b close to W_a; however, the unemployment benefit multiplier may be negative. Net wages respond to the unemployment benefit according to: $\frac{dW - s}{db} = (2 - \eta_{L,W}) \frac{dW}{db} - \frac{u}{1-u}$, which may be positive or negative.

Under economy-wide bargaining:

$$\frac{ds}{db} = \frac{u}{1-u} + (\eta_{L,W} - 1)\,\frac{dW}{db} \tag{32}$$

The required unit tax rate to insure (22) may increase or decrease with the unemployment benefit and one can show that it has the same sign as $\frac{dW}{db}$. Also,

$$\frac{dW}{ds} = \frac{\frac{1}{u} L'(W)}{2\,L'(W) + (W - s - b)\,L''(W) + L'(W)\,\frac{(\eta_{L,W} - 1)(1 - u)}{u}} \tag{33}$$

$\frac{dW}{ds}$ is always positive and around the solution that insures the budget constraint may be larger or smaller than (18). It is expected to be larger than (28).

$\frac{dU^*}{db} = (M - L) - L\frac{ds}{db} = -L\,(\eta_{L,W} - 1)\frac{dW}{db}$: the union´s utility and wages move in opposite direction when the unemployment benefit changes. As under industry bargaining, unions´ welfare moves in the opposite direction of the unit tax rate and $\frac{dU^*}{ds} < 0$.

4. If we consider more or less coordination in union bargaining we see that the comments made in section 2. about the ratio $\frac{W_I}{W_G}$ apply now to the ratio between the aggregate wage bill $- u_i$, i = I, G, is the unemployment rate in the economy under each bargaining arrangement - in the two cases:

$$\frac{W_I\,(1 - u_I)}{W_G\,(1 - u_G)} = \frac{W_I\,L_I}{W_G\,L_G} = \frac{1 - \frac{1}{\eta_G}}{1 - \frac{1}{\eta_I u_I}} \tag{34}$$

It is therefore likely that the wage bill is higher in the decentralized case. Moreover, when comparing different economies – represented by different membership size, or different populations – the relation between the expected wage or gross earnings per member established in (34) is still valid (even if not in terms of the wage bill).

Again, (34) and the previous statement is valid if demand shapes differ in the two cases. Suppose aggregate demand at industry level must be represented by the same function that at global bargaining, i.e., $L_I = L(W_I)$, as $L_G = L(W_G)$. We are working in an elastic portion of demand - or, if isoelastic, L(W) must have wage elasticity larger than one; hence

$$W_I L_I > W_G L_G \quad \text{implies} \quad W_I < W_G \tag{35}$$

However, as $L'(W_I^s)(W_I^s - s_I^s - W_a^s) + L(W_I^s) = 0$ – first-order conditions that drive (24) - and $W_a^s \geq b$, $L'(W_I^s)(W_I^s - s_I^s - b) + L(W_I^s) < 0$; if the denominator of the first part of (31) is negative at W_I^s – if equilibrium wages increase with the unemployment benefit -, $W_G^s < W_I^s$. Then, economy-wide bargaining will still lead to a lower wage than industry bargaining – yet, to a higher wage bill, once wage-elasticity of demand is higher than one. (34) is smaller than 1.

If the denominator of (31) is positive, the reverse happens: $W_I^s < W_G^s$, i.e., economy-wide bargaining yields a higher wage and unemployment levels, and (35) holds; (34) is larger than 1.

The intuition behind this second result is that at industry-level bargaining the budget constraint effect is transmitted *ex post* to the alternative wage, W_a. Given that industry unions respond to this wage, if the *ex post* maximand is convex in W around the optimal solution, the larger perceived income alternative is wage depressant because, as labor demand is very elastic, it implies a lower unit tax rate; hence, the solution ends up by not being as employment-reducing as with global bargaining.

Then:

Assuming unions are utilitarian and members are risk neutral:

Proposition 3. If taxes are levied on employed members to pay for the unemployment compensation, and the budget equivalence is not recognized by unions:

3.1. The "bargained real wage curve" may be positively sloped (at high levels of unemployment). It will be positively sloped if there is coordination in union negotiations (corporate or global bargaining).

3.2. Corporate bargaining will lead to a higher after-tax wage than gross wage is in the case where before-tax earnings are considered in union´s optimisation.

3.3. The wage bill in industry-wide bargaining will be higher than with corporate bargaining if

$$\eta_G > \eta_I u_I$$

It is then likely that in this case the industry-wide bargaining wage will be lower than global bargaining one. If labor demand features are preserved under the two bargaining arrangements, this statement is only valid when wages respond negatively to the unemployment benefit; if they respond positively, wages will still be higher under industry-wide bargaining.

3.4. 1.2 of Proposition 1. may still hold, provided gross wages respond positively to the unemployment benefit; this will occur if and only if the required unit taxes to insure the balanced budget move in the same direction of the unemployment benefit. Equilibrium wages and the union´s objective function respond in opposite directions to changes in the unemployment benefit.

5. Note that we did not assume that the union fully internalises the budget constraint in its decision, which is likely with economy-wide bargaining. If we did - and this point can be found in Layard, Nickell & Jackman (1991), p. 129-131 -, the union would act as if maximizing the wage bill, set wages at unitary labor demand, and would not respond to the unemployment benefit [17]. Then, the

union acts as if W_a in problem (1) is 0. As the wage in the economy increases with W_a – according to (3) -, the unions fully internalising the budget constraint works as a decrease in W_a; hence, wages and unemployment will be lower than implied by (29). Or, due to (9), necessarily positive at $W_a = 0$, a lower wage elasticity is chosen, hence lower wages and unemployment level.

Conclusions

This article inspects the response of labor market equilibrium aggregates to the degree of coordination in union bargaining by analysing the features of the implicit wage curve.

The research assumes (closed-shop) monopoly unions that maximize collective earnings. It distinguishes the cases of unions responding to before and after-tax wages (or that always respond to after-tax wages and when wages are not and are levied on employed workers), and where (unit) taxes finance the unemployment benefit bill under a balanced budget constraint.

It was shown that if unions respond to gross wages, under plausible conditions (and as some empirical evidence seems to support), economy-wide bargaining will exhibit a lower wage than industry-wide bargaining – resulting from the fact that the probability of employment is perceived as higher under industry-wide bargaining.

If unions respond to net wages and earnings taxes finance the unemployment insurance fund, the reverse can occur. Under a balanced budget constraint, industry-wide bargaining will be consistent with a negatively sloped bargained real wage curve for low levels of unemployment, and positively sloped when unemployment is high – as empirical wage curves based on local disaggregation seem to be. Corporate bargaining will lead to a positively sloped wage curve. Under both bargaining systems, wages and unemployment are higher than when taxes were absent or unions respond to gross earnings.

In any event, a positive reaction of wages and unemployment to the unemployment benefit is magnified with centralized bargaining relative to industry bargaining. However, under a balanced budget - worker financed - unemployment insurance scheme, that response may become negative.

Notes

[1] See, for example, Calmfors & Driffill (1988). Also, Tarantelli (1986). A recent survey of international evidence can be found in Flanagan (1999).

[2] Such theoretical refinements in centralization bargaining have been applied and developed in studies such as those of Calmfors & Driffil (1988), Davidson (1988) and Dowrick (1989 and 1993).

[3] As in Oswald (1982). Holmund & Lundborg (1989) analyse the changes in the equilibrium induced by different unemployment insurance funding schemes.

[4] Efficient bargaining would lead to $P\ F_L = W_a$, with employment being determined by the relative strength of the union with respect to the employer one. We shall always assume monopoly unions.

[5] If labor demand is linear or concave, second-order conditions are always satisfied. For a standard constant elasticity demand, $L(W) = A\ W^{-\eta}$ – which is convex in W -, second-order conditions are satisfied around the optimal solution (i.e., provided $\eta > 1$, required by first-order conditions).

[6] See Carlin & Soskice (1990), page 391, for the derivation of this expression - the "bargained real wage curve".

[7] The empirically observed negative relation between the local unemployment rates and wages - well documented in Blanchflower & Oswald (1994) - is, therefore, consistent with monopoly union models. And, because the local framework is considered, would apply to local (the analog to industry-wide) bargaining.

[8] This is the case for $L(W, \eta_{L,W}) = A\ W^{-\eta}$ provided $W \geq 1$.

[9] When we compare economies with different labor demand schedules, (9) does not apply; (10) and (11) do not apply, in general. Nevertheless, (3) must hold.

[10] Tarantelli (1986) explains his findings of a negative relation between the Okun´s misery index (rate of growth of consumer prices plus rate of unemployment) and the degree of neocorporatism as being due to "lower risks of free riding" in a "more centralized system of industrial relations", thus originating "greater price stability". He argues that free riding in less centralized systems leads to a higher real wage level and higher unemployment. Layard, Nickell & Jackman (1991) comparing centralized and decentralized union bargaining also arrive at this relation between the corresponding outcomes - also explaining the fact as being due to the unemployment benefit which is seen as the alternative with coordinated union behavior - and complete coverage - is in place. Their argument differs from ours in that they conclude that for

given coverage there is no reason to believe that "intermediate levels of centralization are bad" - we are comparing scenarios with the same (complete) coverage, therefore, we enlarge their conclusions to intermediate centralization. These authors focus on the corner solution - full employment - in the centralized bargaining case.

[11] We could not find any sensitivity of these conclusions to the imposition of (22).

[12] Note that also $W = \dfrac{s}{u - \dfrac{1}{\eta_{L,W}}}$. We assume that the government sets b, its policy target, and endogenously determines the tax rate under (22) to ensure the balanced budget. Hence, the bargained real wage curve is seen as (24) and not this expression in s.

[13] Once wages decrease with the tax rate - see (21) – this was to be expected.

[14] Hart & Moutos (1995), section 5.4, discuss the sign effects of the unemployment benefit increase on the wage rate under efficient bargaining in a two-sector model. They find that the sign of the multiplier on total employment becomes positive as a balanced budget constraint, recognized by unions and firms, analogous to (22) is introduced.

[15] This comparison is always valid around the same equilibrium of the cases of both sections.

[16] This was implied by (18).

[17] In this context, if unions are utilitarian and union members risk averse, Oswald (1982) proves that the wage will move in the same direction as the unemployment benefit (He considers a proportional income tax).

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4 Human capital earnings functions: The Portuguese case

Introduction

The acquisition of human capital through education and On-The-Job Training has been generally viewed as an investment. After major developments in the theory - in which Schultz, Becker and Mincer have been pioneers -, a substantial number of empirical papers have been produced intending to highlight the features of such an investment.

It is the purpose of this research to use available Portuguese cross-section information on earnings, schooling and an indirect measure of experience of the working population to

1. offer some estimates of the pattern of the (human capital) rates of return to schooling and general On-the-Job-Training.

2. illustrate some possible applications of the methodology used, namely in discrimination issues and evaluation of the results of some education policies [1].

3. infer some aspects of the current equilibrium in the Portuguese human capital market by comparison with international evidence.

We start by introducing the reader to some theoretical background in section II. In section III we present and discuss the log-earnings regression results for the male population. In section IV we analyze the male-female differentials in the implied pattern of compensation. Section V deals with the comparison of the general high-school with the extinct technical school system. Further extensions of the specifications of sections III and IV to include higher degree polynomial terms in experience are presented in section VI. Some final remarks are put forward in section VII. We conclude by summarizing the main results in section VIII.

Theoretical background

1. An individual has a potential working life of T+1 years. Assume that if he chooses 0 years of schooling, he will earn (in real terms) E_t^0 from t=0 to T; if he chooses s years, he will earn E_t^s, from t=s to T, incurring in costs C_t^s, during the schooling period, t = 0 to s-1. Then, the internal rate of return to human capital will be given by the rate r that equates:

$$\Sigma_{t=0}^{T} E_t^0/(1+r)^t = -\Sigma_{t=0}^{s-1} C_t^s/(1+r)^t + \Sigma_{t=s}^{T} E_t^s/(1+r)^t \qquad (1.1)$$

The individual will choose to go to school if the rate of return to the human capital investment is higher than the borrowing rate he faces. Now, assume that money costs are zero - only opportunity costs in the use of time are involved in going to school [2]- and that the earnings streams are constant for both options - E^0 and E^s. Then, for $T \to \infty$ we obtain [3]:

$$E^0 \{1/[1-1/(1+r)]\} = E^s \{1/[1-1/(1+r)]\} / (1+r)^s \qquad (1.2)$$

Thus:

$$E^s = (1+r)^s E^0 \qquad (1.3)$$

Taking logarithms:

$$\ln E^{s} = \ln E^{0} + s \log(1+r) \qquad (1.4)$$

By Taylor's expansion and small values of r, $\log(1+r) \approx r$. The approximation to the rate of return to schooling can therefore be [4]:

$$\ln E^{s} = \ln E^{0} + s\,r \qquad (1.5)$$

If we use Taylor's expansion to a higher order term, we can get:

$$\ln E^{s} = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \ldots \qquad (1.6)$$

A possible interpretation of the derivative of the expression with respect to s is the rate of return to human capital at each level of schooling [5].

$$d(\ln E^{s})/ds = r(s) \qquad (1.7)$$

For example, if we take only to the term education squared,

$$d(\ln E^{s})/ds = a_1 + 2 a_2 s \qquad (1.8)$$

Then, a value of $a_1 > 0$ and of $a_2 < 0$ will imply a diminishing rate of return to education.

Notice that we can therefore estimate the rate of return to human capital in an economy if we have (cross-section) data on earnings of individuals of different schooling years by simply regressing the logarithms of earnings on the schooling period.

2. In terms of interpretation, the rate in (1.1) through (1.5) - and the one we get from the regression (1.5) - is the equilibrium rate of the economy.

Psacharopoulos (1981) reports estimates of the average private rate of return from log-earnings regressions of 14,4% for the LDC's, 9,7% for intermediate countries and 7,7% for advanced countries[6]; the ranking of those rates seems to be maintained, as suggested by later surveys (Psacharopoulos 1985 and 1994). This would imply underinvestment in human capital in less advanced economies, being the basis for public support of the education systems - not only due to external effects associated with it but simply due to the major difficulty in access to credit for the investment. That is, people find it difficult to get credit from the bank to subsidize their schooling years, once there is no guarantee (or knowledge of the future intentions of the individual - usually he has no credit history) to the bank of the future payment of an eventual loan.

Empirical findings also suggest a declining rate as the level of schooling increases. This could be related to decreasing credit constraints. That is, individuals will engage in schooling if the rate of return to the investment is at least equal to the borrowing rate they face. Therefore, people that face lower interest rates will have higher levels of schooling. The observed equilibrium pattern would then be of a declining rate of return to schooling relative to the schooling level. Also, people who acquire more schooling may have relatively higher "taste for studying" - implying they receive utility from studying, compensating, in equilibrium, the smaller money-yielding returns of people that choose smaller schooling levels.

3. Apart from the investment in (general) human capital through schooling, the enhancement of the ability to earn may be acquired through On-the-Job Training (O.J.T.). Let t denote experience in the labor market, k_0 be the proportion of earnings potential (or time-equivalent units) invested at time 0 in the market and T the experience level till which investment is made. Then, if the ratio of investment to earnings potential declines linearly [7], we can write the natural logarithm of the observed earnings for experience level t and schooling s, Y_t^s as:

$$\ln Y_t^s = \ln E^0 + s\,r + r_t\,k_0\,t - (r_t\,k_0/2T)\,t^2 + \ln(1-k_0 + k_0\,t/T) \qquad (1.9)$$

An approximation to the pattern of earnings profiles will be obtained if we regress:

$$\ln Y_t^s = a_0 + a_1\,s + b_1\,t + b_2\,t^2 \qquad (1.10)$$

If the proportion of potential earnings invested declines exponentially and at a rate b, (in which case there is investment through all the lifetime), we arrive to the Gompertz specification [8]:

$$\ln Y_t^s = \ln E^0 + s\, r + r_t\, k_0/b - (r_t k_0/b)\, e^{-bt} + \ln(1-k_0\, e^{-bt}) \qquad (1.11)$$

Then, an approximation of the function can be obtained through the regression

$$\ln Y_t^s = a_0 + a_1\, s + b_1\, e^{-bt} + b_2\, e^{-2bt}\,, \qquad (1.12)$$

where

$$b_1 = -\,(k_0 + r_t\, k_0/b)\,, \quad \text{and} \qquad (1.13)$$

$$b_2 = -\,(k_0^2/2)\,. \qquad (1.14)$$

Assume we have data on earnings, experience and schooling. For specific levels of b, e^{-bt} and e^{-2bt} can be computed and simple linear regression can yield estimates for a_0, a_1, b_1, and b_2.

4. If in expression (1.1) we enter private expenditure and (net or after-tax) returns, then we will obtain the private rate of return to education. If we considered the gross returns and effective implied total (public and private) expenditure with education, we talk about social returns to education. In practice, it has been observed that the actual social rate estimated for other countries is lower than the private one, due to the subsidization of schooling all over the world.

Notice that this seems somehow odd. In theory, social rates should be higher than private rates whenever there are positive externalities coming from a particular investment. In fact, the opposite has been observed; this occurs because in the actual estimates of social rates we do not include - because they are very difficult to measure - external benefits (and indirect costs).

Estimates of the social and private internal rates of return to schooling for different education levels using cost-benefit analysis - that is, for specific (adjacent) schooling categories for which there is data on labor income streams for people in different years in working lifetime - have already been obtained for Portugal [9]. We will use the same data, and combine information on age to derive experience levels and take the log-earnings approach to infer about the structure of the rates of return.

Some estimates of rates of return to education for Portugal using log earnings regressions were derived in Silva (1985), and more recently, in Kiker & Santos (1991 and 1997). They use different data sets and our methodology differs from theirs in several ways - namely in the differentials approach followed, the experience coefficients interpretations, age group decomposition of profiles and the extensions here considered - and also, in some questions we aim to answer.

Rates of return to human capital: Male population

1. In Table II.1 we reproduce some regressions of the log of monthly earnings on schooling and experience for the Portuguese male sample (see Appendix 1 for a description of the sample and data used).

Equation (1) refers to the estimates of equation (1.5) and equation (2) to those of (1.10). Equation (2). implies a rate of 7,3%. This rate is somewhat smaller than the one derived in Silva (1985) - whose estimates ranged between 9,1 and 9,3% -, or Kiker & Santos (1991). For the U.S.[10], Mincer (1974) obtained (for data of 1959) a higher level: 10,3% [11]. However, for the Nordic countries, much smaller rates than ours seem to have been found [12].

The relation between log of earnings and experience in (2) suggests that a peak is reached at experience level of 27 years - 34 years for the U.S.

Equation (3) allows for a second-order term in the coefficient of education and an interdependence between experience and education, that is, r(s,t) is of the form

$$r(s,t) = d(\ln E_t^S)/ ds = a_1 + 2\, a_2\, s + a_3\, t \qquad (2.1)$$

The corresponding estimates yield:

$$r(s,t) = d(\ln E_t^S)/ ds = 0{,}099 + 0{,}002\ s - 0{,}001\ t \quad (2.2)$$

This implies - as we can see in Table II.2 - a convex profile for log of earnings with respect to schooling, contrary to what was found for other countries (diminishing rates of return). However, a_2 is not significant at the 5% significance level (although it is at 10%), thus suggesting that the rate of return is independent of the schooling level.

The interaction between experience and schooling is significantly negative - the same having been reported for the U.S.[13], for which:

$$r(s,t) = d(\ln E_t^S)/ ds = 0{,}255 - 0{,}0058\ s - 0{,}0043\ t \quad (2.3)$$

Table II.1. *Human Capital Earnings Functions: Men*

Regres	Int.	Educ	Educ2	Exp	Exp2	Exp*Edu	$e^{-.05x_t}$	$e^{-2*.05x_t}$
(1)	8.83	0.065						
		(0.003)						
	N = 392		R^2 =	0.52	F =	422.3	SSR =	36.304
(2)	8.02	0.073		0.054	-0.001			
		(0.002)		(0.002)	(3.8E-5)			
	N = 392		R^2 =	0.87	F =	884.5	SSR =	9.646
(3)	7.64	0.099	0.001	0.076	-0.001	-0.001		
		(0.006)	(2.8E-4)	(0.003)	(3.6E-5)	(1.1E-4)		
	N = 392		R^2 =	0.91	F =	793.9	SSR =	6.702
(4)	9.69	0.075		-0.013			-1.5	-0.384
		(0.002)		(0.004)			(0.489)	(0.313)
	N = 392		R^2 =	0.89	F =	759.7	SSR =	8.542
(5)	9.79	0.053	0.001	-0.014			-1.591	-0.327
		(0.005)	(2.9E-4)	(0.004)			(0.477)	(0.305)
	N = 392		R^2 =	0.89	F =	643.5	SSR =	8.1

Taking s=0, the maximum earnings are obtained at 38 years, very close to the implied level for the U.S. - 41 years.

2. Several Gompertz specifications were tried - with b from 0,05 to 0,30. The best results (adjusted R^2) were obtained for the smallest rates - 0,05 and 0,10. For the U.S., the best results

corresponded to values of 0,10 and 0,15 - the rate of decline would seem to be slightly lower in Portugal.

We report the estimates corresponding to 0,05, once they resulted in meaningful signs for the interpretations implied in (1.13) and (1.14). That is, we have implied estimates of general O.J.T. rates of return (r_t) and the initial proportion of earnings capacity devoted to training (k_0). In the regression we also included experience, which coefficient is interpreted as the depreciation rate of human capital [14].

Table II.2. *Implicit Rates of Return to Human Capital Investments: Men. (%)*

	r_s by Years of Schooling						k_0	T	r_x	d
Regression	0	4	6	9	11	16		(years)		
(2)	7.3	7.3	7.3	7.3	7.3	7.3	74.0	27		
(3)										
Exp.										
0	9.9	10.7	11.1	11.7	12.1	13.1		38		
5	9.4	10.2	10.6	11.2	11.6	12.6		38		
10	8.9	9.7	10.1	10.7	11.1	12.1		38		
20	7.9	8.7	9.1	9.7	10.1	11.1		38		
30	6.9	7.7	8.1	8.7	9.1	10.1		38		
40	5.9	5.7	7.1	7.7	8.1	9.1		38		
50	4.9	4.7	6.1	6.7	7.1	8.1		38		
(4)	7.5	7.5	7.5	7.5	7.5	7.5	87.6		13.6	1.3
(5)	5.3	6.1	6.5	7.1	7.5	8.5	80.9		14.8	1.4

The numbers (Tables II.1 and II.2) suggest that the initial investment is very high - 80 to 88%. (Also, for the U.S., for b = 0,10, k_0 = 56%; for b = 0,15, k_0 = 81%). The rate of return to O.J.T. is rather high: 13,6-14,8% (for the U.S., for b = 0,10, r_t = 13,1%; for b = 0,15, r_t = 6,7%). The depreciation rate is 1,3-1,4%. (1,2% for the U.S.)

The Portuguese schooling profile - equation (5) - is again convex. There would seem to be underinvestment at high schooling levels [15], or more important signaling effects for increasing schooling levels in Portugal.

Table II.3. *Human Capital Earnings Functions by Age Group: Men.*

Age Group	Int.	Educ	Educ2	Exp	Exp2	Exp*Educ
14-24	7.32	0.121		0.083	4.60E-4	
		(0.007)		(0.016)	(0.001)	
	N = 66	R^2 = 0.845		F = 112.5	SSR =	1.399
14-24	5.83	0.332	-0.007	0.338	-0.009	-0.018
		(0.051)	(0.002)	(0.053)	(0.002)	(0.004)
	N = 66	R^2 = 0.892		F = 99.34	SSR =	0.971
25-34	8.04	0.083		0.046	-3.90E-4	
		(0.004)		(0.009)	(2.70E-4)	
	N = 80	R^2 = 0.893		F = 212.4	SSR =	0.657
25-34	7.24	0.137	-0.001	0.123	-0.002	-0.003
		(0.071)	(0.002)	(0.067)	(0.001)	(0.003)
	N = 80	R^2 = 0.899		F = 131.5	SSR =	0.623
35-44	8.9	0.068		-0.003	1.82E-4	
		(0.004)		(0.012)	(2.28E-4)	
	N = 80	R^2 = 0.94		F = 394.5	SSR =	0.469
35-44	9.39	0.011	0.002	-0.024	3.86E-4	0.001
		(0.084)	(0.001)	(0.08)	(0.001)	(0.002)
	N = 80	R^2 = 0.943		F = 243.7	SSR =	0.445
45-54	9.51	0.069		-0.031	4.69E-4	
		(0.005)		(0.021)	(2.98E-4)	
	N = 80	R^2 = 0.916		F = 277.5	SSR =	0.798
45-54	10.15	-0.002	0.002	-0.051	0.001	0.001
		(0.143)	(0.002)	(0.136)	(0.002)	(0.003)
	N = 80	R^2 = 0.92		F = 169.6	SSR =	0.766
55-65	10.72	0.051		-0.058	4.78E-4	
		(0.007)		(0.039)	(4.22E-4)	
	N = 86	R^2 = 0.807		F = 114.1	SSR =	2.129
55-65	9.69	0.089	-3.33E-4	-0.02	1.29E-4	-0.001
		(0.231)	(0.002)	(0.217)	(0.002)	(0.004)
	N = 86	R^2 = 0.807		F = 66.81	SSR =	2.128

Also, experience profiles would seem to have peaks at the same level of experience - being, of course, lower but also flatter in Portugal [16].

3. Table II.3 presents the regression results of the log-earnings equations of linearly decreasing investment in h.c. through O.J.T. - that is, of form (2) and (3) of tables II.1 and II.2. We were trying to see whether different cohorts might result in different regression patterns.

One of the results implied - comparing results of form (2) - is a decreasing rate of return to schooling by age group, from 12,1% for people aged 14-24 to 5,1% for people aged 55-65. This reminds us of the negative coefficient, found for the total population in the U.S., of (2.3). It may be related to nonlinear depreciation of human capital. Usually, the experience profile shows switched signs relative to the US results- even if one of the coefficients is, in general, not statistically significant for Portugal.

Simultaneously, the intercept, associated with earnings at level 0 of schooling, consistently increases. This would suggest that somehow for older cohorts the returns to education are lower but individuals start(ed) at higher levels of earnings. Whether compensation patterns are more related to age than experience (or tenure - or compensation "rules" in implicit contracting terms), it is difficult to measure here, once our proxy uses age and education (we cannot distinguish the effects of age and experience). Possibly, compensation patterns are also more diffuse (less disperse and, thus, less dependent on schooling levels) at higher age levels.

The performance of form (3), allowed us to get "correct" signs - as compared to the U.S. - of the parameters for ages 14-34. The "wrong" signs of some coefficients for the group aged 35-44 years are not significant. And for the last two groups the results for this regression were usually poor, suggesting, rather a form of type (2).

The results relative to the experience terms led us to believe about the convenience of experimenting with higher order polynomials in t - performed in section V. There is, nevertheless, some reason to believe that different cohorts, which made their decision in the past, may have suffered different economic and schooling conditions, which somehow may have influenced the set of results thus obtained. (This consideration is not the same as those made in the cohort differentials literature, usually associated with differences in the returns to schooling in different points in time[17].)

Male-female earnings differentials

1. In Tables III.1 to III.2 we present the results for women equivalent to those of Table II.1 to II.2. The quality of the estimation is, in general, poorer than for men, as expected: the

female labor force is usually characterized by a break in participation in the fertility period, which, with the corresponding depreciation in human capital, makes the use of the proxy for experience as calculated a little inapplicable. Therefore, more interesting - correct - conclusions might be drawn from the cohort disaggregation (here much more than in the male case).

Table III.1. Human Capital Earnings Functions: Women.

Regres	Int.	Educ	Educ2	Exp	Exp2	Ex*Edu	$e^{-.05x_t}$	$e^{-2*.05x_t}$
(1)	8.61	0.07						
		(0.003)						
	N = 308		R^2 =	0.64	F =	533.8	SSR =	17.819
(2)	8.05	0.08		0.033	-4.13E-4			
		(0.002)		(0.003)	(4.7E-5)			
	N = 308		R^2 =	0.81	F =	425.5	SSR =	9.407
(3)	7.78	0.11	-0.001	0.047	-0.001	-0.001		
		(0.01)	(4.5E-4)	(0.004)	(5.6E-5)	(2.0E-4)		
	N = 308		R^2 =	0.82	F =	275.1	SSR =	8.805
(4)	8.86	0.081		-0.003			-0.476	-0.52
		(0.002)		(0.005)			(0.663)	(0.441)
	N = 308		R^2 =	0.82	F =	335	SSR =	9.012
(5)	8.88	0.078	2.21E-4	-0.003			-0.484	-0.517
		(0.007)	(4.2E-4)	(0.005)		(0.664)	(0.442)	
	N = 308		R^2 =	0.82	F =	267.7	SSR =	9.004

The implied estimates of rates of return to schooling are usually higher for women than men. This would correspond - the estimated rate is the prevailing equilibrium rate - to less women with advanced schooling than men. Women would deter from going to school, which would drive their rate of return up. Or, signaling effects of schooling are stronger for women than for men. The fact is that lately, women seem to have entered the education systems in at least equal number as men, suggesting a response to that high rate of return.

Other studies of human capital earnings functions for Portugal did not find such a relation between rates of return to human capital for men and women, but rather the inverse - for example, Silva (1985) found rates between 8,4 and 9,0% for women while (recall from section II) rates of 9,1 to 9,3% for men in similar regressions. That is also an unusual finding in contrast with some international evidence, where female rates of return are usually lower than men's [18] - being such evidence in favor of the discrimination hypothesis. Nevertheless other international surveys report – see Psacharopoulos (1994) - report the pattern here presented; Kiker & Santos (1991), using 1985 data, also find such result for Portugal.

Table III.2. *Implicit Rates of Return to Human Capital Investments: Women (%)*

	r_S by Years of Schooling						k_0	T	r_X	d
Regression	0	4	6	9	11	16		(years)		
(2)	8.0	8.0	8.0	8.0	8.0	8.0	41.3	40		
(3) Years of Experience:										
0	11.0	10.2	9.8	9.2	8.8	7.8		24		
5	10.5	9.7	9.3	8.7	8.3	7.3		24		
10	10.0	9.2	8.8	8.2	7.8	6.8		24		
20	9.0	8.2	7.8	7.2	6.8	5.8		24		
30	8.0	7.2	6.8	6.2	5.8	4.8		24		
40	7.0	6.2	5.8	5.2	4.8	3.8		24		
50	6.0	5.2	4.8	4.2	3.8	2.8		24		
(4)	8.1	8.1	8.1	8.1	8.1	8.1	102.0		7.33	0.3
(5)	7.8	8.0	8.1	8.2	8.3	8.5	101.7		7.38	0.3

The other distinguishing feature relative to males is the "correct signs" relative to the U.S. (male population).

Table III.3. *Human Capital Earnings Functions by Age Group: Women*

Age Group	Int.	Educ	Educ2	Exp	Exp2	Exp*Educ
14-24	7.28	0.138		0.063	0.001	
		(0.008)		(0.017)	(0.001)	
	N = 54	R^2 = 0.873		F = 114.3	SSR =	0.94
14-24	6.44	0.213	-0.001	0.236	-0.006	-0.01
		(0.061)	(0.002)	(0.057)	(0.002)	(0.004)
	N = 54	R^2 = 0.919		F = 108.4	SSR =	0.601
25-34	8.15	0.083		0.024	-1.98E-4	
		(0.005)		(0.011)	(3.32E-4)	
	N = 72	R^2 = 0.886		F = 175.7	SSR =	0.822
25-34	9.57	-0.049	0.003	-0.093	0.002	0.005
		(0.088)	(0.002)	(0.083)	(0.002)	(0.004)
	N = 72	R^2 = 0.889		F = 106.2	SSR =	0.795

35-44	7.67	0.084		0.059	-0.001	
		(0.007)		(0.03)	(0.001)	
	N = 68	R^2 = 0.834		F = 107.3	SSR =	1.391
35-44	11.22	-0.097	0.002	-0.162	0.003	0.006
		(0.17)	(0.003)	(0.167)	(0.003)	(0.005)
	N = 68	R^2 = 0.842		F = 65.86	SSR =	1.329
45-54	8.62	0.071		0.005	-5.77E-5	
		(0.01)		(0.044)	(0.001)	
	N = 66	R^2 = 0.766		F = 67.53	SSR =	2.214
45-54	12.25	-0.001	-0.001	-0.181	0.002	0.002
		(0.288)	(0.004)	(0.272)	(0.003)	(0.007)
	N = 66	R^2 = 0.781		F = 42.72	SSR =	2.073
55-65	9.73	0.078		-0.049	0.001	
		(0.012)		(0.07)	(0.001)	
	N = 48	R^2 = 0.742		F = 42.23	SSR =	1.951
55-65	12.12	0.096	-0.002	-0.151	0.002	2.42E-4
		(0.382)	(0.004)	(0.378)	(0.004)	(0.007)
	N = 48	R^2 = 0.754		F = 25.72	SSR =	1.864

The experience profiles are, as expected, flatter for women than for men; our proxy for experience, however, extremely overestimates the true experience of women - once they may have non-participating periods much larger than men - and, possibly, increasingly with the age group considered, once female participation has increased over the years. Even if this was not the case, implicit contracting, due to the smaller attachment of women to the labor market, may cause "true" experience profiles to be flatter for women.

Table III.4. *Human Capital Earnings Functions: Sex Differentials*

Age Group/ Regression	Sex	Sex*Educ	Sex*Exp	FTest1*	FTest2**	FTest3***
All (1)	-0.227	0.005		39.50	-	39.50
	(0.041)	(0.005)		(2, 696)	-	(2, 696)
	N = 700	R^2 = 0.603		F = 352.3	SSR =	54.12
All (2)	-0.12	0.006	-0.004	71.58	27.93	62.75
	(0.04)	(0.003)	(0.001)	(3, 693)	(1, 692)	(4, 692)
	N = 700	R^2 = 0.855		F = 678.8	SSR =	19.82

All (3)	-0.086	0.005	-0.005	97.11	14.83	58.89
	(0.037)	(0.003)	(0.001)	(3, 691)	(3, 688)	(6, 688)
	N = 700	$R^2 = 0.879$		F = 626.8	SSR =	16.51
All (4)	-0.125	0.006	-0.004	78.86	13.31	54.32
	(0.038)	(0.003)	(0.001)	((3, 692)	(2, 690)	(5, 690)
	N = 700	$R^2 = 0.866$		F = 640.3	SSR =	18.23
All (5)	-0.13	0.007	-0.004	79.39	10.54	46.61
	(0.038)	(0.003)	(0.001)	(3, 691)	(3, 688)	(6, 688)
	N = 700	$R^2 = 0.869$		F = 571.8	SSR =	17.89

Notes: * Testing the joint hypothesis of null sex dummies. ** Comparison of the regression with freeing all parameters (that is, against the hypothesis of different regressions for men and women). *** Comparison of Regression without sex dummies against the hypothesis of different regressions for men and women (Chow test).

Therefore, we see that the rate of return to experience in the Gompertz specification is much lower than for men.

Table III.5.1. *Human Capital Earnings Functions: Sex Differentials.*

Age Group/ Regression	Sex	Sex*Educ	Sex*Exp	FTest1*	FTest2**	FTest3***
14-24 (1)	-0.087	0.018	-0.007	4.90	0.34	3.73
	(0.136)	(0.01)	(0.01)	(3, 113)	(1, 112)	(4, 112)
	N = 120	$R^2 = 0.857$		F = 112.8	SSR =	2.346
14-24 (2)	-0.169	0.024	-0.002	7.95	1.99	4.98
	(0.116)	(0.009)	(0.008)	(3, 111)	(3, 108)	(6, 108)
	N = 120	$R^2 = 0.899$		F = 123.8	SSR =	1.652
25-34 (1)	0.074	-0.001	-0.016	44.94	0.19	33.57
	(0.138)	(0.007)	(0.006)	(3, 145)	(1, 144)	(4, 144)
	N = 152	$R^2 = 0.902$		F = 222.6	SSR =	1.481
25-34 (2)	0.062	-8.59E-5	-0.015	44.53	1.38	23.14
	(0.138)	(0.007)	(0.006)	(3, 143)	(3, 140)	(6, 140)
	N = 152	$R^2 = 0.903$		F = 167.3	SSR =	1.46
35-44 (1)	-0.568	0.017	0.008	51.36	3.69	40.17
	(0.229)	(0.008)	(0.007)	(3, 141)	(1, 140)	(6, 140)
	N = 148	$R^2 = 0.903$		F = 218.0	SSR =	1.909
35-44 (2)	-0.567	0.017	0.008	50.29	2.64	27.74
	(0.231)	(0.008)	(0.007)	(3, 139)	(3, 136)	(6, 136)
	N = 148	$R^2 = 0.904$		F = 163.1	SSR =	1.888

Notes: * Testing the joint hypothesis of null sex dummies. ** Comparison of the regression with freeing all parameters (that is, against the hypothesis of different regressions for men and women). *** Comparison of regression without sex dummies against the hypothesis of different regressions for men and women (Chow test).

By age group, we see the same pattern for women and men in what refers to the returns to schooling - specification (2) also shows a decreasing rate by age group, starting at 14% for the 14-24 years group and being 7-8% for the 45-65 years group. The estimates of the experience coefficients have very large standard errors.

Table III.5.2. *Human Capital Earnings Functions: Sex Differentials*

Age Group/ Regression	Sex	Sex*Educ	Sex*Exp	FTest1*	FTest2**	FTest3***
45-54 (1)	-0.246	0.001	-0.001	41.96	0.73	31.59
	(0.384)	(0.01)	(0.009)	(3, 139)	(1, 138)	(4, 138)
	N = 146	R^2 = 0.87		F = 154.4	SSR =	3.028
45-54 (2)	-0.23	0.001	-0.001	41.46	2.67	22.83
	(0.388)	(0.01)	(0.009)	(3, 155)	(3, 134)	(6, 134)
	N = 146	R^2 = 0.87		F = 114.9	SSR =	3.009
55-65 (1)	-1.142	0.028	0.016	15.37	0.03	11.44
	(0.567)	(0.013)	(0.01)	(3, 127)	(1, 126)	(4, 126)
	N = 134	R^2 = 0.824		F = 98.81	SSR =	4.081
55-65 (2)	-1.067	0.025	0.014	15.05	0.63	7.77
	(0.575)	(0.013)	(0.011)	(3, 125)	(3, 122)	(6, 122)
	N = 134	R^2 = 0.825		F = 73.52	SSR =	4.054

Notes: * Testing the joint hypothesis of null sex dummies. ** Comparison of the regression with freeing all parameters (that is, against the hypothesis of different regressions for men and women). *** Comparison of regression without sex dummies against the hypothesis of different regressions for men and women (Chow test).

2. A set of tests were performed trying to evaluate the significance of the difference between female and male earnings. The method involves the use of sex-dummies (Sex_i=1 if i refers to woman, 0 if man). The results are presented in Tables III.4 and III.5. In general, and as expected, different regressions are advisable for the two sexes.

A negative coefficient is found for Sex. In logarithms, the difference found corresponds - approximately - to the percentage difference of women's earnings relative to the men's level. Thus, the sex-dummy (Table III.4) reads a 22,7% wage differential between women and men - the initial earnings start at a level 22,7% lower than that of men. As we include more explanatory variables, this differential decreases to 12% (formulation (2)) and

8,6% with formulation (3). The Gompertz specification indicates a value of 13%.

In the age-group decomposition, the sex-dummy yields maximum differentials for women 55-65 and 35-44, being lower at young ages, and higher for older women. As expected, results worsen with the cohorts´age; it is possible that the situation has improved in more recent periods: some of the differentials of earlier years may have persisted even if not affecting posterior earnings growth.

The sex dummy interacted with education yields a positive bias in favor of women, which was already discussed. The differential is negative for ages 25-34, but not significant. Simultaneously, the bias is negative with respect to experience. Again, differentials may be overstated due to the experience proxy used, specially if education and ("true") experience are positively correlated - which we cannot measure with the data we have available: then, part of the (positive) influence would be captured in the education coefficient for women.

Finally, different specifications are advisable for men and women - not only for both separately (from FTest2 in table III.4, we conclude that just including the dummies is not enough to account for the differences), but also, as we will see below, higher order polynomial forms may fit the data more accurately.

3. Discrimination studies usually decompose the wage gap between men and women in the following way[19]:

$$W_F - W_M = (Z_F - Z_M) B_F + (B_F - B_M) Z_F \quad (3.1)$$

where Z_j refers a vector of (mean) levels of productivity characteristics (say, schooling and experience) of an individual of group j and B_j the coefficients representing the contribution of those characteristics to the determination of the productivity of group j. Alternatively, we can write:

$$W_F - W_M = (Z_F - Z_M) B_M + (B_F - B_M) Z_M \quad (3.2)$$

The first term evaluates the differences of the values of the Z's at the price of women in (3.1) and of men in (3.2). The second term, evaluates the sex price differential at the value of the women's set of characteristics in (3.1), and of men's in (3.2). This second term is, thus a measure of market discrimination.

We have no information on the mean levels of the characteristics for the male and female samples. We can, however, construct the series of differentials in earnings for given characteristics - schooling and experience. We can infer the dimension of those second coefficients, i.e., the price differentials, through the regression:

$$(W_{Fi} - W_{Mi}) = (B_F - B_M)\, Z_{Mi} = B_{F-M}\, Z_{Mi} \tag{3.3}$$

where i refers to a specific level of schooling and experience. We can therefore see how the prices differ for each category, that is, estimate the vector B_{F-M}. We performed several regressions of the earnings differentials on the characteristics available. We used W_{ji}, j=F,M, in logarithms, to conform with the previous results - also, the estimated price differences, captured in the intercept and in the coefficients, have the advantage of allowing an interpretation in percentage difference terms.

Table III.6.1. *Female-Male Earnings Differentials. (%)*

	Years of Schooling						
Experience:	0	4	6	9	11	14	16
0	10,2	7,8	6,6	4,8	3,6	1,8	0,6
5	12,7	10,3	9,1	7,3	6,1	4,3	3,1
10	15,2	12,7	11,6	9,8	8,6	6,8	5,6
20	20,2	17,8	16,6	14,8	13,6	11,8	10,6
30	25,2	22,8	21,6	19,8	18,6	16,8	15,6
40	30,2	27,8	26,6	24,8	23,6	21,8	20,6
50	35,2	32,8	31,6	29,8	28,6	26,8	25,6

Firstly, the average differential in the sample is of about 18,9% (women's earnings being about 18,9% lower than those of men)[20].

The regression of the log-earnings differential on education and experience yielded:

$$\text{DifLogWage} = -\,0{,}102 + \underset{(0{,}002)}{0{,}006\ s} - \underset{(0{,}001)}{0{,}005\ t} \qquad (3.4)$$

$$R^2 = 0{,}2 \quad F_{(2,305)} = 38{,}213$$

This would suggest that the gap would start (no schooling, no experience) at a female 10,2% disadvantage relative to the male earnings level; a one year increase in education would diminish the earnings gap by 0,6%, but an extra year of experience would increase it by 0,5%. Both coefficients are highly significant.

In table III.6.1 we can analyze the evolution implied by these numbers: the differential for high levels of education is very small (0,6% for the highest degree of education) at the starting levels, but increases with the schooling levels and range between 20% to 35% at high levels of experience.

Table III.6.2. *Female-Male Earnings Differentials. (%)*

	Years of Schooling						
Experience:	0	4	6	9	11	14	16
0	-13,0	-22,2	-25,6	-29,2	-30,6	-31,2	-30,6
5	8,0	-1,2	-4,6	-8,2	-9,6	-10,2	-9,6
10	21,9	12,7	9,3	5,7	4,3	3,7	4,3
20	37,3	28,1	24,7	21,1	19,7	19,1	19,7
30	46,9	37,7	34,3	30,7	29,3	28,7	29,3
40	56,3	47,1	43,7	40,1	38,7	38,1	38,7
50	63,4	54,2	50,8	47,2	45,8	45,2	45,8

Other specifications were, thus, tried:

$$\text{DifLogWage} = 0{,}13 + \underset{(0{,}002)}{0{,}027\ s} \underset{(0{,}001)}{-0{,}001\ s^2} - \qquad (3.5)$$

$$\underset{(0{,}013)}{-\,0{,}051\ t} + \underset{(0{,}001)}{0{,}002\ t^2} - \underset{(2{,}315E\text{-}5)}{4{,}191E\text{-}5\ t^3} + \underset{(1{,}992E\text{-}7)}{3{,}239E\text{-}7\ t^4}$$

$$R^2 = 0{,}344 \qquad F_{(6,301)} = 26{,}313$$

The implied pattern of wage differentials can be examined in table III.6.2. At the initial experience levels (till 5 years) the estimated differential does not favor men. As we increase experience, the gap increases - decreasing with the schooling level - ranging between 38 and 63% in the highest experience levels.

Table III.6.3. *Female-Male Earnings Differentials. (%)*

	Years of Schooling							
	0	4	6	9	9T	11	14	16
	29,1	29,8	14,5	12,3	12,8	14,3	11,1	20,0
Experience:								
0	-16,1	-13,6	-28,1	-29,5	-28,5	-24,4	-27,2	-18,2
5	4,4	6,9	-7,6	-9,0	-8,0	-3,9	-6,7	2,3
10	17,6	20,1	5,6	4,2	5,2	9,3	6,5	15,5
20	30,7	33,2	18,7	17,3	18,3	22,4	19,6	28,6
30	36,3	38,8	24,3	22,9	23,9	28,0	25,2	34,2
40	40,0	42,5	28,0	26,6	27,6	31,7	28,9	37,9
50	40,3	42,8	28,3	26,9	27,9	32,0	29,2	38,2

Other specifications included the use of dummy variables for education. The simple use of education dummies (no experience variables included) originated the pattern of the first line of Table III.6.3. This measures the simple mean differential between schooling categories. In the following lines we report the results when we include as well the quartic representation of experience (the 4-th term is almost significant at the 10% level) - of the several regressions performed in the log-earnings differentials, this showed the highest adjusted R^2.

In terms of schooling differentials only (first line) the highest gap is found at no schooling or primary school (29-30%). The gap decreases till 9 years, jumps to 14% at 11 years, decreases at 11 years and jumps again to 20% at "licenciatura".

Controlling also for experience (the interaction between schooling and experience was not significant), using a quartic approximation for the experience profile, we see that schooling is not the cause of the differential in earnings (we have negative values at the initial experience). Rather, experience seems to be the cause of the negative differential Interestingly, the differential seems to decrease with schooling (by experience level), but we see some increases at the highest levels as suggested by the results of the first line. Differences between high-school degree holders and technical school are not significant.

Whether a segmented labor market interpretation for these findings may be appropriate is not explored here. Rather, given the limited information we have, only a quantitative statement of the observed differentials is presented [21].

Technical versus high-school systems

1. In Table IV.1. and IV.2 we can see some evidence concerning differentials between high-school and technical systems. The unified system abolished the technical schools. We use a dummy variable (D91=1 if the individual completed the technical-school degree, 0 otherwise)

Table IV.1. *Male Earnings: Technical versus High-School Systems.*

	Independent Variables										
Group	Int.	Educ	Educ2	Exp	Exp2	Exp3	Edu*Ex	D91	D91Exp	$D91Ex^2$
1 Male	8.02	0.073		0.054	-0.001			-0.021		
		(0.002)		(0.002)	(4E-5)			(0.024)		
	N= 392		R^2 =	0.873		F =	663.3		SSR =	9.626
2 Male	7.64	0.098	0.001	0.076	-0.001		-0.001	0.019	0.001	-2.5E-5
		(0.006)	(3E-4)	(0.003)	(4E-5)		(1E-4)	(0.063)	(0.006)	(1E-4)
	N= 392		R^2 =	0.912		F =	493.6		SSR =	6.686
3 Male	8.42			0.099	-0.003	2.28E-5				
S=9 years				(0.007)	(3E-4)	(4E-6)				
	N= 100		R^2 =	0.913		F =	334.5		SSR =	0.798
4 Male	8.42			0.097	-0.003	2.28E-5		0.007	0.003	-9.0E-5
S=9 years				(0.007)	(3E-4)	(4E-6)		(0.057)	(0.005)	(1E-4)
	N= 100		R^2 =	0.915		F =	167.1		SSR =	0.776
5 Male	8.45			0.091	-0.002	1.90E-5				
S=90 years				(0.009)	(4E-4)	(5E-6)				
(H.Sc.)	N= 50		R^2 =	0.927		F =	194.6		SSR =	0.329
6 Male	8.40			0.106	-0.003	2.66E-5				
S=91 years				(0.01)	(5E-4)	(6E-6)				
(T.Sc.)	N= 50		R^2 =	0.905		F =	146.2		SSR =	0.439

The dummy coefficient seems to indicate that individuals with a technical school degree earn slightly more than those with the equivalent high-school years, even if not significantly. (Notice that the dummy D91 aggregates differences in the rate of return and in the intercept - initial earnings.) The technical school experience profiles, however, seem to be flatter, but not significantly, for men; the contrary seems to occur for women.

Table IV.2. *Female Earnings: Technical versus High-School Systems*

					Independent Variables					
Group	Int.	Educ	Educ2	Exp	Exp2	Exp3	Edu*Ex	D91	D91Exp	D91Ex2
1	8.05	0.079		0.033	-4.1E-4			0.044		
Female		(0.02)		(0.003)	(5E-5)			(0.029)		
	N= 308		R^2 =	0.809		F =	321.0		SSR =	9.337
2	7.78	0.107	-3.0E-4	0.047	-0.001		-0.001	0.071	-0.003	6.98E-5
Female		(0.011)	(5E-4)	(0.004)	(6E-5)		(2E-4)	(0.094)	(0.009)	(2E-4)
	N= 308		R^2 =	0.822		F =	172.4		SSR =	8.711
3	8.66			0.061	-0.002	1.35E-5				
Female				(0.014)	(0.001)	(8E-6)				
S=9 years	N= 85		R^2 =	0.571		F =	35.92		SSR =	1.886
4	8.60			0.069	-0.002	1.60E-5		0.081	-0.01	-2.2E-4
Female				(0.015)	(0.001)	(8E-6)		(0.114)	(0.01)	(2E-4)
S=9 years	N= 85		R^2 =	0.581		F =	18.05		SSR =	1.84
5	8.54			0.082	-0.002	2.36E-5				
Female				(0.019)	(0.001)	(1E-5)				
S=90 years(H.Sc.)	N= 42		R^2 =	0.598		F =	18.88		SSR =	0.844
6	8.75			0.046	-0.001	6.47E-6				
Female				(0.02)	(0.001)	(1E-5)				
S=91 years (T.Sc.)	N= 42		R^2 =	0.576		F =	17.70		SSR =	0.971

International evidence – collected in Psacharopoulos (1985) – suggests a smaller rate of return on technical/vocational curricula relative to the general, academic type. We found no such difference – if some is present, it seems to go in the opposite direction. One cannot infer from the findings here presented for Portugal whether the extinct technical school system was or not irrelevant - the results may simply indicate an equilibrium situation achieved with the simultaneous systems, each performing each function.

2. We also used the log-earnings differentials approach used in section III. The earnings differentials are not very high, and neither the significance of the regressions performed for each sex:

For men, workers with a technical school degree earned about 1,8% more than people with the equivalent high-school years. The earnings differential was negatively correlated with experience - even if not significantly. The regression of the difference of log-earnings gave the following results:

$$DLEarn(Tech\text{-}High) = 0{,}047 - \underset{(0{,}001)}{0{,}001}\ Exp$$

$$R^2 = 0{,}024 \qquad F_{(1,48)} = 1{,}185$$

Using the Gompertz specification, the coefficients showed higher significance but the regression was not significant at the 10% significance level:

$$DLEarn(Tec\text{-}Hig) = -0{,}068 + \underset{(0{,}245)}{0{,}511}\, e^{-0{,}05Ex} - \underset{(0{,}258)}{0{,}513}\, e^{-0{,}05Ex^2}$$

$R^2 = 0{,}085$ $\quad F_{(2,47)} = 2{,}188$

For women, the differential was 1,5% and positively (not significantly at the 10% level) correlated with experience. A quadratic term in experience proved significant:

$$DLEarn(Tech\text{-}High) = 0{,}199 - \underset{(0{,}013)}{0{,}026}\, Exp + \underset{(2{,}734E\text{-}4)}{0{,}001}\, Exp^2$$

$R^2 = 0{,}15$ $\quad F_{(2,34)} = 2{,}999$ (significant at 10% level)

The best regression for women with the Gompertz terms was significant at 10% but not at the 5% significance level:

$$DLEarn(Tech\text{-}High) = -1{,}114 + \underset{(0{,}641)}{1{,}365}\, e^{-0{,}05Exp} + \underset{(0{,}012)}{0{,}027}\, Exp$$

$R^2 = 0{,}132$ $\quad F_{(2,34)} = 2{,}576$

Empirical specifications of the experience-earnings profiles

Another application of log-earnings functions included the study of more complex experience profiles. If we may to some extent lose the interpretation in the above formulations, we may get more accurate descriptions of the patterns implied.

A recent study [22] concluded that in the U.S. a 4-th degree polynomial in t (experience) - a quartic rather than a quadratic specification - would give a more adequate representation of the male earnings profiles.

Table V.1.1. *Experience Profiles: Men*

	Independent Variables								
Regr./ Educ.	Int.	Exp	Exp2	Exp3	Exp4	Exp5	Exp6	Ed	Ed*Exp
(1)	7.425	0.116	-0.003	4.60E-5	-2.86E-7			0.109	-0.001
All		(0.009)	(0.001)	(2E-5)	(1E-7)			(0.003)	(1E-4)
	N =	392	R^2 =	0.921	F =	748.27		SSR =	5.972
(2)	6.367	0.296	-0.012	2.3E-4	-1.55E-6				
0		(0.031)	(0.002)	(4E-5)	(2.6E-7)				
	N =	52	R^2 =	0.932	F =	160.013		SSR =	0.203
(3)	2.682	1.196	-0.094	0.004	-8.10E-5	8.9E-7	-3.9E-9		
0		(0.128)	(0.012)	(0.001)	(1.5E-5)	(2E-7)	(9.E-10)		
	N =	52	R^2 =	0.978	F =	326.48		SSR =	0.067
(4)	7.558	0.154	-0.005	7.05E-5	-3.9E-7				
4		(0.02)	(0.001)	(3E-5)	(2.6E-7)				
	N =	52	R^2 =	0.96	F =	280.023		SSR =	0.201
(5)	7.821	0.073	0.003	-2.6E-4	5.77E-6	-4.2E-8			
4		(0.042)	(0.004)	(2E-4)	(2.9E-6)	(2E-8)			
	N =	52	R^2 =	0.963	F =	242.25		SSR =	0.183
(6)	7.54	0.215	-0.009	1.88E-4	-1.4E-6				
6		(0.015)	(0.001)	(3E-5)	(2.5E-7)				
	N =	52	R^2 =	0.974	F =	438.95		SSR =	0.187

Table V.1.2. Experience Profiles: Men

	Independent Variables								
Regr./ Educ.	Int.	Exp	Exp2	Exp3	Exp4	Exp5	Exp6	Ed	Ed*Exp
(7)	8.344	0.128	-0.005	9.8E-5	-7.4E-7				
9		(0.013)	(0.001)	(3E-5)	(3E-7)				
	N = 100		R^2 =	0.918	F =	265.588		SSR =	0.75
(8)	8.22	0.191	-0.014	0.001	-1.0E-5	7.47E-8			
9		(0.023)	(0.003)	(1E-4)	(2.9E-6)	(2E-8)			
	N = 100		R^2 =	0.926	F =	236.197		SSR =	0.674
(9)	8.6	0.121	-0.006	1.4E-4	-1.4E-6				
11		(0.032)	(0.003)	(8E-5)	(8.2E-7)				
	N = 48		R^2 =	0.808	F =	45.275		SSR =	0.867
(10)	8.807	0.01	0.009	-0.001	1.72E-5	-1.5E-7			
11		(0.056)	(0.007)	(4E-4)	(8.0E-6)	(7E-8)			
	N = 48		R^2 =	0.83	F =	41.091		SSR =	0.767
(11)	9.251	0.023	0.001	-2.9E-5	1.45E-7				
14		(0.033)	(0.003)	(1E-4)	(1.2E-6)				
	N = 44		R2 =	0.662	F =	19.055		SSR =	0.926
(12)	9.182	0.044	-0.001						
14		(0.007)	(2E-4)						
	N = 44		R2 =	0.649	F =	37.981		SSR =	0.959

(13)	9.227	0.101	-0.004	7.08E-5	-6.4E-7				
16		(0.035)	(0.003)	(1E-4)	(1E-6)				
	N = 44		R^2 =	0.732	F =	26.568		SSR =	1.021
(14)	9.313	0.07	-0.001						
16		(0.007)	(2E-4)						
	N = 44		R^2 =	0.722	F =	53.361		SSR =	1.056

In fact, such finding in no way diminishes the explanatory power of human capital theory, only suggests that other patterns of investment rather than the linear or exponential patterns are observed. Moreover, Taylor expansion to higher degree for both forms would yield higher degree polynomials. The advantage of the second-degree polynomial forms is that they provide readily interpretations for the coefficients and implied estimates of initial investment, or rates of return to O.J.T., etc. In order to describe the pattern of compensation, other nonlinear forms can be applied[23].

Table V.2.1. *Experience Profiles: Women*

				Independent Variables				
Regr./ Educ.	Int.	Exp	Exp2	Exp3	Exp4	Exp5	Educ	Ed*Exp
(1)	7.714	0.067	-0.001	1.115E-5			0.1	-0.001
All		(0.007)	(2.7E-4)	(3.1E-6)			(0.005)	(1.8E-4)
	N = 308		R^2 =	0.827	F =	287.872	SSR =	8.481
(2)	8.034	0.052	-0.001	1.102E-5				
0		(0.017)	(0.001)	(4.9E-6)				
	N = 49		R^2 =	0.371	F =	8.861	SSR =	0.259
(3)	4.63	0.686	-0.044	0.001	-2.00E-5	1.12E-7		
0		(0.152)	(0.01)	(3.3E-4)	(4.9E-6)	(2.8E-8)		
	N = 49		R^2 =	0.554	F =	10.683	SSR =	0.184
(4)	7.716	0.104	-0.003	2.97E-5				
4		(0.011)	(4.3E-4)	(4.8E-6)				
	N = 52		R^2 =	0.867	F =	104.442	SSR =	0.407
(5)	7.851	0.126	-0.004	3.58E-5				
6		(0.014)	(0.001)	(7.4E-6)				
	N = 47		R^2 =	0.873	F =	98.411	SSR =	0.561
(6)	7.572	0.203	-0.01	1.99E-4	-1.52E-6			
6		(0.032)	(0.002)	(6.3E-5)	(5.8E-7)			
	N = 47		R^2 =	0.891	F =	85.458	SSR =	0.483
(7)	8.657	0.061	-0.002	1.35E-5				
9		(0.014)	(0.001)	(7.9E-6)				
	N = 85		R^2 =	0.571	F =	35.919	SSR =	1.886

Therefore, some estimates were performed using higher order polynomials, for men and women for all the individuals and for each education group. The regression results are presented in

Tables V.1 (men) and V.2 (women). The Gompertz specifications were also enlarged - Tables V.3 and V.4.

Table V.2.2. *Experience Profiles: Women*

	Independent Variables							
Regr./ Educ.	Int.	Exp	Exp2	Exp3	Exp4	Exp5	Educ	Ed*Exp
(8)	8.792	0.055	-0.001	-2.05E-6				
11		(0.021)	(0.001)	(1.5E-5)				
	N = 36		R^2 =	0.596	F =	15.727	SSR =	1.049
(9)	8.52	0.218	-0.026	0.001	-3.64E-5	3.17E-7		
11		(0.077)	(0.01)	(0.001)	(1.3E-5)	(1.1E-7)		
	N = 36		R^2 =	0.682	F =	12.874	SSR =	0.825
(10)	9.397	0.023	-0.002	5.93E-5				
14		(0.055)	(0.003)	(5.3E-5)				
	N = 21		R^2 =	0.446	F =	4.566	SSR =	0.682
(11)	10.052	-0.207	0.021	-0.001	1.090E-5			
14		(0.118)	(0.011)	(4.08E-4)	(5.1E-6)			
	N = 21		R^2 =	0.57	F =	5.308	SSR =	0.529
(12)	9.539	0.041	-0.001	1.421E-5				
16		(0.027)	(0.001)	(2.2E-5)				
	N = 18		R^2 =	0.304	F =	2.041	SSR =	0.376
(13)	9.588	0.025	-0.001					
16		(0.011)	(2.6E-4)					
	N = 18		R^2 =	0.284	F =	2.975	SSR =	0.387

For each regression performed to the males data, we present the 4-th order polynomial in the experience proxy and (when a different degree offered better estimates) the polynomial form that gave the best fit (adjusted R^2). The 4-th order gave the best fit for the sample as a whole, and only for the education group with 6 years of schooling (secondary school). For people with

- no schooling, 6-th degree gave the best fit
- primary schooling, 5-th degree gave the best fit
- high-school (9-years), 5-th degree gave the best fit
- complementary high-school (11 years), 5-th degree gave the best fit
- B.A. and "licenciatura" (14 and 16 years), 2nd degree gave the best fit.

Table V.3.1. *Gompertz - Men*

	Int	E	E^2	Exp	k_0	r_x (%)
Years of Education:						
0	8.441	2.531	-4.789	0.008	3.095	0.91
		(0.921)	(0.826)	(0.005)		
	N = 52	R^2 = 0.927		F = 202.664	SSR =	0.217
4	10.317	-1.725	-1.093	-0.019	1.479	10.83
		(0.741)	(0.544)	(0.005)		
	N = 52	R^2 = 0.958		F = 364.29	SSR =	0.21
6	10.072	-0.386	-2.136	-0.014	2.067	5.93
		(0.678)	(0.45)	(0.005)		
	N = 52	R^2 = 0.97		F = 520.191	SSR =	0.214
9	10.052	-0.536	-1.189	-0.01	1.542	6.74
		(0.653)	(0.404)	(0.005)		
	N = 100	R^2 = 0.919		F = 364.03	SSR =	0.738
11	10.649	-1.849	-0.118	-0.016	0.486	24.02
		(1.664)	(1.005)	(0.014)		
	N = 48	R^2 = 0.798		F = 57.898	SSR =	0.913
14	12.534	-5.536	2.292	-0.051		
		(2.007)	(1.103)	(0.019)		
	N = 44	R^2 = 0.663		F = 26.265	SSR =	0.921
16	12.087	-2.909	0.048	-0.042		
		(2.156)	(1.181)	(0.02)		
	N = 44	R^2 = 0.731		F = 36.182	SSR =	1.024

For women, the equivalent regressions suggested the use of a 3rd-degree polynomial for the sample as a whole, 4 years and 9 years of schooling. For the others:

- no schooling, 5th degree
- 6 years, 3rd degree
- 11 years, 5th degree
- B.A. (14 years), 4th degree
- "licenciatura" (16 years), 2nd degree.

The Gompertz regressions were first performed by education group in order to see whether some implied pattern for the (general) O.J.T. rate of return could somehow be deduced.

The implied estimates for the male sample show a mixed pattern, with some trade-off with the initial endowment. For high schooling levels no real solution was found for k_0 and r_x. A 4th degree polynomial in the exponential term gave the best fit for the population as a whole, no schooling workers and 6 years of schooling.

Table V.3.2.1. *Gompertz - Men*

Years of Ed.	Inter.	E	E^2	E^3	E^4	E^5	E^6	Ed	Ex	Ed*Ex
All	11.62	-9.572	16.738	-20.04	8.814			0.109	-0.038	-0.001
		(3.63)	(7.921)	(8.86)	(3.72)			(0.003)	(0.01)	(1.E-4)
	N =	392	R^2 =	0.922		F =	648.76		SSR =	5.895
0	9.468	1.851	-23.92	62.097	-53.28				-0.009	
		(4.82)	(14.9)	(23.5)	(14.0)				(0.012)	
	N =	52	R^2 =	0.982		F =	510.32		SSR =	0.053
4	13.574	-18.46	48.322	-72.00	38.185				-0.063	
		(6.559)	(16.5)	(21.4)	(10.5)				(0.021)	
	N =	52	R^2 =	0.971		F =	307.12		SSR =	0.145
4	33.941	-159.3	764.76	-2339	4067.5	-3681	1339.4		-0.313	
		(20.88)	(94.2)	(270)	(442)	(379)	(131)		(0.04)	
	N =	52	R^2 =	0.993		F =	955.99		SSR =	0.033
6	14.876	-19.999	40.153	-46.02	18.728				-0.084	
		(6.49)	(14.8)	(17.4)	(7.671)				(0.022)	
	N =	52	R^2 =	0.976		F =	372.51		SSR =	0.173

Table V.3.2.2. *Gompertz – Men*

Years of Ed.	Inter.	E	E^2	E^3	E^4	E^5	E^6	Ex	Ed	Ed*Ex
9	10.378	-0.687	-4.506	7.399	-4.412			-0.016		
		(7.13)	(15.02)	(16.4)	(6.799)			(0.03)		
	N =	100	R^2 =	0.923		F =	224.80		SSR =	0.705
9	11.454	-5.055	5.024	-3.201				-0.032		
		(2.34)	(3.12)	(1.60)				(0.01)		
	N =	100	R^2 =	0.922		F =	282.61		SSR =	0.708
11	16.206	-23.75	46.132	-50.31	20.616			-0.099		
		(19.75)	(40.3)	(43.2)	(17.59)			(0.08)		
	N =	48	R^2 =	0.804		F =	34.519		SSR =	0.884
11	41.665	-143.3	395.0	-650.3	543.52	-178.26		-0.456		
		(52.75)	(149)	(251)	(216)	(73.5)		(0.17)		
	N =	48	R^2 =	0.829		F =	33.086		SSR =	0.773
14	17.51	-23.99	38.618	-37.33	14.539			-0.127		
		(26.6)	(49.0)	(47.9)	(18.0)			(0.12)		
	N =	44	R^2 =	0.669		F =	15.395		SSR =	0.904
14	44.663	-140.6	343.26	-513.8	396.53	-120.7		-0.524		
		(77.07)	(195)	(300)	(238)	(75.09)		(0.27)		
	N =	44	R2 =	0.691		F =	13.796		SSR =	0.845
16	6.881	18.763	-48.98	56.008	-23.63			0.033		
		(28.58)	(52.1)	(50.6)	(18.97)			(0.13)		
	N =	44	R2 =	0.755		F =	23.431		SSR =	0.932

16	16.379	-15.211	14.751	-6.793				-0.115		
		(8.566)	(9.985)	(4.58)				(0.05)		
	N =	44	R^2 =	0.745		F =	28.499		SSR =	0.97

For women (Tables V.4.1 and V.4.2), a 2nd-degree term usually performed better. The results imply a pattern of the rate of return to O.J.T. and initial investment which indicates (as expected) that the pattern may not be of the Gompertz type.

A 1st-degree polynomial seems to be sufficient for the sample as a whole. For women with no schooling and a B.A., a 4-th degree polynomial is best, and a 5-th term is includable for women with primary schooling. For these categories, the experience term is positive. This suggests the use of a form with the experience terms, rather than with the Gompertz specification.

The implied pattern for all women suggests (Table V.4.2) an initial investment of 74,7% of initial earnings potential and a rate of return to O.J.T of 12,9% - which compares with 100% (for initial investment) and 7,3% (rate of return) implied by specification (4) of tables III.1 and III.2, where we got a much smaller rate - the interaction of experience with education yields, thus, different estimates.

Table V.4.1. *Gompertz – Women*

Years of Education	Int	E	E^2	Exp	k_0	r_x (%)
0	8.084	2.234	-3.162	0.009	2.515	0.56
		(1.332)	(1.343)	(0.007)		
	N = 49	R^2 = 0.412		F = 10.503	SSR =	0.242
4	7.669	3.195	-3.539	0.022	2.660	-1.01
		(1.019)	(0.748)	(0.007)		
	N = 52	R^2 = 0.871		F = 107.766	SSR =	0.397
6	8.404	2.563	-3.457	0.014	2.629	0.13
		(1.287)	(0.869)	(0.01)		
	N = 47	R^2 = 0.891		F = 117.678	SSR =	0.479
9	9.842	-0.697	-0.546	-0.009	1.045	8.33
		(1.396)	(0.881)	(0.011)		
	N = 85	R^2 = 0.572		F = 36.141	SSR =	1.879
11	12.357	-5.167	1.635	-0.058		
		(2.665)	(1.514)	(0.025)		
	N = 36	R^2 = 0.592		F = 15.449	SSR =	1.06

14	4.59	8.008	-3.178	0.115	2.521	-10.88
		(7.346)	(4.061)	(0.074)		
	N = 21	R^2 = 0.401		F = 3.798	SSR =	0.737
16	10.742	-1.209	0.018	-0.021		
		(3.194)	(1.723)	(0.031)		
	N = 18	R^2 = 0.299		F = 1.987	SSR =	0.379

Table V.4.2. *Gompertz - Women*

Years Educ.	Inter	E	E^2	E^3	E^4	E^5	Ex	Ed	Ed*Ex
All	9.122	-1.182	-0.279				-0.006	0.101	-0.001
		(0.664)	(0.432)				(0.005)	(0.005)	(1.8E-4)
	N = 308		R^2 =	0.827	F =	288.126		SSR =	8.475
All	9.294	-1.601					-0.009	0.101	-0.001
		(0.149)					(0.002)	(0.005)	(1.8E-4)
	N = 308		R^2 =	0.826	F =	360.75		SSR =	8.487
0	3.766	30.219	-117.44	232.626	-173.34		0.062		
		(12.98)	(44.38)	(79.12)	(53.61)		(0.031)		
	N = 48		R^2 =	0.591	F =	12.406		SSR =	0.168
4	-6.456	81.258	-283.62	584.55	-612.93	249.89	0.207		
		(26.22)	(91.76)	(188.5)	(195.9)	(79.55)	(0.065)		
	N = 52		R^2 =	0.895	F =	63.757		SSR =	0.323
14	-92.586	330.39	-563.7	532.56	-199.36		1.7		
		(114.2)	(206.8)	(206.3)	(81.45)		(0.547)		
	N = 21		R^2 =	0.631	F =	5.133		SSR =	0.454

Finally, we point out the fact that the education coefficient of the best regression for men and women now show a smaller rate of return to schooling for females than for males at each schooling level.

Some final remarks

1. We interpreted the results in terms of standard human capital theory, associating

- schooling with investment through education.
- experience with general O.J.T.

Earnings functions may have other interpretation than that advanced in section I. [24]. The positive effect of schooling in earnings can be associated with signaling effects. A positive relation between tenure (and, thus, because we have no data on tenure, experience which is positively correlated with it) and earnings can be explained by implicit contract theories without human capital theory. Therefore, our results may have something of both sources. Also, hedonic wage functions may have been found, in which case, we have a mixture of supply and demand considerations in the formation of an empirically observed relation between the variables.

Also, as observed before, some segmented labor market interpretations maybe applicable for the earnings and returns differentials observed [25]. We preferred to offer a quantitative account of the latter.

2. The results may suffer from ability bias (usually considered to bias estimates of rates of return upwards), self-selection (usually causing a downward bias), or other problems [26], some explained along the exposition and in the Appendix. Nevertheless, the results seem to show a good fit to the data - and most biases cannot be corrected without more information regarding other variables.

Summary and conclusions

We performed some estimates - made for other countries - of log-earnings regressions specifications with the available data for the Portuguese labor force. We can summarize the main conclusions as follows:

1. The estimates of the rates of return to schooling for the male sample indicate a convex pattern - that is, an increasing rate of return with the schooling level. The average rate of return to schooling is (was in 1977) about 7,3%.

2. Male experience profiles indicate much higher rates of return for O.J.T. -13-15%. (Notice, however that general and specific O.J.T may be causing these high values, once we have no information on tenure and a positive correlation must exist between tenure and experience.)

3. Results for women indicated an average rate of 8% and a decreasing profile of schooling rates of return. Rates of return to O.J.T. are much lower than for men - which may be partly attributed to the bad proxy for female experience available.

4. Wage differentials by sex indicate a substantial difference between men and women, the data suggesting the negative

difference relative to men comes from experience price and not from the schooling reward. (Again, these results are clouded by the bad proxy for experience of women. Caution must therefore be taken when interpreting these issues as symptoms of discrimination.)

5. When we consider estimates by age groups, we get a declining pattern of returns to schooling with groups age; this occurs even when age/experience within the group is controlled for.

6. The extinct technical school system does not show significant differences in the pattern of rewards relative to the high-school system.

7. A quartic representation of the earnings-experience profile seemed to be adequate for the male sample. A cubic representation was achieved for women.

Notes

[1] Although the primary interpretations adopted in the paper are in consonance with standard neo-classical theory, some complementary references of segmented labor market literature are also presented in the treatment of this subject.

[2] This seems a reasonable assumption (specially in private terms), once education is heavily subsidized. Also, compared to opportunity costs - that is, of foregone income due to the fact that the individual is using time to study instead of working -, those costs are small. Notice that these costs are sometimes (at least partly) supported by parents, and not by the individual himself; so in private terms the assumption is even more accurate (assuming a setting where schooling of children is viewed as utility-yielding-consumption by parents and not in an intergenerational transfer context). Part-time income of students is also not accounted for, which would also off-set part of the money costs of schooling.

[3] Using the fact that we are summing the terms of an infinite geometric series.

[4] See Mincer (1974).

[5] See, for example, Weiss (1986). As is well known, earnings regressions maybe interpreted in an hedonic framework - see Rosen (1974) and Willis (1986).

[6] The estimates from earnings regressions are usually lower than those obtained by direct methods - see Willis (1986).

[7] See Mincer, op. cit.

[8] See Mincer, op. cit.

[9] See Soares, Pedro & Magalhães (1984).

[10] In the comparisons we will present, we sometimes refer to results for the U.S. in which log of weeks worked during the year by the individual were included in the regression. We have no corresponding information - and use mean data on individuals, assuming the mean week would be the same for every class considered.

[11] Notice that we cannot exclude the possibility of 7,3% being still a very high rate. Kula (1985) presents an estimate of 7,2% for the Portuguese rate of time preference (However, the interest rate maybe higher than this rate - see MaCurdy (1981)).

[12] See Asplund, Barth, Le Grand, Mastekaasa & Westergård-Nielsen (1991), were rates of return for Denmark, Sweden, Norway and Finland are reported. Only for Finland do the estimates approximate our result of 7%, with 4 and 5% for the other countries.

[13] See Mincer, op. cit.

[14] See Mincer, op. cit.

[15] See the Appendix: this could come from the fact that we are not using after-tax earnings, the tax system being convex.

[16] Experience-earnings profiles as implied by Human Capital theory could also be explained in an implicit contract theory context. The increasing earnings over the life-cycle would be associated with the need to keep workers in their jobs, delaying payments (but rewarding them at the prevailing interest rate). Specific human capital investments, associated with tenure on the job - related to experience in the labor market and of which both us and Mincer had no information about - would thus be preserved. Then, the flatter profile in Portugal could be due to implicit contracts implying a smaller necessity for postponing payments to keep employees, due to the smaller (job) mobility of the Portuguese labor force.

[17] See Freeman (1986) for a discussion of the problem and a survey of relevant literature.

[18] See, for example Asplund et al, op. cit.

[19] See Cain (1986). See also and Tzannatos (1990) for a survey of the literature.

[20] Recall that the data we have - see the Appendix - refers to mean earnings for each category. To know the correct mean difference, we should weight by the percentage of people in each cell, but such information is not available.

[21] See Taubman & Wachter (1986) and McNabb & Ryan (1990) for recent surveys. Also, see Hartog & Vriend (1990), and Magnac (1991) for an attempt to distinguish the two hypothesis (neo-classical and segmented labor market). This same comment applies to all the differential approaches.

[22] Murphy & Welch (1990).

[23] We could have also experimented using earnings - and not log of earnings - as de dependent variable. Foreign evidence suggests that the log-earnings specifications are the ones that have provided the best predicting power of earnings profiles. See Freeman (1986) for further references.

[24] See Willis, op. cit.

[25] See footnote 21.

[26] See Willis, op. cit.

Appendix

1. The data used was published in Soares, São Pedro & Magalhães (1984). There, we find tables - one for men and another one for women - of mean monthly earnings of individuals by age and schooling categories for 1977. The data is based on the annual survey "Quadros de Pessoal" conducted by the Labor Ministry.

2. We constructed experience - as in Mincer (1974) - by subtracting, from the age, the schooling years and 6 (initial schooling age). That is for individuals of class i:

$$\text{Experience}_i = \text{Age}_i - 6 - \text{Years of Education}_i \quad \text{(A.1)}$$

3. We considered the following equivalence in terms of schooling years relative to the category of schooling for which mean earnings level by age was reported:

No Schooling	0 years
"Primário"	4 years
"Preparatório"	6 years
"Secundário Liceal"	9 years
"Secundário Técnico"	9 years
"Secundário Complementar"	11 years
"Universitário (3 anos)"	14 years
"Universitário (5 years)"	16 years

In the text, we used the term B.A. for the 14 years degree - even if the correspondence is not completely accurate. The higher degree (16 years) is referred to as "licenciatura".

Notice that nowadays an extra schooling year has been created - the "12º Ano" - between High-School and University. However, such was not the case at (till) the time - 1977.

We abstracted from the fact that some people may have taken more than those years to complete that schooling level - therefore experience may be over-estimated by the index (A.1), even if people always worked since leaving school. If the over-approximation is homogeneously (additively) distributed among the population, this will not affect the estimates.

Also, some people did not complete a schooling category and have more years of schooling than the adjacent lower category. This implies that we overestimate schooling and, implicitly, underestimate experience. Again, this will only affect the estimates of the rates of return in some special cases.

4. The information on earnings reported does not correspond to net private earnings - recall §.4 of section I -, but, to our knowledge, they are the only available (published) information at the moment. Being the tax system progressive, this will yield a tendency towards overestimation of the rates of return to schooling. This could also be related to the finding of a positive correlation between the rate of return to schooling and the schooling level for men, the overestimation being, thus, increasing with the schooling level.

5. The data refers to monthly earnings - the earnings that we should use would be annual. We assume that annual earnings are a fixed multiple of monthly earnings, that is:

$$E_{annual} = E_{monthly} \times 14 \qquad (A.2)$$

Therefore, in log-earnings terms:

$$\ln E_{annual} = \ln E_{monthly} + \text{constant}. \qquad (A.3)$$

This implies that, apart from the intercept, the interpretation of the coefficients in the regressions will not be altered.

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